COMPLETE SCHOOL ALGEBRA

HAWKES-LUBY-TOUTON

TEACHERS' EDITION

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PREFACE

The busy teacher of high-school algebra often finds that limitations of time forbid the actual working out of all the exercises and problems in a text. This teachers' edition is designed to meet such a condition. In its plan it is brief and schematic rather than complete and detailed.

Many exercises are so simple that the answers only are given in this teachers' edition. The solutions of numerous other exercises are complete in outline only. For this reason they should not be regarded as models of the form in which the pupil should present his work.

Circumstances may make imperative, and at times justify, the use of such a book as this; but continued reliance upon it, however, with class after class will result in teaching which is faulty and lacking both in freedom and life.

The authors and the publishers desire the coöperation of teachers in keeping this book out of the hands of pupils.

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Page 1

1. $4h + 3m = 4(3600) + 3(60) = 14,580$ seconds.

2. $5y + 4f = 5(36) + 4(12) = 228$ inches.

3. $4q + 6d = 4(25) + 6(10) = 160$ cents.

4. $4t + 6h = 4(2000) + 6(100) = 8600$ pounds.

5. $3x + 5x = 8x$.

6. $4x + 5x = 9x$.

7. $2x + 3x + 6x = 11x$.

8. $2x + 2 + 3x + 4 = 5x + 6$.

9. $x + x + 2 + x + x + 2 = 4x + 4$.

10. $n + n + 1 + n + 2 = 3n + 3$.

11. $5a + 18 - 3a - 7 = 2a + 11$.

12. $8x - 3 + 18 - 5x = 3x + 15$.

13. $4w - 8 + 3w + 20 = 7w + 12$.

14. $y + 5m = 17m$.

15. $2y + 7f = 13f$.

16. $4q + 3n = 23n$.

17. $2d + 15h = 63h$.

18. $15h + 50m = 950m$.

19. Area, $5 \cdot 5 \cdot 1$ square inch $= 25$ square inches; perimeter, $4 \cdot 5$ inches $= 20$ inches.

20. Perimeter, $4s$ inches; area, $s \cdot s$ square inches.

21. Area, $12 \cdot 4 \cdot 1$ square foot $= 48$ square feet; perimeter, $2 \cdot 12$ feet $+ 2 \cdot 4$ feet $= 32$ feet.

22. Area, ab square feet; perimeter, $2a$ feet $+ 2b$ feet $= (2a + 2b)$ feet.

23. (a) $2x$ inches; (b) $2 \cdot x + 2 \cdot 2x = 6x$ inches; (c) $2x \cdot x$ square inches.

24. $3 \cdot y$ years.

25. $(30 + y)$ years.

26. Length, $(12 + w)$ feet; perimeter, $w + w + 12 + w + 12 + w = (24 + 4w)$ feet.

27. Length, $(18 + w)$ feet; perimeter, $w + w + 18 + w + 18 + w = (36 + 4w)$ feet.

28. Let w feet = the width.

Then $2w + 4$ feet = the length,

and the perimeter $= w + w + 2w + 4 + 2w + 4$,

$= (6w + 8)$ feet.

Page 5

1. Greater number + less number = 160.

If we represent the less by l, then $4l$ must represent the greater, and the above statement becomes

$$4l + l = 160.$$

Whence $$l = 32,$$

and $$4l = 128.$$

Therefore the greater number is 128 and the less is 32.

2. The number + five times the number = 216.

If we represent the number by n, the above statement becomes

$$n + 5n = 216,$$

or $$6n = 216.$$

Whence $$n = 36.$$

3. Greater number + less number = 72.

If we represent the less by l, then $7l$ must represent the greater, and the above statement becomes

$$7l + l = 72.$$

Whence $$l = 9,$$

and $$7l = 63.$$

4. 1st number + 2d number + 3d number = 105.

If we let t represent the 3d number, $2t$ must represent the 1st number and $4t$ the 2d. Then the above statement becomes

$$2t + 4t + t = 105.$$

Whence $$t = 15,$$

$$2t = 30,$$

and $$4t = 60.$$

5. 1st number + 2d number + 3d number = 117.

If we represent the 1st number by n, $2n$ must represent the 2d number and $6n$ the 3d. Then the above statement becomes

$$n + 2n + 6n = 117.$$

Whence $$n = 13,$$

$$2n = 26,$$

and $$6n = 78.$$

6. 1st number + 2d number + 3d number = 192.

If we represent the 2d number by n, $2n$ must represent the 1st number and $3n$ the 3d. Then the above statement becomes

$$2n + n + 3n = 192.$$

Whence $$n = 32,$$

$$2n = 64,$$

and $$3n = 96.$$

7. 1st number + 2d number + 3d number = 324.

If we represent the 1st number by n, $5n$ must represent the 2d number and 30 the 3d. Then the above statement becomes

$$n + 5n + 30n = 324.$$

Whence $$n = 9,$$

$$5n = 45,$$

and $$30n = 270.$$

8. 1st number + 2d number + 3d number = 104.

If we represent the 1st number by n, $3n$ must represent the 2d number and $4n$ the 3d. Then the above statement becomes

$$n + 3n + 4n = 104.$$

Whence $$n = 13,$$

$$3n = 39,$$

and $$4n = 52.$$

10. From arithmetic, principal + interest = amount.

Principal + .06 principal = \$265.

If p represents the principal, this last statement becomes

$$p + .06p = 265.$$

Whence $$p = 250.$$

Therefore the sum is \$250.

11. Principal + .12 principal = \$700.

If p represents the principal, the above statement becomes

$$p + .12p = 700.$$

Whence $$p = 625.$$

Therefore the required sum is \$625.

12. $225 \times .06 \cdot$ (number of years) = 27.

If we represent the number of years by n, this statement becomes

$$225 \times .06 \cdot n = 27,$$

or $$13.5n = 27.$$

Whence $$n = 2.$$

13. $520 \times .065 \cdot$ (number of years) = 169.

If n represents the number of years, this statement becomes

$$520 \times .065 \cdot n = 169,$$

or $$33.8n = 169.$$

Whence $$n = 5.$$

15. Let x = the rate of interest.

Then \$825x = the interest for one year,

and \$3300x = the interest for four years.

But $\$165 =$ the interest for four years.
Whence $3300x = 165,$
and $x = \frac{1}{20}.$
Therefore the money is lent at 5%.

16. Let $x =$ the rate of interest.
Then $\$250x =$ the interest for one year,
and $\$1500x =$ the interest for six years.
But $\$317.50 - \$250$, or $\$67.50 =$ the interest for six years.
Whence $1500x = 67.50,$
and $x = .045.$
Therefore the money is lent at $4\frac{1}{2}\%$.

17. Let $x =$ the number of years.
Then $\$8x =$ the interest at 4%.
But $\$200 =$ the interest at 4%.
Whence $8x = 200,$
and $x = 25.$
Therefore the time required is 25 years.

18. $\$150 \times .05 \cdot$ (number of years) $= \$450 - \$150.$
If we represent the number of years by n, this statement becomes

$$3.50n = 300.$$

Whence $n = 40.$
Therefore the required time is 40 years.

19. Let $s =$ a side of the square in feet.
Then $4s =$ the perimeter, or 160 feet.
Whence $4s = 160,$
and $s = 40.$
Therefore the square is 40 feet on a side.

20. Let $s =$ the number of feet in the width of the rectangle.
Then $3s =$ the number of feet in the length of the rectangle,
and $2s + 2 \cdot 3s$, or $8s =$ the number of feet in the perimeter of the rectangle, or 256 feet.
Whence $8s = 256.$
Therefore $s = 32,$
and $3s = 96.$

21. Let $s =$ the side of either square in inches.
Then $2s =$ the length of the rectangle in inches,

and $\quad 2 \cdot s + 2 \cdot 2s$, or $6s =$ the perimeter of the rectangle, or 198 inches.

Therefore $\quad 6s = 198.$

Whence $\quad s = 33,$

and $\quad 4s = 132,$ the perimeter.

22. Let $\quad x =$ the side of each square in inches.

Then $\quad 4x =$ the perimeter of each square, or 120 inches.

Therefore $\quad 4x = 120.$

Whence $\quad x = 30,$

and $\quad 6x = 180,$ the perimeter.

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1. $3a + 5b.$
2. $5b - 3a.$
3. $b^2 - a^2.$
4. $3a^2 - 2a^2.$
5. $\frac{a}{b}.$
6. $4a^2b.$
7. $\frac{a+b}{a-b}$, or $\frac{a+b}{b-a}.$
8. $a(2b - c).$
9. $a(b + c).$
10. $7x - (a - b).$
11. $(a + b)(a - b)$, or $(a+b)(b-a).$
12. $\sqrt{5a} + \sqrt[3]{7b}.$
13. $\sqrt{7x}(x - y).$
14. $(a + b)^2.$
15. $(a-b)^2$, or $(b-a)^2.$
16. $\frac{3ab^2}{4ca^3}.$
17. $\frac{a}{3x} + \frac{4y}{c}.$

Page 10

1. $20 - 5 + 6 - 10 = 15 + 6 - 10 = 21 - 10 = 11.$
2. $16 - (8 - 2) = 16 - 6 = 10.$
3. $14 - (16 - 8) + (12 - 4) = 14 - 8 + 8 = 14.$
4. $6 \div 3 - 2 = 2 - 2 = 0.$
5. $8 \cdot 6 \div 3 - 10 = 48 \div 3 - 10 = 16 - 10 = 6.$
6. $18 \div (2 \cdot 3) = 18 \div 6 = 3.$
7. $(6 - 3)(17 - 2 \cdot 5) = 3(17 - 10) = 21.$
8. $23 - 2 \cdot 6 - 4 \div 2 + 16 = 23 - 12 - 2 + 16 = 11 + 14 = 25.$
9. $18 \div (9 - 3) = 18 \div 6 = 3.$
10. $(10 - 3)(16 - 3 \cdot 2 + 8 \div 4) = 7(16 - 6 + 2) = 7 \cdot 12 = 84.$
11. $14 - 3 \cdot (16 - 2 \cdot 5) \div 6 + 8 \cdot 2 = 14 - (16 - 10) \div 2 + 16 = 30 - 3 = 27.$
12. $(18-2) - 6 \div (4 + 2 \cdot 8 - 18 \div 9) = 16 - 6 \div (4 + 16 - 2) = 16 - 6 \div 18 = 15\frac{2}{3}.$
13. $(16 - 6)(18 - 8) \div 100 \cdot 5 - 5 = 10 \cdot 10 \div 100 \cdot 5 - 5 = 5 - 5 = 0.$
14. $(5 + 3)(5 - 3) \div 5 - 3 = 8 \cdot 2 \div 5 - 3 = 3\frac{1}{5} - 3 = \frac{1}{5}.$

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If $a = 3$, $b = 1$, $c = 5$, $d = 7$, and $e = 2$:

1. $4a + 3d = 4 \cdot 3 + 3 \cdot 7 = 33.$
2. $a^2 + 2c = 3^2 + 2 \cdot 5 = 19.$
3. $ab + cd = 3 \cdot 1 + 5 \cdot 7 = 38.$
4. $c^2 - 5ab = 25 - 5 \cdot 3 \cdot 1 = 10.$

5. $abcd - 4e^2 = 3 \cdot 1 \cdot 5 \cdot 7 - 4 \cdot 4 = 89.$

6. $\dfrac{d+c}{2} = \dfrac{7+5}{2} = 6.$

7. $\dfrac{cd\cancel{e}}{5\cancel{e}} + \dfrac{\cancel{d}ce}{2\cancel{d}} = \dfrac{5 \cdot 7}{5} + \dfrac{5 \cdot 2}{2} = 12.$

8. $\dfrac{10b}{e} - \dfrac{c}{b} = \dfrac{10 \cdot 1}{2} - \dfrac{5}{1} = 0.$

9. $\dfrac{1}{c} + \dfrac{1}{d} + \dfrac{1}{e} = \dfrac{1}{5} + \dfrac{1}{7} + \dfrac{1}{2} = \dfrac{14}{70} + \dfrac{10}{70} + \dfrac{35}{70} = \dfrac{59}{70}.$

10. $\dfrac{e^2}{ac} + \dfrac{4cd}{ae} = \dfrac{4}{3 \cdot 5} + \dfrac{4 \cdot 5 \cdot 7}{3 \cdot 2} = \dfrac{4}{15} + \dfrac{70}{3} = \dfrac{354}{15} = 23\frac{3}{5}.$

11. $\dfrac{cd + ae + ce}{a - b + c} = \dfrac{35 + 6 + 10}{3 - 1 + 5} = \dfrac{51}{7} = 7\frac{2}{7}.$

12. $\dfrac{a^2 + b^2 + c^2 + d^2}{a + b + c + d} = \dfrac{9 + 1 + 25 + 49}{3 + 1 + 5 + 7} = \dfrac{84}{16} = 5\frac{1}{4}.$

13. $\dfrac{a^3 - ce^2 + 3cd}{d - a + c} = \dfrac{27 - 5 \cdot 4 + 3 \cdot 35}{7 - 3 + 5} = \dfrac{112}{9} = 12\frac{4}{9}.$

14. $5b^3 + 4b^2 - 2b - 5 = 5 \cdot 1 + 4 \cdot 1 - 2 \cdot 1 - 5 = 9 - 7 = 2.$

15. $3b^5 - 14b^4 + 11b^3 + 11b^2 + 13b - 20 = 3 - 14 + 11 + 11 + 13 - 20 = 4.$

16. $a^e = 3^2 = 9.$

17. $c^a = 5^3 = 125.$

18. $d^e + e^a = 7^2 + 2^3 = 57.$

19. $b^c + c^a = 1^5 + 5^3 = 126.$

20. $d^e - c^2 + b^3 = 7^2 - 25 + 1 = 25.$

21. $e^2 \cdot c^a = 4 \cdot 5^3 = 500.$

22. $\dfrac{d + e^c}{2c + a} = \dfrac{7 + 2^5}{2 \cdot 5 + 3} = \dfrac{7 + 32}{13} = 3.$

23. $\dfrac{a^a + 3b}{c^e - d - e - b} = \dfrac{3^3 + 3 \cdot 1}{5^2 - 7 - 2 - 1} = \dfrac{30}{25 - 10} = 2.$

If $a = 4$, $b = 0$, $c = 5$, $d = 7$, and $e = 8$:

24. $\dfrac{4a + 3b + 2d}{c + e + 2} = \dfrac{4 \cdot 4 + 3 \cdot 0 + 2 \cdot 7}{5 + 8 + 2} = \dfrac{16 + 14}{15} = 2.$

25. $\dfrac{b}{a + c + d} = \dfrac{0}{4 + 5 + 7} = 0.$

26. $abc + acd - be = 4 \cdot 0 \cdot 5 + 4 \cdot 5 \cdot 7 - 0 \cdot 8 = 140.$

27. $\dfrac{ab}{c} + \dfrac{bd}{a} + \dfrac{be}{cd} = \dfrac{0}{5} + \dfrac{0}{4} + \dfrac{0}{35} = 0.$

28. $\dfrac{a^2 - b^2 + c^3}{3d - 2e + c} = \dfrac{16 - 0 + 125}{21 - 16 + 5} = 14\frac{1}{10}.$

29. $\sqrt{a} + \sqrt{2e} = \sqrt{4} + \sqrt{16} = 6.$

30. $2\sqrt{a} + \sqrt[3]{e} = 2\sqrt{4} + \sqrt[3]{8} = 6.$

31. $b\sqrt{a} + cd + \sqrt[3]{e} \cdot \sqrt{a} = 0 + 35 + 2 \cdot 2 = 39.$

32. $c\sqrt[2]{2\,ae} + d\sqrt[3]{2\,ae} = 5\sqrt[2]{2\cdot 4\cdot 8} + 7\sqrt[3]{2\cdot 4\cdot 8} = 40 + 28 = 68.$

33. $\sqrt{b^2 + c^2 + d + a} = \sqrt{0 + 25 + 7 + 4} = 6.$

34. $\sqrt[3]{3\,d + ec + a - 1} = \sqrt[3]{21 + 40 + 4 - 1} = 4.$

35. $de + ac\sqrt{ac^2 + 2\,e + c} = 56 + 20\sqrt{100 + 16 + 5} = 56 + 220 = 276.$

36. $(a + c)c = (4 + 5)5 = 45.$

37. $(a + e)(c + d) = (4 + 8)(5 + 7) = 144.$

38. $ab(a + b) = 4\cdot 0(4 + 0) = 0.$

39. $acd(a + c + d) = 4\cdot 5\cdot 7(4 + 5 + 7) = 140\cdot 16 = 2240.$

40. $ac(e - c + b + d) = 4\cdot 5(8 - 5 + 0 + 7) = 20\cdot 10 = 200.$

41. $(a + d)^2 = (4 + 7)^2 = 121.$

42. $(e - a)^2 = (8 - 4)^2 = 16.$

43. $(d - a)^3 = (7 - 4)^3 = 27.$

44. $\dfrac{3\,ad(3\,a - 2\,c)^2(e - c)^3}{6\,ed} = \dfrac{\cancel{3}\cdot\cancel{4}\cdot\cancel{7}(12 - 10)^2(8 - 5)^3}{\cancel{6}\cdot\underset{4}{\cancel{8}}\cdot\cancel{7}} = \dfrac{4\cdot 27}{4} = 27.$

45. $\dfrac{ab(c + d)(a + b)}{(e - a)} = \dfrac{4\cdot 0(5 + 7)(4 + 0)}{(8 - 4)} = 0.$

46. $c^3 - 3\,c^2a + 3\,ca^2 - a^3 = 125 - 3\cdot 25\cdot 4 + 3\cdot 5\cdot 16 - 64 = 1.$

47. If $x = 2$ and $y = 3$, $3\,x + 5\,y = 21$ becomes $3\cdot 2 + 5\cdot 3 = 21$. Yes.

48. If $x = 8$, $7\,x - 9 = 3\,x + 25$ becomes $7\cdot 8 - 9 = 3\cdot 8 + 25$, or $47 = 49$, which is false. No.

49. If $x = 2$, $x^2 + 5\,x + 6 = 0$ becomes $4 + 10 + 6 = 0$, which is false. No.
If $x = 3$, it becomes $9 + 15 + 6 = 0$, which is false. No.
If $x = 4$, it becomes $16 + 20 + 6 = 0$, which is false. No.

50. If $x = 5$, $3\,x^2 - 14\,x - 5 = 0$ becomes $3\cdot 25 - 14\cdot 5 - 5 = 0$. Yes.
If $x = \frac{1}{3}$, it becomes $3\cdot\frac{1}{9} - \frac{14}{3} - 5 = 0$, which is false. No.
If $x = 6$, it becomes $3\cdot 36 - 14\cdot 6 - 5 = 0$, which is false. No.
If $x = 4$, it becomes $3\cdot 16 - 14\cdot 4 - 5 = 0$, which is false. No.

Page 14

1. $4 + 2 = 6.$
2. $-2 + 4 = 2.$
3. $-3 + 5 = 2.$
4. $-3 + 3 = 0.$
5. $-4 + 2 = -2.$
6. $-7 + 5 = -2.$
7. $5 - 2 = 3.$
8. $2 - 5 = -3.$
9. $4 - 6 = -2.$
10. $-2 - 4 = -6.$
11. $-4 - 3 = -7.$
12. $-3 - 2 = -5.$

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1. (*a*) $10° - 5° = 5°$; (*b*) $10° - 10° = 0°$; (*c*) $10° - 18° = -8°$.

2. (*a*) $-12° + 7° = -5°$;
(*b*) $-12° + 12° = 0°$;
(*c*) $-12° + 25° = +13°$.

3. (*a*) $-12° - 7° = -19°$;
(*b*) $-12° - 12° = -24°$;
(*c*) $-12° - 25° = -37°$.

4. $13° + 7° = 20°$ or 69 miles $\cdot$ 20 = 1380 miles.

5. $20° - 4° = 16°$, at end of first day.
$20° - 2 \cdot 4° = 12°$, at end of second day.
$20° - 3 \cdot 4° = 8°$, at end of third day.
$20° - 4 \cdot 4° = 4°$, at end of fourth day.
$20° - 5 \cdot 4° = 0°$, at end of fifth day.
$20° - 6 \cdot 4° = -4°$, at end of sixth day.
$\frac{20 + 16}{4} = 9$ days.

6. (*a*) Property $= + \$5200$.
Debts $= - \$2300$.
(*b*) Standing $= \$5200 - \$2300 = + \$2900$.

7. (*a*) Property $= + \$2300$.
Debts $= - \$5200$.
(*b*) Standing $= \$2300 - \$5200 = - \$2900$.

8. $-12° + 3° \cdot 3 = -3°$ at 9.00 A.M.
$-12° + 3° \cdot 4 = 0°$ at 10.00 A.M.
$-12° + 3° \cdot 7 = +9°$ at 1.00 P.M.

Page 16

1. $+3 + (+2) = +5$.
2. $-3 + (-2) = -5$.
3. $+3 + (-2) = +1$.
4. $-3 + (+2) = -1$.
5. $7 + (-4) = +3$.
6. $-7 + (+5) = -2$.
7. $-6 + (-4) = -10$.
8. $6 + (-5) = +1$.
9. $4 + (-7) = -3$.
10. $-3 + (-6) = -9$.

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1. $+4 + (+7) = +11$.
2. $-4 + (-7) = -11$.
3. $+4 + (-7) = -3$.
4. $-4 + (+7) = +3$.
5. $+8 + (-5) = +3$.
6. $-8 + (+5) = -3$.
7. $-12 + (-9) = -21$.
8. $-6 + 6 = 0$.
9. $-6 + (-6) = -12$.
10. $-4 + (+3) + (+6) = +5$.
11. $3 + (-7) + (5) + (-4) = -3$.
12. $8 + (-2) + (-4) + (+6) = +8$.
13. $6 + (+3) = 9$.
14. $6 + (-4) = 2$.
15. $8 + (+4) = 12$.
16. $8 + (-4) = 4$.
17. $-8 + (-2) = -10$.
18. $-8 + (+2) = -6$.
19. $-10 + (-6) = -16$.
20. $-10 + (+17) = 7$.
21. $12 + (-8) = 4$.
22. $-12 + (+16) = 4$.

Page 18

Subtracting:

1. $8 - 5 = 3$.
2. $13 - 9 = 4$.
3. $5 - 8 = -3$.
4. $9 - 13 = -4$.
5. $-8 - (-5) = -3$.
6. $8 - (-5) = 13$.
7. $-10 - 5 = -15$.
8. $-4 - (-6) = 2$.
9. $-18 - 12 = -30$.
10. $+13 - 25 = -12$.

Changing signs and adding:

11. $8 + (-5) = 3$.
$13 + (-9) = 4$.
$5 + (-8) = -3$.
$9 + (-13) = -4$.
$-8 + (+5) = -3$.
$8 + (+5) = 13$.
$-10 + (-5) = -15$.
$-4 + (+6) = 2$.
$-18 + (-12) = -30$.
$13 + (-25) = -12$.

Yes. The answers are the same as those obtained before.

Page 19

1. $8 - (+5) = 8 + (-5) = 3$.
2. $+8 - (-5) = 8 + (+5) = 13$.
3. $+5 - (-8) = 5 + (+8) = 13$.
4. $+5 - (+8) = 5 + (-8) = -3$.
5. $12 - (+9) = 12 + (-9) = 3$.
6. $12 - (-9) = 12 + (+9) = 21$.
7. $-12 - (9) = -12 + (-9) = -21$.
8. $-12 - (-9) = -12 + (9) = -3$.
9. $+6 - (+6) = 6 + (-6) = 0$.
10. $-6 - (-6) = -6 + (+6) = 0$.
11. $6 - (-6) = 6 + (6) = 12$.
12. $-6 - (+6) = -6 + (-6) = -12$.
13. $-14 - (-19) = -14 + (19) = 5$.
14. $0 - (+1) = 0 + (-1) = -1$.
15. $-0 - (-1) = +(+1) = 1$.
16. $1 - (-2) = 1 + (+2) = 3$.
17. $12 - (+3) - (+2) = 12 + (-3) + (-2) = 7$.
18. $-12 - (+3) - (+2) = -12 + (-3) + (-2) = -17$.
19. $-12 - (-3) - (-2) = -12 + (3) + (2) = -7$.
20. $18 - (-5) - (7) = 18 + (5) + (-7) = 16$.
21. $+6 + \mathbf{4} = 10$.
22. $-6 + (-\mathbf{4}) = -10$.
23. $-3 + \mathbf{3} = 0$.
24. $+6 + (-\mathbf{6}) = 0$.
25. $+6 + (-\mathbf{2}) = 4$.
26. $-8 + \mathbf{5} = -3$.
27. $-8 + \mathbf{3} = -5$.
28. $-7 + \mathbf{14} = 7$.
29. $9 - \mathbf{6} = 3$.
30. $-7 - (-\mathbf{2}) = -5$.
31. $+5 - \mathbf{11} = -6$.
32. $-7 - (-\mathbf{11}) = 4$.
33. $-5 - (-\mathbf{5}) = 0$.
34. $2 - \mathbf{2} = 0$.
35. $4 - (-\mathbf{14}) = 18$.
36. $12 + (3) - (5) = 12 + (3) + (-5) = 10$.
37. $12 - (-3) + 6 = 12 + (3) + 6 = 21$.
38. $12 - (-4) + (-6) = 12 + (4) + (-6) = 10$.
39. $18 + (-6) - (+7) = 12 + (-7) = 5$.
40. $-16 + (-10) - (+11) = -26 + (-11) = -37$.
41. $-13 - (8) + (-14) = -27 + (-8) = -35$.

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1. $+3 \cdot +4 = +12$.
2. $+4 \cdot +12 = +48$.
3. $-5 \cdot +4 = -20$.
4. $+6 \cdot -6 = -36$.
5. $-7 \cdot +8 = -56$.
6. $-7 \cdot -3 = +21$.
7. $-12 \cdot +9 = -108$.
8. $+6 \cdot -4 = -24$.
9. $-6 \cdot -6 = +36$.
10. $+5 \cdot -10 = -50$.
11. $+0 \cdot +4 = 0$.
12. $-7 \cdot 0 = 0$.
13. $+4 \cdot -5 \cdot +6 = -120$.
14. $+4 \cdot -5 \cdot -6 = +120$.
15. $-4 \cdot -5 \cdot -6 = -120$.
16. $12 \cdot +0 \cdot -5 = 0$.
17. $9 \cdot -10 \cdot -0 = 0$.
18. $-4 \cdot +3 \cdot -6 = +72$.
19. $-3 \cdot -2 \cdot -5 = -30$.
20. $2 \cdot -3 \cdot +5 = -30$.

Page 23 (First set)

1. $+10 \div +2 = +5$.
2. $-10 \div -2 = +5$.
3. $-15 \div +3 = -5$.
4. $+14 \div -7 = -2$.
5. $-18 \div -2 = +9$.
6. $-7 \div +7 = -1$.
7. $0 \div +5 = 0$.
8. $0 \div -5 = 0$.
9. $+18 \div +3 \div -2 = -3$.
10. $+45 \div -5 \div -3 = 3$.
11. $-64 \div +8 \div -2 = +4$.
12. $+96 \div -6 \div +4 = -4$.
13. $72 \div +9 \div -4 = -2$.
14. $60 \div -5 \div -12 = 1$.
15. $48 \div +3 \div -4 = -4$.
16. $\dfrac{-10}{-5} = 2$.
17. $\dfrac{-12}{6} = -2$.
18. $\dfrac{+16}{+2} = 8$.

Page 23 (Second set)

1. $(7) + (5) = 12$.
2. $(7) - (5) = 2$.
3. $(7) + (-5) = 2$.
4. $(7) - (-5) = 12$.
5. $(-7) + (5) = -2$.
6. $-7 + 5 = -2$.
7. $(-9) - (4) = -13$.
8. $-9 - 4 = -13$.
9. $-11 + (-13) = -24$.
10. $-6 - (-10) = 4$.
11. $-6 + 10 = 4$.
12. $8 + (-10) = -2$.
13. $12 - 18 = -6$.
14. $-18 - 12 = -30$.
15. $15 - 14 = 1$.
16. $+7 - 0 = 7$.
17. $-0 - 3 = -3$.
18. $(-3)(6) = -18$.
19. $(-5)6 = -30$.
20. $(7)(-5) = -35$.
21. $8(-3) = -24$.
22. $(-5)(-12) = 60$.
23. $-3(-8) = 24$.
24. $-5 \cdot 4 = -20$.
25. $5 \cdot 0 = 0$.
26. $0 \cdot (-9) = 0$.
27. $4 \cdot 8 = 32$.
28. $-3 \cdot 6 = -18$.
29. $12 \div (-2) = -6$.
30. $-12 \div 2 = -6$.

31. $-39 \div (-3) = 13.$

32. $45 \div (-15) = -3.$

33. $0 \div (-6) = 0.$

34. $0 \div 3 = 0.$

35. $-27 \div 9 = -3.$

36. $3 - 5 + 6 = 4.$

37. $-4 + 6 - 2 + 1 = 1.$

38. $-4 + 6 + 2 - 1 = 3.$

39. $2 - 3 + 4 - 5 - 6 = -8.$

40. $\begin{array}{r} 7 \\ -2 \\ 3 \\ \underline{-5} \\ 3 \end{array}$ 41. $\begin{array}{r} 6 \\ -2 \\ -3 \\ \underline{4} \\ 5 \end{array}$ 42. $\begin{array}{r} -8 \\ 6 \\ 2 \\ \underline{-5} \\ -5 \end{array}$ 43. $\begin{array}{r} 4 \\ -9 \\ -3 \\ \underline{6} \\ -2 \end{array}$

44. $3 \cdot 6 \div 3 = 6.$

45. $-4(7) \div (-2) = 14.$

46. $3(-6) \div 2 = -9.$

47. $4 \cdot 6(-8) \div (-16) = 12.$

48. $18 \div (-3) \cdot 6 \div 4 = -9.$

49. $3^2 = 9.$

50. $(-3)^2 = 9.$

51. $2^3 = 8.$

52. $(-2)^3 = -8.$

53. $(-4)^3 + 4^2 = -64 + 16 = -48.$

54. $(-1)^2 + (-1)^3 + (-2)^2 + (-2)^3 = 1 - 1 + 4 - 8 = -4.$

55. $6 + 3 \cdot 2 + 18 \div (-3) = 6 + 6 - 6 = 6.$

56. $5^2 - 4 \div (-2) + 6(-3) = 25 + 2 - 18 = 9.$

57. $3 \cdot 6 \div 9 - 2 \cdot 6 \div 4 + (-3)^2 = 2 - 3 + 9 = 8.$

58. $6 + 6 \cdot 3^2 - 5^2 \cdot 2 \div 10 = 6 + 54 - 5 = 55.$

If $x = 3$ and $y = -2$:

59. $y^2 = 4.$

60. $y^3 = -8.$

61. $y^4 = 16.$

62. $y^5 = -32.$

63. $2y^2 = 8.$

64. $2y^3 = -16.$

65. $5x^2y^2 = 5 \cdot 9 \cdot 4 = 180.$

66. $4x^2y^4 = 4 \cdot 9 \cdot 16 = 576.$

67. $x^2 - y^2 = 9 - 4 = 5.$

68. $x^3 - y^3 = 27 - (-8) = 35.$

69. $x^2 + 2xy + y^2 = 9 + 2 \cdot 3(-2) + 4 = 1.$

70. $x^2 - 2xy + y^2 = 9 - 6(-2) + 4 = 25.$

71. $(x + y)(x - y) = 1 \cdot 5 = 5.$

72. $x^3 + 3x^2y + 3xy^2 + y^3 = 27 + 3 \cdot 9(-2) + 3 \cdot 3 \cdot 4 - 8 = 1.$

73. $y^3 - 3xy^2 + 3x^2y - x^3 = -8 - 3 \cdot 3 \cdot 4 + 3 \cdot 9(-2) - 27 = -125.$

74. If $x = 5$, $4x - 2 = 2x + 8$ becomes $4 \cdot 5 - 2 = 2 \cdot 5 + 8$, or $18 = 18.$ Yes.

75. If $x = -9$, $3x - 5 = 2x + 8$ becomes $-9 \cdot 3 - 5 = -9 \cdot 2 + 8$, but $-32 \neq -10.$ No.

76. If $x = 4$, $x^2 - x - 12 = 0$ becomes $16 - 4 - 12 = 0$, or $0 = 0.$ Yes.
If $x = -8$, it becomes $64 + 8 - 12 = 0$, but $60 \neq 0.$ No.
If $x = -4$, it becomes $16 + 4 - 12 = 0$, but $8 \neq 0.$ No.

77. If $x = \frac{2}{3}$, $3x^2 + 19x = 14$ becomes $3 \cdot \frac{4}{9} + \frac{38}{3} = 14$, or $14 = 14.$ Yes.
If $x = 2$, it becomes $3 \cdot 4 + 2 \cdot 19 = 14$, but $50 \neq 14.$ No.
If $x = -7$, it becomes $3 \cdot 49 - 7 \cdot 19 = 14$, or $14 = 14.$ Yes.

78. $15° - 3° \cdot 2 = +9°$, at 9.00 A.M.
$15° - 3° \cdot 5 = 0°$, at noon.
$15° - 3° \cdot 8 = -9°$, at 3.00 P.M.

79. Average height $= \dfrac{30 + (-7) + 18 + (-10) + 16}{5}$, or $9\frac{2}{5}$ feet.

The average height was $9\frac{2}{5}$ feet above the assumed level.

80. Mean height $= \dfrac{9\frac{2}{5} \cdot 5 + (-38)}{6}$, or $1\frac{1}{2}$ feet above the level.

81. Let $x =$ the height of the seventh point.

Then $\dfrac{1\frac{1}{2} \cdot 6 + x}{7} = 0$, or $x = -9$.

The last point was -9 feet high.

82. Euclid lived about -300; Sir Isaac Newton died in $+1727$.

83. The date -450 means 450 B.C.; the date $+1910$ means 1910 A.D. The difference, $1910 - (-450)$, is 2360 years.

84. Rate upstream, $(8 - 1\frac{1}{2})$ miles per hour; rate downstream, $(8 + 1\frac{1}{2})$ miles per hour.

85. Rate toward bow, $(12 + 3)$ miles per hour; rate toward stern, $(12 - 3)$ miles per hour.

86. Balloon, -500 pounds; men, 1500 pounds; both, $[1500 + (-500)]$, or 1000 pounds.

87. (*a*) $(2 + 1\frac{1}{2})$ miles per hour; (*b*) $(2 - 1\frac{1}{2})$, or $-\frac{1}{2}$ mile per hour; i.e. he is being carried downstream $\frac{1}{2}$ mile per hour.

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1. $18 - 5 + 6 = 19$.
2. $18a - 5a + 6a = 19a$.
3. $12 - 7 + 3 + 4 = 12$.
4. $12a^2 - 7a^2 + 3a^2 + 4a^2 = 12a^2$.
5. $6 - 4 - 17 + 20 = 5$.
6. $6ab - 4ab - 17ab + 20ab = 5ab$.
7. $15 - 17 + 8 - 12 - 25 = -31$.
8. $8abc - 17abc - 4abc + 15abc - 12abc = -10abc$.
9. $11 + 5 - 9 - 3 + 16 - 25 = -5$.
10. $16ac - 9ac + 5ac - 2ac - 3ac + 11ac = 18ac$.
11. $7x + 4x - 15x - 8x + 3x = -9x$.
12. $12y - 17y + 10y + 20y - 25y = 0$.
13. $4xy - 8xy - 12xy + 13xy - xy = -4xy$.
14. $14x^2 - 13x^2 + x^2 - 5x^2 + 4x^2 = x^2$.
15. $5y^2 - 2y^2 - 11y^2 + y^2 - 7y^2 = -14y^2$.
16. $7a^2b - 5a^2b + 8a^2b - a^2b + 9a^2b = 18a^2b$.

Page 29

1. The sum of a, $3\,b$, and $-c = a + 3\,b - c$.
2. The sum of $4\,x$, $-2\,b$, $3\,y$, and $10 = 4\,x - 2\,b + 3\,y + 10$.
3. The sum of $3\,ab^2$, $2\,bx$, $-cy$, and $4\,a^2b = 3\,ab^2 + 2\,bx - cy + 4\,a^2b$.
4. The sum of $5\,x^3y$, $-5\,xy^3$, c^3y, and $-2\,cy^3$
$$= 5\,x^3y - 5\,xy^3 + c^3y - 2\,cy^3.$$
5. The sum of $4\,x$, $-3\,a$, $2\,b$, $-5\,x$, and $3\,y = -x - 3\,a + 2\,b + 3\,y$.
6. The sum of $5\,a$, $-4\,b$, $+3\,c$, $6\,b$, and $-2\,c^2 = 5\,a + 2\,b + 3\,c - 2\,c^2$.
7. The sum of $3\,a^2$, $+2\,b^2$, $-5\,c^2$, $-4\,b^2$, and $5\,a^2 = 8\,a^2 - 2\,b^2 - 5\,c^2$.
8. The sum of $4\,a^3b$, $-4\,ab^3$, $-3\,a^2b$, $3\,ab^2$, $4\,a^3b$, and $2\,ab^3$
$$= 8\,a^3b - 2\,ab^3 - 3\,a^2b + 3\,ab^2.$$
9. $-4\,a^2b + 6\,a^2b^2 - 15\,a^2b^2 + 3\,a^2b^2 + 0\,a^2b^2 = -4\,a^2b - 6\,a^2b^2$.
10. $-b^3 - 23\,b^3 + 17\,b^3 + b^3 - 0\,b^3 + 13\,b^3 = 7\,b^3$.
11. $12\,a^3b + 6\,a^3b - a^3b + 16\,a^3b - 13\,a^3b - 25\,a^3b = -5\,a^3b$.
12. $11\sqrt{a} - 14\sqrt{a} + 21\sqrt{a} - \sqrt{a} = 17\sqrt{a}$.
13. $3\sqrt{x-y} - \sqrt{x-y} + 9\sqrt{x-y} - 7\sqrt{x-y} = 4\sqrt{x-y}$.
14. $3\,(a+b) - 2\,(a+b) + 8\,(a+b) = 9\,(a+b)$.
15. $-7\,(a-2\,b) + (a-2\,b) + 12\,(a-2\,b) = 6\,(a-2\,b)$.
16. $8\,(a+b) + 3\,(a-b) - 4\,(a+b) - 2\,(a-b) = 4\,(a+b) + (a-b)$.
17. $4\,(x-y) - 3\,(x+3) - 6\,(x-y) + 5\,(x-3)$
$$= -2\,(x-y) - 3\,(x+3) + 5\,(x-3).$$
18. $(2\,x-y)^2 - 3\,(2\,x-y)^2 + 4\,(2\,x-y)^2 - 7\,(2\,x-y)^2 = -5\,(2\,x-y)^2$.

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1.
$$\begin{array}{r} x + y + z \\ x - 2\,y + 3\,z \\ 3\,x + 4\,y - 7\,z \\ \hline 5\,x + 3\,y - 3\,z \end{array}$$

2.
$$\begin{array}{r} 2\,x + 5\,y - z \\ 3\,x - 8\,y + 6\,z \\ x - y - z \\ \hline 6\,x - 4\,y + 4\,z \end{array}$$

3.
$$\begin{array}{r} 3\,x + 5\,y \\ 4\,x - 7\,y + 6\,z \\ 3\,x - 3\,y - 3\,z \\ \hline 10\,x - 5\,y + 3\,z \end{array}$$

4.
$$\begin{array}{r} 7\,x - y + 3\,z \\ 5\,x - 4\,z \\ 2\,x + 6\,y - 5\,z \\ \hline 14\,x + 5\,y - 6\,z \end{array}$$

5.
$$\begin{array}{r} 4\,x - 3\,y - 5\,z \\ 6\,y - 2\,z \\ 7\,x - 6\,y - 4\,z \\ \hline 11\,x - 3\,y - 11\,z \end{array}$$

6.
$$\begin{array}{r} x + 3\,y + 2\,z \\ x + y - 3\,z \\ -2\,x - 4\,y + z \\ \hline 0 \quad + 0 \quad + 0 \end{array}$$

7.
$$\begin{array}{r} 5\,x - y + 3\,z \\ x + 2\,y - 11\,z \\ -7\,y + 9\,z \\ \hline 6\,x - 6\,y + z \end{array}$$

8.
$$\begin{array}{r} 8\,a - 7\,b - 6\,c \\ -4\,a - 3\,b + 5\,c \\ 3\,b + 7\,c \\ \hline 4\,a - 7\,b + 6\,c \end{array}$$

9.
$$\begin{array}{r} 9\,ac - bc \\ -4\,ac + 8\,ab \\ -ac - 12\,ab \\ \hline 4\,ac - bc - 4\,ab \end{array}$$

10.
$$\begin{array}{r} a^2 - 4\,a + 10 \\ -6\,a^2 + 5\,a + 4 \\ 2\,a^2 + 3\,a - 16 \\ \hline -3\,a^2 + 4\,a - 2 \end{array}$$

11.
$$\begin{array}{r} -a^2 + a + 9 \\ 6\,a^2 - 4\,a - 7 \\ 5\,a^2 - a \\ \hline 10\,a^2 - 4\,a + 2 \end{array}$$

12.
$$\begin{array}{r} \frac{1}{2}x - \frac{2}{3}y + \frac{2}{5}z \\ x - y + 2\,z \\ \frac{1}{8}x - \frac{1}{2}y + \frac{1}{10}z \\ \hline 1\frac{5}{6}\,x - 2\frac{1}{6}\,y + 2\frac{1}{2}\,z \end{array}$$

13.
$$\begin{array}{r}
\tfrac{1}{4}x \qquad\qquad - \tfrac{3}{20}z \\
\tfrac{1}{8}x + \tfrac{4}{9}y \qquad\qquad \\
- \tfrac{5}{3}y + \tfrac{3}{5}z \\
\hline
\tfrac{3}{8}x - 1\tfrac{2}{9}y + \tfrac{9}{20}z
\end{array}$$

14.
$$\begin{array}{r}
a - 3(x-y) + z \qquad \\
-10a + 4(x-y) \qquad + 5 \\
-2(x-y) \qquad + 6 \\
\hline
-9a - (x-y) + z + 11
\end{array}$$

15.
$$\begin{array}{r}
a + 2(b-c) + d \\
-12a + 7(b-c) + 6d \\
11a - 5(b-c) \qquad \\
\hline
4(b-c) + 7d
\end{array}$$

16.
$$\begin{array}{r}
7a^2 - 13b^2 + 12c^2 \\
-a^2 + 15b^2 - 7c^2 \\
3b^2 + 5c^2 \\
\hline
6a^2 + 5b^2 + 10c^2
\end{array}$$

17.
$$\begin{array}{r}
x^2 - 2xy + y^2 \qquad \\
-4x^2 - 4xy - y^2 \qquad \\
-16x^2 - 8xy + y^2 + y^3 \\
\hline
-19x^2 - 14xy + y^2 + y^3
\end{array}$$

18.
$$\begin{array}{r}
-a^2 + 5ab + b^2 \\
5a^2 - 4ab - 9b^2 \\
-2ab + 2b^2 \\
\hline
4a^2 - ab - 6b^2
\end{array}$$

19.
$$\begin{array}{r}
3x^2 - 6x + 11 \\
-4x - 8 \\
-3x^2 + 5x - 16 \\
+13x \qquad \\
\hline
8x - 13
\end{array}$$

20.
$$\begin{array}{r}
8b^2 - 10bc + 12c^2 \qquad \\
-6b^2 - bc - c^2 \qquad \\
-11c^2 + c^3 \\
\hline
2b^2 - 11bc \qquad\qquad + c^3
\end{array}$$

21.
$$\begin{array}{r}
4x^2y - xy + y^2 \qquad\qquad \\
-2x^2y - 3xy \qquad\qquad + 4xy^2 \\
x^2y - 3xy + 4y^2 \qquad\qquad \\
\hline
3x^2y - 7xy + 5y^2 + 4xy^2
\end{array}$$

22.
$$\begin{array}{r}
\tfrac{1}{2}a + \tfrac{1}{3}b - \tfrac{1}{4}c \qquad \\
-\tfrac{2}{3}a + \tfrac{1}{2}b - c \qquad \\
-b + \tfrac{1}{2}c \qquad \\
-\tfrac{3}{4}a \qquad\qquad\qquad + 7 \\
\hline
-\tfrac{11}{12}a - \tfrac{1}{6}b - \tfrac{3}{4}c + 7
\end{array}$$

23. $ax + bx + cx = (a + b + c)x.$

24. $2ax - 3x + bx = (2a - 3 + b)x.$

25. $3ax - 4cx + x = (3a - 4c + 1)x.$

26. $3ax^2 - bx^2 - x^2 + a^2x^2 = (3a - b - 1 + a^2)x^2.$

27. $by - 4cy - y - 4by = -4cy - y - 3by = (-4c - 1 - 3b)y.$

28. $a(b + c) + 3(b + c) = (a + 3)(b + c).$

29. $4(a - x) - 3b(a - x) = (4 - 3b)(a - x).$

30. $8a(a + 3b) - 1(a + 3b) = (8a - 1)(a + 3b).$

31. $7b(x^2 + y^2) - a(x^2 + y^2) + (x^2 + y^2) = (7b - a + 1)(x^2 + y^2).$

Page 35 (First set)

1.	$x + 5 = 11.$	
Subtracting 5 from each member,	$x = 6.$	Ax. II
2.	$x - 4 = 12.$	
Adding 4 to each member,	$x = 16.$	Ax. I
3.	$3x + 10 = 28.$	
Subtracting 10 from each member,	$3x = 18.$	Ax. II
Dividing each member by 3,	$x = 6.$	Ax. IV

4. $5x - 6 = 19.$
Adding 6 to each member,
$$5x = 25.$$
Whence $x = 5.$

5. $9x - 12 = 6.$
Adding 12 to each member,
$$9x = 18.$$
Whence $x = 2.$

6. $4x = 12 + x.$
Whence $x = 4.$

7. $6x = 20 + 2x.$
Whence $x = 5.$

8. $9n = 40 - n.$
Whence $n = 4.$

9. $13n = -5n + 36.$
Whence $n = 2.$

10. $-4n = -13n + 27.$
Whence $n = 3.$

11. $3y + 2 = y + 8.$
Whence $y = 3.$

12. $5 + 4y = 3y + 20.$
Whence $y = 15.$

13. $2y - 3 = 17 - y.$
Whence $y = 6\frac{2}{3}.$

14. $8x - 15 = 6x - 15.$
Adding 15 to each member,
$$8x = 6x.$$
Whence $x = 0.$

15. $-7h + 19 = 25 - 9h.$
Whence $h = 3.$

16. $-7x + 18 = 4x + 18.$
Whence $x = 0.$

17. $5k - 4 = 3k + 18.$
Whence $k = 11.$

18. $8 - 6x + 12 + 10x = 26.$
Whence $x = 1\frac{1}{2}.$

19. $x + 2x + 18 + x + 2x + 18 = 116.$
Whence $x = 13\frac{1}{3}.$

20. $4x - 3 + 8x - 17 = 40 + 6x - 54.$
Collecting, $12x - 20 = 6x - 14.$
Whence $x = 1.$

Page 35 (Second set)

1. Length, $3x$; perimeter, $2 \cdot x + 2 \cdot 3x$, or $8x$.
2. Length, $x + 10$ feet; perimeter, $2 \cdot x + 2(x + 10)$, or $4x + 20$ feet.
3. Width, $x - 18$ feet; perimeter, $2 \cdot x + 2(x - 18)$, or $4x - 36$ feet.
4. Width, x feet; length, $2x + 4$ feet; perimeter, $2x + 2(2x + 4)$, or $6x + 8$ feet.
5. Greater by 1.
6. $n + 1$.
7. $n + 1$ and $n + 2$; sum is $2n + 3$.
8. $n + 1$, $n + 2$, $n + 3$; the sum of the four $= 4n + 6$.
9. Greater by 2.
10. $n + 2$; $n - 2$.
11. n, $n + 2$, $n + 4$; their sum $= 3n + 6$.
12. n, $n + 2$, $n + 4$, $n + 6$; their sum $= 4n + 12$.

13. First, n; second, $2n$; third, $6n$.
14. First, n; second, $n + 10$; third, $n + 10 - 7$, or $n + 3$.
15. $x - 4$; $x + 6$.
16. (*a*) B, x years; A, $2x$ years. (*b*) B, $x - 7$ years; A, $2x - 7$ years. (*c*) B, $x + 12$ years; A, $2x + 12$ years.

17. $8 + x = 5$.
18. $x = 10 - 2$.
19. $x = 3 + 5$.
20. $x = 5 + y$.
21. $3x = 21$.
22. $4a = 18 + 2$.
23. $8 + x = 34 - x$.
24. $2x - 9 = 14 + x$.
25. $2x + 3x = 48 + x$.
26. $3x - 12 = 50 + x$.

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1. Greater number + less number = 135.

If we represent the less number by n, $2n$ must represent the greater, and the above statement becomes

$$2n + n = 135.$$

Whence $$n = 45,$$

and $$2n = 90.$$

2. Greater number + less number = 105.

If we represent the less number by n, $4n$ must represent the greater, and the above statement becomes

$$4n + n = 105.$$

Whence $$n = 21,$$

and $$4n = 84.$$

3. Greater number − less number = 52.

If we let n represent the less number, $5n$ must represent the greater, and the above statement becomes

$$5n - n = 52.$$

Whence $$n = 13,$$

and $$5n = 65.$$

4. Greater number + less number = 129.

If we let n represent the less number, $n + 5$ must represent the greater, and the above statement becomes

$$n + 5 + n = 129.$$

Whence $$n = 62,$$

and $$n + 5 = 67.$$

5. Greater number + less number = 168.

If we represent the less number by n, $n + 18$ must represent the greater, and the above statement becomes

$$n + 18 + n = 168.$$

Whence $n = 75,$

and $n + 18 = 93.$

6. Twice the width + twice the length = 156 feet.

If we let w represent the width in feet, $5w$ must represent the length, and the above statement becomes

$$2 \cdot w + 2 \cdot 5w = 156.$$

Whence $w = 13,$

and $5w = 65.$

7. 2 · length + 2 · width = 96 feet.

If we represent the width in feet by w, $12 + w$ must represent the length, and the above statement becomes

$$24 + 2w + 2w = 96.$$

Whence $w = 18,$

and $12 + w = 30.$

8. 2 · length + 2 · width = 98 feet.

If we represent the width in feet by w, $2w + 4$ must represent the length, and the above statement becomes

$$4w + 8 + 2w = 98.$$

Whence $w = 15,$

and $2w + 4 = 34.$

9. 2 · length + 2 · width = 88 feet.

If we let w represent the width in feet, $3w - 20$ must represent the length, and the above statement becomes

$$6w - 40 + 2w = 88.$$

Whence $w = 16,$

and $3w - 20 = 28.$

10. 1st number + 2d number + 3d number = 45.

If we represent the first number by n, $n + 1$ and $n + 2$ must represent the next two numbers greater, and the above statement becomes

$$(n) + (n + 1) + (n + 2) = 45.$$

Whence $n = 14,$

$n + 1 = 15,$

and $n + 2 = 16.$

11. 1st number + 2d number + 3d number + 4th number = 106.

If we represent the first number by n, $n + 1$, $n + 2$, and $n + 3$ must represent the three consecutive numbers greater, and the above statement becomes

$$(n) + (n + 1) + (n + 2) + (n + 3) = 106.$$

Whence $n = 25,$

$$n + 1 = 26,$$
$$n + 2 = 27,$$
and $$n + 3 = 28.$$

12. The sum of five consecutive numbers $= 85$.

If we represent the first number by n, $n+1$, $n+2$, $n+3$, $n+4$ must represent the next four numbers greater, and the statement above becomes

$$(n)+(n+1)+(n+2)+(n+3)+(n+4)=85.$$

Whence $$n = 15,$$
$$n + 1 = 16,$$
$$n + 2 = 17,$$
$$n + 3 = 18,$$
and $$n + 4 = 19.$$

13. 1st odd number + next odd number + next odd number $= 291$.

If we represent the first number by n, the others must be represented by $n+2$ and $n+4$ respectively, and the above statement becomes

$$(n) + (n + 2) + (n + 4) = 291.$$

Whence $$n = 95,$$
$$n + 2 = 97,$$
and $$n + 4 = 99.$$

14. 1st even number + next even number + next even number $= 66$.

If we represent the first number by n, $n+2$ and $n+4$ must represent the other numbers, and the above statement becomes

$$(n) + (n + 2) + (n + 4) = 66.$$

Whence $$n = 20,$$
$$n + 2 = 22,$$
and $$n + 4 = 24.$$

15. The sum of five consecutive odd numbers $= 315$.

Representing the first number by n, we must then represent the others by $n+2$, $n+4$, $n+6$, $n+8$ respectively, and the above statement becomes

$$(n)+(n+2)+(n+4)+(n+6)+(n+8)=315.$$

Whence $$n = 59,$$
$$n + 2 = 61,$$
$$n + 4 = 63,$$
$$n + 6 = 65,$$
and $$n + 8 = 67.$$

16. A's age + B's age $= 43$ years.

If we represent B's age in years by x, $x - 2$ must represent A's age, and the above statement becomes

$$x - 2 + x = 43.$$

Whence $$x = 22\tfrac{1}{2},$$
and $$x - 2 = 20\tfrac{1}{2}.$$

17. A's age + 6 years + B's age + 6 years = 54 years.

If we represent A's age in years by x, $2x$ must represent B's age, and the above statement becomes

$$x + 6 + 2x + 6 = 54.$$

Whence $x = 14,$

and $2x = 28.$

18. Mileage of United States + mileage of Europe = 402,000 miles.

If we represent the mileage of Europe by m, $m + 52{,}000$ must represent that of United States, and the above statement becomes

$$m + 52{,}000 + m = 402{,}000.$$

Whence $m = 175{,}000,$

and $m + 52{,}000 = 227{,}000.$

19. Length of Nile + length of Amazon + length of Mississippi = 11,500 miles.

If we represent the length of the Amazon in miles by l, $l + 500$ must represent the length of the Nile, and $l + 500 + 300$, or $l + 800$, the length of the Mississippi. The above statement becomes

$$(l + 500) + (l) + (l + 800) = 11{,}500.$$

Whence $l = 3400,$

$l + 500 = 3900,$

and $l + 800 = 4200.$

20. Height of Mount McKinley + height of Pike's Peak = 34,607 feet.

If we represent the height of Pike's Peak in feet by h, $h + 6313$ must represent the height of Mount McKinley, and the statement above becomes

$$(h + 6313) + (h) = 34{,}607.$$

Whence $h = 14{,}147,$

and $h + 6313 = 20{,}460.$

The height of Mount Everest = 34,607 feet − 5605 feet = 29,002 feet.

21. Horse power of a freight engine + that of a passenger engine + that of a tractor = 11,200 H.P.

If we represent the H.P. of the freight engine by x, $x - 1800$ must represent the H.P. of the tractor and $x + 1000$ that of the passenger engine. The above statement becomes

$$(x) + (x + 1000) + (x - 1800) = 11{,}200.$$

Whence $x = 4000,$

$x + 1000 = 5000,$

and $x - 1800 = 2200.$

Page 40 (First set)

1. $3x - 2x = x.$
$2x - 3x = -x.$

2. $3x - 4x = -x.$
$4x - 3x = x.$

3. $-3x - (-2x) = -x.$
$-2x - (-3x) = x.$

4. $-3x - (-5x) = 2x.$
$-5x - (-3x) = -2x.$

5. $4x - (-x) = 5x.$
$-x - 4x = -5x.$

6. $-3x - (-x) = -2x.$
$-x - (-3x) = 2x.$

7. $5c - (-c) = 6c.$
$-c - 5c = -6c.$

8. $-5ac - (-ac) = -4ac.$
$-ac - (-5ac) = 4ac.$

9. $-11a^2c - 8a^2c = -19a^2c.$
$8a^2c - (-11a^2c) = 19a^2c.$

10. $-6x^2y^2 - 6x^2y^2 = -12x^2y^2.$
$6x^2y^2 - (-6x^2y^2) = 12x^2y^2.$

11. $0 - 3x = -3x.$
$3x - 0 = 3x.$

12. $0 - (-4ab) = 4ab.$
$-4ab - 0 = -4ab.$

13. $b - a.$
$a - b.$

14. $2x - c.$
$c - 2x.$

15. $-4y - x.$
$x - (-4y) = x + 4y.$

16. $2b - (-3a) = 2b + 3a.$
$-3a - (+2b) = -3a - 2b.$

17. $-5b - (-2a) = -5b + 2a.$
$-2a - (-5b) = -2a + 5b.$

18. $-2a - (-2a) = 0.$
$-2a - (-2a) = 0.$

Page 40 (Second set)

1. $\begin{array}{r} a+3 \\ a+2 \\ \hline 1 \end{array} \quad \begin{array}{r} a+2 \\ a+3 \\ \hline -1 \end{array}$

2. $\begin{array}{r} a-2 \\ a-4 \\ \hline 2 \end{array} \quad \begin{array}{r} a-4 \\ a-2 \\ \hline -2 \end{array}$

3. $\begin{array}{r} 3a+5 \\ 2a-3 \\ \hline a+8 \end{array} \quad \begin{array}{r} 2a-3 \\ 3a+5 \\ \hline -a-8 \end{array}$

4. $\begin{array}{r} 9-4a \\ 7+3a \\ \hline 2-7a \end{array} \quad \begin{array}{r} 7+3a \\ 9-4a \\ \hline -2+7a \end{array}$

5. $\begin{array}{r} 4a-3 \\ 4a \\ \hline -3 \end{array} \quad \begin{array}{r} 4a \\ 4a-3 \\ \hline 3 \end{array}$

6. $\begin{array}{r} 5c \\ -5c+2ab \\ \hline 10c-2ab \end{array} \quad \begin{array}{r} -5c+2ab \\ 5c \\ \hline -10c+2ab \end{array}$

7. $\begin{array}{r} 5xy \\ 3xy-a \\ \hline 2xy+a \end{array} \quad \begin{array}{r} 3xy-a \\ 5xy \\ \hline -2xy-a \end{array}$

8. $\begin{array}{r} x+3 \\ 0 \\ \hline x+3 \end{array} \quad \begin{array}{r} 0 \\ x+3 \\ \hline -x-3 \end{array}$

9. $\begin{array}{r} 0 \\ 2x-3 \\ \hline -2x+3 \end{array} \quad \begin{array}{r} 2x-3 \\ 0 \\ \hline 2x-3 \end{array}$

10. $\begin{array}{r} 2x-2y+z \\ x-2y+3z \\ \hline x \qquad -2z \end{array}$

11. $\begin{array}{r} 4x-5y+3z \\ 4x-8y-2z \\ \hline 3y+5z \end{array}$

12. $\begin{array}{r} 4a-b+2c \\ 3a-2b \\ \hline a+b+2c \end{array}$

13. $\begin{array}{r} 3a-5b \\ 5a-4b-3c \\ \hline -2a-b+3c \end{array}$

14. $\begin{array}{r} b-x+y \\ a \quad -x-y \\ \hline b-a \quad +2y \end{array}$

15. $\begin{array}{r} 4a-b+5c \\ 3a-2b-c+6 \\ \hline a+b+6c-6 \end{array}$

16. $$\begin{array}{r} a - 3b \\ 2a - 2b - 2c + 4 \\ \hline -a - b + 2c - 4 \end{array}$$

17. $$\begin{array}{r} c - d + e \\ -c \qquad + a + b \\ \hline 2c - d + e - a - b \end{array}$$

18. $$\begin{array}{r} 2x + 5y - 3z \\ x - 2y + z \\ \hline x + 7y - 4z \end{array}$$

19. $$\begin{array}{r} 5x - 2y - 4z \\ 7x - 9y + 3z \\ \hline -2x + 7y - 7z \end{array}$$

20. $$\begin{array}{r} 6a + 3b \\ 5a - 4b - 6c \\ \hline a + 7b + 6c \end{array}$$

21. $$\begin{array}{r} 5 \qquad\qquad \\ +a + b - 2c \\ \hline 5 - a - b + 2c \end{array}$$

22. $$\begin{array}{r} 3xy + z \qquad\qquad \\ + 3ab + c \\ \hline 3xy + z - 3ab - c \end{array}$$

23. $$\begin{array}{r} 0 \qquad\qquad \\ 2x - 4y - z \\ \hline -2x + 4y + z \end{array}$$

24. $$\begin{array}{r} xy^2 + x^2y + z \\ -3xy^2 + 2x^2y \quad \\ \hline 4xy^2 - x^2y + z \end{array}$$

25. $$\begin{array}{r} x - 2y + z \\ 3x + 2y - z \\ \hline -2x - 4y + 2z \end{array}$$

26. $$\begin{array}{r} x^2 - 7x + 10 \\ 3x^2 + 14x - 8 \\ \hline -2x^2 - 21x + 18 \end{array}$$

27. $$\begin{array}{r} x - y + z \\ 5x + 3y - 8z \\ \hline -4x - 4y + 9z \end{array}$$

28. $$\begin{array}{r} -8x^2 \qquad + 4 \\ x^2 - 5x + 6 \\ \hline -9x^2 + 5x - 2 \end{array}$$

29. $$\begin{array}{l} 3a - 5b + c \\ 0 \\ \hline 3a - 5b + c \end{array}$$

30. $$\begin{array}{r} 4a - 6b + 8y \qquad\qquad \\ + x - 12 \\ \hline 4a - 6b + 8y - x + 12 \end{array}$$

31. $$\begin{array}{lrl} & a^2 - 2ab + b^2 & (1) \\ & a^2 - 12ab + \quad 20 & (2) \\ \cline{2-2} (1)+(2), & 2a^2 - 14ab + b^2 + 20 & (3) \end{array} \qquad \begin{array}{lrl} & a^2 - 13a + 30 & (4) \\ & 2a^2 \qquad + 20 - 14ab + b^2 & (3) \\ \cline{2-2} (4)-(3), & -a^2 - 13a + 10 + 14ab - b^2 & \end{array}$$

32. $$\begin{array}{lrl} & a - 3b + c & (1) \\ & 4a + 5b - 6c + 4 & (2) \\ \cline{2-2} (1)+(2), & 5a + 2b - 5c + 4 & (3) \end{array} \qquad \begin{array}{lrl} & a - b + c - x & (4) \\ & 5a + 2b - 5c \qquad + 4 & (3) \\ \cline{2-2} (4)-(3), & -4a - 3b + 6c - x - 4 & \end{array}$$

33. $$\begin{array}{lrl} & 5x + 3x^2y - 15xy^2 & (1) \\ & -6x + 7x^2y - 12xy^2 & (2) \\ \cline{2-2} (1)+(2), & -x + 10x^2y - 27xy^2 & (3) \\ & 11x - 5x^2y + 7xy^2 & (4) \\ \cline{2-2} (3)-(4), & -12x + 15x^2y - 34xy^2, & \text{or } 15x^2y - 12x - 34xy^2. \end{array}$$

34. $$\begin{array}{lrl} & 4abc^2 - 3ab^2c + 2a^2bc \qquad\qquad & (1) \\ & 6abc^2 - 5ab^2c \qquad\qquad - 4a^2b^2c & (2) \\ \cline{2-2} (1)+(2), & 10abc^2 - 8ab^2c + 2a^2bc - 4a^2b^2c & (3) \\ & 2abc^2 + 7ab^2c - 3a^2bc \qquad\qquad & (4) \\ \cline{2-2} (3)-(4), & 8abc^2 - 15ab^2c + 5a^2bc - 4a^2b^2c & \end{array}$$

35.

$$\begin{array}{rll}
& 3x-4xy-2z & (1) \\
& \underline{-3x+7xy-4z} & (2) \\
& 3xy-6z & (3) \\
& \underline{9x-2xy+4z-7a^2bc} & (6) \\
(3)-(6), & -9x+5xy-10z+7a^2bc &
\end{array}$$

$$\begin{array}{rll}
& 5z-2xy-a^2bc & (4) \\
& \underline{-z\quad -6a^2bc+9x} & (5) \\
(4)+(5), & 4z-2xy-7a^2bc+9x & (6)
\end{array}$$

36.

$$\begin{array}{rll}
& 4x-3y+6 & (1) \\
& \underline{3x+5y-10} & (2) \\
(1)+(2), & 7x+2y-4 &
\end{array}$$

37.

$$\begin{array}{rll}
& 7c+5d-e & (1) \\
& \underline{4c+5d-9e} & (2) \\
(1)-(2), & 3c\quad +8e &
\end{array}$$

38.

$$\begin{array}{rll}
& x^2+2x+5 & (1) \\
& \underline{2x^2+x-10} & (2) \\
(1)+(2), & 3x^2+3x-5 & (3) \\
& \underline{x^2-5x+3} & (4) \\
(3)-(4), & 2x^2+8x-8 &
\end{array}$$

39.

$$\begin{array}{rll}
& x+3y-2z & (1) \\
& \underline{4x-5y+3z} & (2) \\
(1)+(2), & 5x-2y+z & (3) \\
& \underline{3x-2y-6z} & (4) \\
(3)-(4), & 2x\quad +7z &
\end{array}$$

40.

$$\begin{array}{rll}
& 5x+3y-z & (1) \\
& \underline{+4y+7z} & (2) \\
(1)+(2), & 5x+7y+6z & (3) \\
& \underline{x-y+3z} & (4) \\
(3)-(4), & 4x+8y+3z &
\end{array}$$

41.

$$\begin{array}{rll}
& 4x-3y+7 & (1) \\
& \underline{2x-5y-4} & (2) \\
(1)-(2), & 2x+2y+11 & (3) \\
& \underline{4x\quad -8} & (4) \\
(3)+(4), & 6x+2y+3 &
\end{array}$$

42.

$$\begin{array}{rll}
& 3c-5d-e & (1) \\
& \underline{+6d+11e+5} & (2) \\
(1)-(2), & 3c-11d-12e-5 & (3) \\
& \underline{5c\quad +4e} & (4) \\
(3)-(4), & -2c-11d-16e-5 &
\end{array}$$

43.

$$\begin{array}{rll}
& 3a+3b-4c & (1) \\
& \underline{-3b-3c-4} & (2) \\
(1)-(2), & 3a+6b-c+4 & (3) \\
& \underline{4a\quad -8c\quad +x} & (4) \\
(3)-(4), & -a+6b+7c+4-x &
\end{array}$$

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1. $8x-2=6x+6.$
Transposing, $8x-6x=2+6.$
Whence $x=4.$

2. $4x-5=2x+10.$
Transposing,
$4x-2x=5+10.$
Whence $x=7\frac{1}{2}.$

3. $6y-5=9y+2.$
Transposing, $6y-9y=5+2.$
Whence $y=-2\frac{1}{3}.$

4. $7y+3=10+8y.$
Transposing,
$7y-8y=10-3.$
Whence $y=-7.$

5. $5n-3+21=18+4n.$
Combining,
$5n+18=18+4n.$
Transposing,
$5n-4n=18-18.$
Whence $n=0.$

6. $6 + 4n - 15 = 15 - n.$
Transposing,
$$4n + n = 15 + 9.$$
Whence $n = 4\frac{4}{5}.$

7. $5n + 3 - 2n = 7 - 4.$
Combining, $3n + 3 = 3.$
Whence $n = 0.$

8. $3k + 9 + 5k + 31 = 0.$
Combining, $8k + 40 = 0.$
Whence $k = -5.$

9. $6k + 3 - 2k = 27.$
Whence $k = 6.$

10. $3x - 6 = 34 + 8x.$
Transposing,
$$-5x = 40.$$
Whence $x = -8.$

11. $2x - 14 - 5x + 4 = 0.$
Whence $x = -3\frac{1}{3}.$

12. $x + 12 - 11x = -15x + 22.$
Transposing,
$$15x - 10x = 22 - 12.$$
Whence $x = 2.$

13. $5y + 3 = 17 + 3y + 8.$
Transposing,
$$5y - 3y = 25 - 3.$$
Whence $y = 11.$

14. $3y + 5 + 8y + \frac{1}{2} = 0.$
Whence $y = -\frac{1}{2}.$

15. $2 - 4h = 3 - 8h + 8.$
Transposing,
$$8h - 4h = 11 - 2.$$
Whence $h = 2\frac{1}{4}.$

16. $3 - 5h + 2 = 7h + 5.$
Whence $h = 0.$

17. $3h - 25 + 8h - 20 = 0.$
Whence $h = 4\frac{1}{11}.$

18. $14x - 6x = 22 + 17x - 11x.$
Transposing,
$$8x - 6x = 22.$$
Whence $x = 11.$

19. $7x - 13 + 8 = x - 27 - 5x.$
Transposing,
$$7x + 4x = 5 - 27.$$
Whence $x = -2.$

20. $4x - 15 - 11x - 18 + 16x - 17 = 0.$
Whence $x = 5\frac{5}{9}.$

21. $5y - 6 + 3y + 18 - 2y - 25 + 1 = 0.$
Whence $y = 2.$

22. $0 = 9x - 3 - 4x + 27 + 16x + 18.$
Whence $x = -2.$

23. $7n - 5 - 4n + 8 = 3n + 18 - 2n - 3.$
Collecting, $3n + 3 = n + 15.$
Whence $n = 6.$

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1. $n + 5.$
2. $n + a.$
3. $n - 3.$
4. $n - b.$
5. $4n.$
6. $a + b + 3.$
7. $a + b + c.$
8. $2a - b - 5.$
9. $2a - b - c.$
10. $2n - b.$
11. $5n + a.$
12. $4n - 7.$
13. $3n + 8.$
14. $10 - 6$, or $4.$
15. $x - 4.$
16. $12 - y.$
17. $x - a.$
18. $a - x.$
19. $x + y - z.$
20. $18 - 7$, or $11.$

21. $18 - n$.
22. $34 - 18$, or 16.
23. $n - 18$.
24. $30 - b$.
25. $a - 7$.
26. $a - x$.

27. $6 - d$, if 6 is the greater number. $6 + d$, if 6 is the less number.

28. $n - d$, if n is the greater number. $n + d$, if n is the less number.

29. $10 - 4 = 6$; $10 - x$.

30. $25 - 9 = 16$; $25 - y$; $16 - (a + b)$.

31. $40 - 27 = 13$; $40 - 40 = 0$; $40 - a$; $a - b$.

32. $36 - 22 = 14$; $a - 14$; $y - x$.

33. $a + 6 - (a - 6) = 12$; $a + 6 - (b - 6) = a - b + 12$;
$4x - 3 - (3x - 4) = x + 1$.

34. $n + 4$ years; $n + x$ years; $n - 3$ years.

35. $2n - 3 + 10$ years $= 2n + 7$ years; $2n - 3 + a$ years $= 2n - 3 + a$ years;
$2n - 3 - 8$ years $= 2n - 11$ years; $2n - 3 - a$ years.

36. A, $x - 4$ dollars; B, $x + 4$ dollars.

37. A, $x + 50 + y$ dollars; B, $x + 50 - y$ dollars.

38. (*a*) $x + 30 + 3x - 4 = 200$.
(*b*) $x + 30 = 3x - 4$.
(*c*) $x + 30 + 10 = 3x - 4$.
(*d*) $x + 30 + 100 = 3x - 4 - 50$.

39. (*a*) A, $x + 5$ years; B, $2x + 12$ years; C, $3x - 3$ years.
(*b*) A, $x - 3$ years; B, $2x + 4$ years; C, $3x - 11$ years.
(*c*) $x + 4 + 2x + 7 + 4 = 40$.
(*d*) $(3x - 8 - 6) - (x - 6) = 24$, if C is older than A.
(*e*) $x + 10 = 2x + 7$.
(*f*) $3x - 8 - 4 = 2x + 7 + 10$.
(*g*) $2x + 7 + x = 40$.
(*h*) $x + 2 + 2x + 7 + 2 + 3x - 8 + 2 = 100$.

40. A number less 2 is 8.

41. The sum of a number and 3 is 5.

42. Three times a number is 27.

43. Four times a number less 2 is 16.

44. Subtracting a number from 18 gives the same result as subtracting 4 from the number.

45. Subtracting 4 from three times a number gives the same result as adding 8 to twice the number.

46. The sum of a number and a second number is 20.

47. The result of subtracting a second number from a number is 2.

48. Twice one number has the same value as the sum of a second number and 6.

49. Three times a number less twice a second number is 8.

50. The sum of a number and a second number is a.

51. A number less a second number is b.

52. The sum of a number and a is a second number.

53. A number less b is equal to a second number.

54. Three times a number is equal to twice a second number.

55. A number is equal to twice a second number less 6.

56. The sum of three times a number and 4 is equal to the sum of twice a second number and 4.

57. Eighty less a certain number is equal to the sum of a second number and 30.

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1. Let $n =$ the unknown number.
Then $n + 22 = 50.$
Whence $n = 28.$

2. Let $n =$ the unknown number.
Then $n - 15 = 47.$
Whence $n = 62.$

3. Let $n =$ the unknown number.
Then $n + 9 = 28.$
Whence $n = 19.$

4. Let $n =$ the unknown number.
Then $n - 17 = 35.$
Whence $n = 52.$

5. Let $n =$ the unknown number.
Then $2n - 27 = 49.$
Whence $n = 38.$

6. Let $n =$ the unknown number.
Then $3n - 36 = 2n.$
Whence $n = 36.$

7. Let $n =$ the unknown number.
Then $3n - 17 = 2n - 1.$
Whence $n = 16.$

8. Let $n =$ the unknown number.
Then $5n + 6 = 2n + 15.$
Whence $n = 3.$

9. Let $n =$ the unknown number.
Then $4n + 9 = 7n - 33.$
Whence $n = 14.$

10. Let $n =$ the unknown number.
Then $n + 9 = 71 - n.$
Whence $n = 31.$

11. Let $n =$ the unknown number.
Then $2n + 6 = 4n - 10.$
Whence $n = 8.$

13. Let $n =$ the less number.
Then $n + 12 =$ the greater number.
Therefore $n + n + 12 = 74.$
Whence $n = 31,$
and $n + 12 = 43.$

14. Let $n =$ the less number.
Then $n + 3 =$ the greater number.
Therefore $n + n + 3 = 45.$
Whence $n = 21,$
and $n + 3 = 24.$

15. Let $n =$ the less number.
Then $n + 8 =$ the greater number.
Therefore $n + n + 8 = 44.$
Whence $n = 18,$
and $n + 8 = 26.$

16. Let $n =$ the first number.
Then $n - 4 =$ the second number,
and $n + 9 =$ the third number.
Therefore $n + n - 4 + n + 9 = 83.$
Whence $n = 26,$
$n - 4 = 22,$
and $n + 9 = 35.$

17. Let $n =$ the first number.
Then $n - 3 =$ the second number,
and $n - 3 + 18$, or $n + 15 =$ the third number.
Therefore $n + n - 3 + n + 15 = 66.$
Whence $n = 18,$
$n - 3 = 15,$
and $n + 15 = 33.$

18. Let $n =$ one number.
Then $n + 1 =$ the other.
Therefore $n + n + 1 = 37.$
Whence $n = 18,$
and $n + 1 = 19.$

19. Let $n =$ the first number.
Then $n + 1$ and $n + 2 =$ the next two numbers.

Therefore $n + n + 1 + n + 2 = 39.$
Whence $n = 12,$
$n + 1 = 13,$
and $n + 2 = 14.$

20. Let $n =$ the first number.
Then $n + 1$, $n + 2$, and $n + 3 =$ the next three numbers.
Therefore $n + n + 1 + n + 2 + n + 3 = 90.$
Whence $n = 21,$
$n + 1 = 22,$
$n + 2 = 23,$
and $n + 3 = 24.$

21. Let $n =$ the first even number.
Then $n + 2 =$ the next even number.
Therefore $n + n + 2 = 30.$
Whence $n = 14,$
and $n + 2 = 16.$

22. Let $n =$ the first odd number.
Then $n + 2$ and $n + 4 =$ the next two odd numbers.
Therefore $n + n + 2 + n + 4 = 87.$
Whence $n = 27,$
$n + 2 = 29,$
and $n + 4 = 31.$

23. Let $n =$ the first even number.
Then $n + 2$, $n + 4$, and $n + 6 =$ the next three even numbers.
Therefore $n + n + 2 + n + 4 + n + 6 = 100.$
Whence $n = 22,$
$n + 2 = 24,$
$n + 4 = 26,$
and $n + 6 = 28.$

24. Let $w =$ the width in feet.
Then $w + 3 =$ the length in feet.
Therefore $2w + 2w + 6 = 38.$
Whence $w = 8,$
and $w + 3 = 11.$

25. Let $w =$ the width in feet.
Then $w + 16 =$ the length in feet.
Therefore $2w + 2w + 32 = 128.$
Whence $w = 24,$
and $w + 16 = 40.$

26. Let $w =$ the width in feet.
Then $2w + 7 =$ the length in feet.
Therefore $2w + 4w + 14 = 104.$
Whence $w = 15,$
and $2w + 7 = 37.$

27. Let $w =$ the width in feet.
Then $4w + 5 =$ the length in feet.
Therefore $2w + 8w + 10 = 90.$
Whence $w = 8,$
and $4w + 5 = 37.$

28. Let $x =$ the length of the night in hours.
Then $x + 6\frac{1}{10} =$ the length of the day in hours.
Therefore $x + x + 6\frac{1}{10} = 24.$
Whence $x = 8\frac{19}{20}$ hours, or 8 hours and 57 minutes,
and $x + 6\frac{1}{10} = 15\frac{1}{20}$ hours, or 15 hours and 3 minutes.

29. Let $x =$ A's age in years.
Then $\frac{x}{2} =$ B's age in years,
and $x + 7 =$ C's age in years.
Therefore $x + \frac{x}{2} + x + 7 = 67.$
Whence $x = 24,$
$\frac{x}{2} = 12,$
and $x + 7 = 31.$

30. Let $x =$ B's age in years.
Then $3x =$ A's age in years,
and $x + 10 =$ C's age in years.
Therefore $3x + 5 + x + 5 + x + 10 + 5 = 60.$
Whence $x = 7,$
$3x = 21,$
and $x + 10 = 17.$

31. Let $x =$ B's age in years.
Then $x + 10 =$ A's age in years,
and $x - 6 =$ C's age in years.
Therefore $x + 10 - 4 + x - 4 + x - 6 - 4 = 46.$
Whence $x = 18,$
$x + 10 = 28,$
and $x - 6 = 12.$

32. Let $x =$ B's age in years.
Then $2x + 2 =$ A's age in years,
and $2x - 5 =$ C's age in years.
Therefore $2x + 2 + 6 + x + 6 + 2x - 5 + 6 = 70.$
Whence $x = 11,$
$2x + 2 = 24,$
and $2x - 5 = 17.$

33. Let $x =$ the number of million bushels of oats.
Then $x + 1838 =$ the number of million bushels of corn,
and $x - 120 =$ the number of million bushels of wheat.
Therefore $x + x + 1838 + x - 120 = 3981.$
Whence $x = 754\frac{1}{3},$
$x + 1838 = 2592\frac{1}{3},$
and $x - 120 = 634\frac{1}{3}.$

34. Here $47 =$ the number of degrees in the width of the torrid zone,
and $\frac{47}{2}$, or $23\frac{1}{2} =$ the number of degrees in the width of either frigid zone.
Let $x =$ the number of degrees in the width of either temperate zone.
Therefore $47 + 23\frac{1}{2} + 23\frac{1}{2} + x + x = 180.$
Whence $x = 43.$

35. Let $x =$ the per cent in commerce.
Then $x + 8 =$ the per cent in industries,
and $x + 20 =$ the per cent in agriculture.
Therefore $x + x + 8 + x + 20 = 100 - 24.$
Whence $x = 16,$
$x + 8 = 24,$
and $x + 20 = 36.$

36. Let $x =$ the number of million pounds produced by Montana.
Then $x - 110 =$ the number of million pounds produced by Michigan,
and $x - 139 =$ the number of million pounds produced by Arizona.
Therefore $x + x - 110 + x - 139 = 514.$
Whence $x = 254\frac{1}{3},$
$x - 110 = 144\frac{1}{3},$
and $x - 139 = 115\frac{1}{3}.$

37. Let $x =$ the height in feet of the Washington Monument.
Then $2x - 120 =$ the height in feet of the Eiffel Tower,
$x - 105 =$ the height in feet of the Great Pyramid,
and $x - 107 =$ the height in feet of St. Peter's.

Therefore $x + 2x - 120 + x - 105 + x - 107 = 2443.$

Whence
$$x = 555,$$
$$2x - 120 = 990,$$
$$x - 105 = 450,$$
and
$$x - 107 = 448.$$

38. Let $x =$ the area in square miles of the coal fields of all countries not named.

Then $2x + 38{,}400 =$ the area in square miles of the fields of the United States,

and $2x + 44{,}400 =$ the area in square miles of the fields of China and Japan.

Therefore $x + 2x + 38{,}400 + 2x + 44{,}400 = 471{,}800.$

Whence
$$x = 77{,}800,$$
$$2x + 38{,}400 = 194{,}000,$$
and
$$2x + 44{,}400 = 200{,}000.$$

39. Let $=$ the output of Japan in tons.

Then $5x - 4615 =$ the output of other countries in tons,

$2x - 808 =$ the output of Spain and Portugal in tons,

and $5(2x - 808) - 5573,$

or $10x - 9613 =$ the output of the United States in tons.

Therefore $x + 5x - 4615 + 2x - 808 + 10x - 9613 = 486{,}084.$

Whence
$$x = 27{,}840,$$
$$5x - 4615 = 134{,}585,$$
$$2x - 808 = 54{,}872,$$
and
$$10x - 9613 = 268{,}787.$$

40. Let $x =$ the number of square miles in North America.

Then $2x + 982{,}000 =$ the number of square miles in Asia,

$x - 1{,}186{,}000 =$ the number of square miles in South America,

and $x - 4{,}383{,}000 =$ the area of Europe in square miles.

Therefore $x + 2x + 982{,}000 + x - 4{,}383{,}000 + x - 1{,}186{,}000$
$$= 35{,}692{,}000.$$

Whence
$$x = 8{,}055{,}800,$$
$$2x + 982{,}000 = 17{,}093{,}600,$$
$$x - 1{,}186{,}000 = 6{,}869{,}800,$$
and
$$x - 4{,}383{,}000 = 3{,}672{,}800.$$

41. Let $x =$ the number in the Revolution.

Then $9x - 15{,}621 =$ the number in the Civil War,

and $x + 266{,}841 =$ the number in the War of 1812.

Therefore $x + 9x - 15{,}621 + 266{,}841 + x = 3{,}658{,}811.$

Whence $x = 309{,}781,$

$9x - 15{,}621 = 2{,}772{,}408,$

and $x + 266{,}841 = 576{,}622.$

42. Let $x =$ the capacity of St. Peter's.

Then $x - 29{,}000 =$ the capacity of St. Paul's (London),

$x - 17{,}000 =$ the capacity of Cathedral (Milan),

and $x - 22{,}000 =$ the capacity of St. Paul's (Rome).

Therefore $x + x - 29{,}000 + x - 17{,}000 + x - 22{,}000 = 148{,}000.$

Whence $x = 54{,}000,$

$x - 29{,}000 = 25{,}000,$

$x - 17{,}000 = 37{,}000,$

and $x - 22{,}000 = 32{,}000.$

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1. $14 - (6 - 3) - 5 = 14 - 3 - 5 = 6.$

2. $10 + (7 - 4) - (9 - 7) = 10 + 3 - 2 = 11.$

3. $(7 - 3 + 2) - (6 - 4) + 11 = 6 - 2 + 11 = 15.$

4. $11a - (4a - 9a) + (6a - a) = 11a - (-5a) + 5a = 21a.$

5. $(2a - 5a) - (4a - a - 7a) = -3a - (-4a) = a.$

6. $a - (b - c) + (2b - 3c) = a - b + c + 2b - 3c = a + b - c.$

7. $a - b - (c - d) + (a + b) - (b - c)$

$= a - b - c + d + a + b - b + c = 2a - b + d.$

8. $(x - y) - (2y - 3x) + (x - 4y)$

$= x - y - 2y + 3x + x - 4y = 5x - 7y.$

9. $x - (x - y + 2z) - (3z - y + 4) + (x - 6)$

$= x - x + y - 2z - 3z + y - 4 + x - 6$

$= x + 2y - 5z - 10.$

10. $7 - [8 - (3 - 10)] - (13 - 25) = 7 - [8 + 7] - (-12) = 19 - 15 = 4.$

11. $a + [2a - (3a - 2b)] + (3b - 2a)$

$= a + [2b - a] + 3b - 2a = 5b - 2a.$

12. $(5x - 6y) - [-2x - (4z - y) - 2z]$

$= 5x - 6y - [-2x - 6z + y] = 7x - 7y + 6z.$

13. $[3x - (2y - z)] - [-(3y - 2x) - 5x]$

$= 3x - 2y + z - [-3y - 3x] = 6x + y + z.$

14. $[(a + 3) - (x - 5)] - [a + 3 + (x - 5)]$

$= (a + 3 - x + 5) - (a + 3 + x - 5) = 10 - 2x.$

15. $7 - [-6 - \{-4 + (6 - 10)\} + 11] = 7 - [-6 - \{-8\} + 11]$

$= 7 - [5 + 8] = -6.$

16. $-5x + [+10x - \{+11x - (2x - 7x + 4) - 3x\} - 22]$

$= -5x + [+10x - \{+8x + 5x - 4\} - 22]$

$= -5x + [-3x - 18] = -8x - 18.$

17. $\{4a - [2a - (3a - 2b) + 4a] - (4b - 6)\}$
$$= 4a - [6a - 3a + 2b] - 4b + 6$$
$$= 4a - 3a - 2b - 4b + 6 = a - 6b + 6.$$

18. $2x - 3y - [\{+3z - 7x - (y + 4z) - 9x\} + z]$
$$= 2x - 3y - [3z - 16x - y - 4z + z]$$
$$= 2x - 3y + 16x + y = 18x - 2y.$$

19. $(4y - 7x) - \{3x - [4x + (7y - 4x) - (3y - 3x)]\}$
$$= 4y - 7x - 3x + [4x + 7y - 4x - 3y + 3x]$$
$$= 4y - 10x + [4y + 3x] = 8y - 7x.$$

20. $[(a + b) + c] = [a + b + c].$
$[(a + b) - c] = [a + b - c].$

21. $[4x + (3z - 5y)] = [4x + 3z - 5y].$
$[4x - (3z - 5y)] = [4x - 3z + 5y].$

22. $[(a - 2b) + (3c - d)] = [a - 2b + 3c - d].$
$[(a - 2b) - (3c - d)] = [a - 2b - 3c + d].$

23. $[(4x - 3) + (5y - 7)] = [4x - 3 + 5y - 7] = [4x + 5y - 10].$
$[(4x - 3) - (5y - 7)] = [4x - 3 - 5y + 7] = [4x - 5y + 4].$

24. $[(x^2 - a^2) + (y^2 - 2a^2)] = [x^2 - a^2 + y^2 - 2a^2] = [x^2 + y^2 - 3a^2].$
$[(x^2 - a^2) - (y^2 - 2a^2)] = [x^2 - a^2 - y^2 + 2a^2] = [x^2 - y^2 + a^2].$

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1. $x^2 - a^2 - 2ab - b^2 = (x^2) - (a^2 + 2ab + b^2).$
2. $12ab + x^2 - 9b^2 - 4a^2 = (x^2) - (4a^2 - 12ab + 9b^2).$
3. $y^2 - 4b^2 + 4ab - a^2 = (y^2) - (4b^2 - 4ab + a^2).$
4. $10ab + x^2 - a^2 - 25b^2 = (x^2) - (a^2 - 10ab + 25b^2).$
5. $x^2 - b^2 - 4a^2 + 4y^2 - 4ab - 4xy$
$$= (x^2 - 4xy + 4y^2) - (4a^2 + 4ab + b^2).$$
6. $4ab + x^2 - 4b^2 + y^2 - a^2 - 2xy$
$$= (x^2 - 2xy + y^2) - (a^2 - 4ab + 4b^2).$$
7. $16x^2 - a^2 - 16xy - b^2 + 2ab + 4y^2$
$$= (16x^2 - 16xy + 4y^2) - (a^2 - 2ab + b^2).$$
8. $x^2 - b^2 - 10xy + 12ab - 36a^2 + 25y^2$
$$= (x^2 - 10xy + 25y^2) - (36a^2 - 12ab + b^2).$$

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1. $(3)(-8) = -24.$
2. $(-2)(5) = -10.$
3. $(-7)(-3) = 21.$
4. $(-4x)(3) = -12x.$
5. $(-4x)^2 = (-4x)(-4x) = 16x^2.$
6. $(7)(-5a) = -35a.$
7. $(3a)(-6) = -18a.$
8. $(-2y)^2 = (-2y)(-2y) = 4y^2.$
9. $(-9a)(-10) = 90a.$
10. $(-3ax)^2 = (-3ax)(-3ax)$
$$= 9a^2x^2.$$
11. $(4a)(-2a) = -8a^2.$

12. $(6\,abc)^2 = (6\,abc)(6\,abc)$
$= 36\,a^2b^2c^2.$

13. $(-11\,x)(2\,x) = -22\,x^2.$

14. $(7\,x)(-3\,x) = -21\,x^2.$

15. $(-2\,a)^3 = (-2\,a)(-2\,a)(-2\,a)$
$= -8\,a^3.$

16. $(-2\,a)(-3\,a^2) = 6\,a^3.$

17. $(5\,a^4)(7\,a^3) = 35\,a^7.$

18. $(-4\,x)^3 = -64\,x^3.$

19. $(a^8)(-20\,a) = -20\,a^9.$

20. $(-4\,a^5)(-6\,a^2) = 24\,a.^{7}$

21. $(+6\,y)^3 = 216\,y^3.$

22. $(4\,x)(5\,y) = 20\,xy.$

23. $(-3\,a^2x)^2 = 9\,a^4x^2.$

24. $(3\,x^2)(-y) = -3\,x^2y.$

25. $(5\,x^2y)(-2\,x^3) = -10\,x^5y.$

26. $(-6\,x^3y^2)^2 = 36\,x^6y^4.$

27. $(-x^4y)(-x^2y^4) = x^6y^5.$

28. $(2\,ax^2)^3 = (2\,ax^2)(2\,ax^2)(2\,ax^2)$
$= 8\,a^3x^6.$

29. $(5\,a^3)(-4\,a^2)(-3\,a) = 60\,a^6.$

30. $(3ax)(-4a^2x)(-2ax^3) = 24\,a^4x^5.$

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1. $$\begin{array}{l} x+3 \\ 2\,x \\ \hline 2\,x^2+6\,x \end{array}$$

2. $$\begin{array}{l} 7\,x^2-5 \\ 3\,x^3 \\ \hline 21\,x^5-15\,x^3 \end{array}$$

3. $$\begin{array}{l} 5\,x^2-2\,x \\ -4\,x^2 \\ \hline -20\,x^4+8\,x^3 \end{array}$$

4. $$\begin{array}{l} 7\,xy-z \\ 3\,xy \\ \hline 21\,x^2y^2-3\,xyz \end{array}$$

5. $$\begin{array}{l} -4\,x^2+5\,x-6 \\ 6\,x^3 \\ \hline -24\,x^5+30\,x^4-36\,x^3 \end{array}$$

6. $$\begin{array}{l} x^3-3\,x^2+4 \\ -5\,x^4 \\ \hline -5\,x^7+15\,x^6-20\,x^4 \end{array}$$

7. $$\begin{array}{l} x^2-2\,xy+y^2 \\ -3\,xy \\ \hline -3\,x^3y+6\,x^2y^2-3\,xy^3 \end{array}$$

8. $$\begin{array}{l} a^4-a^2b^2+b^4 \\ -a^2b^2 \\ \hline -a^6b^2+a^4b^4-a^2b^6 \end{array}$$

9. $$\begin{array}{l} -a^2x^2+2\,ax-7\,b^2 \\ -4\,abx \\ \hline 4\,a^3bx^3-8\,a^2bx^2+28\,ab^3x \end{array}$$

10. $$\begin{array}{l} 7\,x^3-8\,x^2-12\,x+6 \\ -\frac{3}{4}\,x^3 \\ \hline -5\frac{1}{4}\,x^6+6\,x^5+9\,x^4-4\frac{1}{2}\,x^3 \end{array}$$

11. $$\begin{array}{l} -9\,a^2-12\,ax+42\,x^2 \\ \frac{7}{3}\,ax^3 \\ \hline -21\,a^3x^3-28\,a^2x^4+98\,ax^5 \end{array}$$

12. $4(2\,x-3) = 8\,x-12.$

13. $2\,x(x-y) = 2\,x^2-2\,xy.$

14. $-8(3\,x-7) = -24\,x+56.$

15. $-9(-4\,a+b) = 36\,a-9\,b.$

16. $-3\,x(2\,x-7) = -6\,x^2+21\,x.$

17. $-3(x^2-2\,x-6) = -3\,x^2+6\,x+18.$

18. $5\,xy(x^2-6\,x+9) = 5\,x^3y-30\,x^2y+45\,xy.$

19. $-3\,x(ax-bx+3\,cx^2) = -3\,ax^2+3\,bx^2-9\,cx^3.$

20. $-7\,ab(ax^2+bx+c) = -7\,a^2bx^2-7\,ab^2x-7\,abc.$

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1.
$$\begin{array}{rl} & x + 4 \qquad (1)\\ & x + 3 \qquad (2)\\ \hline & x^2 + 4x\\ & \qquad + 3x + 12\\ \hline (1)\cdot(2), & x^2 + 7x + 12 \end{array}$$

2.
$$\begin{array}{rl} & 2x + 3 \qquad (1)\\ & x + 3 \qquad (2)\\ \hline & 2x^2 + 3x\\ & \qquad + 6x + 9\\ \hline (1)\cdot(2), & 2x^2 + 9x + 9 \end{array}$$

3.
$$\begin{array}{rl} & 4x + 7 \qquad (1)\\ & 3x + 2 \qquad (2)\\ \hline & 12x^2 + 21x\\ & \qquad + 8x + 14\\ \hline (1)\cdot(2), & 12x^2 + 29x + 14 \end{array}$$

4.
$$\begin{array}{rl} & 3x - 5 \qquad (1)\\ & 3x + 8 \qquad (2)\\ \hline & 9x^2 - 15x\\ & \qquad + 24x - 40\\ \hline (1)\cdot(2), & 9x^2 + 9x - 40 \end{array}$$

5.
$$\begin{array}{rl} & 3x - 2 \qquad (1)\\ & 2x + 3 \qquad (2)\\ \hline & 6x^2 - 4x\\ & \qquad + 9x - 6\\ \hline (1)\cdot(2), & 6x^2 + 5x - 6 \end{array}$$

6.
$$\begin{array}{rl} & -4a + 6 \qquad (1)\\ & 5a - 7 \qquad (2)\\ \hline & -20a^2 + 30a\\ & \qquad + 28a - 42\\ \hline (1)\cdot(2), & -20a^2 + 58a - 42 \end{array}$$

7.
$$\begin{array}{rl} & 2x + y \qquad (1)\\ & x + 3y \qquad (2)\\ \hline & 2x^2 + xy\\ & \qquad + 6xy + 3y^2\\ \hline (1)\cdot(2), & 2x^2 + 7xy + 3y^2 \end{array}$$

8.
$$\begin{array}{rl} & 2x - 3y \qquad (1)\\ & 3x - 2y \qquad (2)\\ \hline & 6x^2 - 9xy\\ & \qquad - 4xy + 6y^2\\ \hline (1)\cdot(2), & 6x^2 - 13xy + 6y^2 \end{array}$$

9.
$$\begin{array}{rl} & 3x - \frac{1}{2} \qquad (1)\\ & 2x - \frac{1}{3} \qquad (2)\\ \hline & 6x^2 - x\\ & \qquad - x + \frac{1}{6}\\ \hline (1)\cdot(2), & 6x^2 - 2x + \frac{1}{6} \end{array}$$

10.
$$\begin{array}{rl} & -3x + 11a \qquad (1)\\ & 5x - a \qquad (2)\\ \hline & -15x^2 + 55xa\\ & \qquad + 3xa - 11a^2\\ \hline (1)\cdot(2), & -15x^2 + 58xa - 11a^2 \end{array}$$

11.
$$\begin{array}{rl} & ax - bx \qquad (1)\\ & cx + dx \qquad (2)\\ \hline & acx^2 - bcx^2\\ & \qquad + adx^2 - bdx^2\\ \hline (1)\cdot(2), & acx^2 - bcx^2 + adx^2 - bdx^2 \end{array}$$

12.
$$\begin{array}{rl} & -cx + d \qquad (1)\\ & bx - cx^2 \qquad (2)\\ \hline & -bcx^2 + bdx\\ & \qquad + c^2x^3 - cdx^2\\ \hline (1)\cdot(2), & -bcx^2 + bdx + c^2x^3 - cdx^2 \end{array}$$

13.
$$\begin{array}{rl} & 4x - \frac{1}{3} \qquad (1)\\ & 6x + \frac{2}{5} \qquad (2)\\ \hline & 24x^2 - 2x\\ & \qquad + 1\frac{3}{5}x - \frac{2}{15}\\ \hline (1)\cdot(2), & 24x^2 - \frac{2}{5}x - \frac{2}{15} \end{array}$$

14.
$$\begin{array}{rl} & x^2 - 5x + 6 \qquad (1)\\ & x - 3 \qquad (2)\\ \hline & x^3 - 5x^2 + 6x\\ & \qquad - 3x^2 + 15x - 18\\ \hline (1)\cdot(2), & x^3 - 8x^2 + 21x - 18 \end{array}$$

15.

$$\begin{array}{lrl}
 & 3x^2 - 3x - 7 & (1)\\
 & 2x + 4 & (2)\\ \hline
 & 6x^3 - 6x^2 - 14x \phantom{{}-28} & \\
 & +12x^2 - 12x - 28 & \\ \hline
(1)\cdot(2), & 6x^3 + 6x^2 - 26x - 28 &
\end{array}$$

17.

$$\begin{array}{lrl}
 & a^2x^2 - 2a^2x + 4a^2 & (1)\\
 & ax + 2a & (2)\\ \hline
 & a^3x^3 - 2a^3x^2 + 4a^3x & \\
 & +2a^3x^2 - 4a^3x + 8a^3 & \\ \hline
(1)\cdot(2), & a^3x^3 \qquad\qquad\qquad + 8a^3 &
\end{array}$$

16.

$$\begin{array}{lrl}
 & x^2 - xy + y^2 & (1)\\
 & x + y & (2)\\ \hline
 & x^3 - x^2y + xy^2 & \\
 & +x^2y - xy^2 + y^3 & \\ \hline
(1)\cdot(2), & x^3 \qquad\qquad + y^3 &
\end{array}$$

18.

$$\begin{array}{lrl}
 & 3x^3 - x^2 - 5x & (1)\\
 & 2x^3 - 5x^2 & (2)\\ \hline
 & 6x^6 - 2x^5 - 10x^4 & \\
 & -15x^5 + 5x^4 + 25x^3 & \\ \hline
(1)\cdot(2), & 6x^6 - 17x^5 - 5x^4 + 25x^3 &
\end{array}$$

19.

$$\begin{array}{lrl}
 & 2x^2 - 7x + 12 & (1)\\
 & x^2 - 3x - 5 & (2)\\ \hline
 & 2x^4 - 7x^3 + 12x^2 & \\
 & -6x^3 + 21x^2 - 36x & \\
 & -10x^2 + 35x - 60 & \\ \hline
(1)\cdot(2), & 2x^4 - 13x^3 + 23x^2 - x - 60 &
\end{array}$$

20.

$$\begin{array}{lrl}
 & a^2 - \frac{1}{2}a + \frac{1}{4} & (1)\\
 & a^2 - a + \frac{1}{3} & (2)\\ \hline
 & a^4 - \frac{1}{2}a^3 + \frac{1}{4}a^2 & \\
 & -a^3 + \frac{1}{2}a^2 - \frac{1}{4}a & \\
 & +\frac{1}{3}a^2 - \frac{1}{6}a + \frac{1}{12} & \\ \hline
(1)\cdot(2), & a^4 - 1\frac{1}{2}a^3 + 1\frac{1}{12}a^2 - \frac{5}{12}a + \frac{1}{12} &
\end{array}$$

21.

$$\begin{array}{lrl}
 & x^2 - xy + y^2 & (1)\\
 & x^2 + xy + y^2 & (2)\\ \hline
 & x^4 - x^3y + x^2y^2 & \\
 & +x^3y - x^2y^2 + xy^3 & \\
 & +x^2y^2 - xy^3 + y^4 & \\ \hline
(1)\cdot(2), & x^4 \qquad + x^2y^2 \qquad + y^4 &
\end{array}$$

22.

$$\begin{array}{lrl}
 & 3x^3 + 5x^2 - x + 2 & (1)\\
 & x^2 - 2x + 1 & (2)\\ \hline
 & 3x^5 + 5x^4 - x^3 + 2x^2 & \\
 & -6x^4 - 10x^3 + 2x^2 - 4x & \\
 & +3x^3 + 5x^2 - x + 2 & \\ \hline
(1)\cdot(2), & 3x^5 - x^4 - 8x^3 + 9x^2 - 5x + 2 &
\end{array}$$

23.

$$\begin{array}{lrl}
 & x^3 - x - 5 & (1)\\
 & 2x^2 - 3x - 4 & (2)\\ \hline
 & 2x^5 \qquad - 2x^3 - 10x^2 & \\
 & -3x^4 \qquad + 3x^2 + 15x & \\
 & -4x^3 \qquad + 4x + 20 & \\ \hline
(1)\cdot(2), & 2x^5 - 3x^4 - 6x^3 - 7x^2 + 19x + 20 &
\end{array}$$

$$\begin{array}{lll}
\textbf{24.} & -x^3+x^2+3x-6 & (1)\\
 & -x^2-3x+5 & (2)\\
\hline
 & x^5-x^4-3x^3+6x^2 & \\
 & \quad +3x^4-3x^3-9x^2+18x & \\
 & \qquad -5x^3+5x^2+15x-30 & \\
\hline
(1)\cdot(2), & x^5+2x^4-11x^3+2x^2+33x-30 &
\end{array}$$

$$\begin{array}{lll}
\textbf{25.} & a^3-5a^2+4a+7 & (1)\\
 & a^3+a^2-a+6 & (2)\\
\hline
 & a^6-5a^5+4a^4+7a^3 & \\
 & \quad +a^5-5a^4+4a^3+7a^2 & \\
 & \qquad -a^4+5a^3-4a^2-7a & \\
 & \qquad\quad +6a^3-30a^2+24a+42 & \\
\hline
(1)\cdot(2), & a^6-4a^5-2a^4+22a^3-27a^2+17a+42 &
\end{array}$$

$$\begin{array}{lll}
\textbf{26.} & 2x^3-5x^2-9x+8 & (1)\\
 & 8x^3+5x-4 & (2)\\
\hline
 & 16x^6-40x^5-72x^4+64x^3 & \\
 & \qquad +10x^4-25x^3-45x^2+40x & \\
 & \qquad\qquad -8x^3+20x^2+36x-32 & \\
\hline
(1)\cdot(2), & 16x^6-40x^5-62x^4+31x^3-25x^2+76x-32 &
\end{array}$$

$$\begin{array}{lll}
\textbf{27.} & x^2y-y^2x & (1)\\
 & -5x^2y+4xy & (2)\\
\hline
 & -5x^4y^2+5x^3y^3 & \\
 & \qquad +4x^3y^2-4x^2y^3 & \\
\hline
(1)\cdot(2), & -5x^4y^2+5x^3y^3+4x^3y^2-4x^2y^3 & (3)\\
 & 3x^2y-7xy^2 & (4)\\
\hline
 & -15x^6y^3+15x^5y^4+12x^5y^3-12x^4y^4 & \\
 & \qquad +35x^5y^4 \qquad -28x^4y^4-35x^4y^5+28x^3y^5 & \\
\hline
(3)\cdot(4), & -15x^6y^3+50x^5y^4+12x^5y^3-40x^4y^4-35x^4y^5+28x^3y^5 &
\end{array}$$

$$\begin{array}{lll}
\textbf{28.} & x^2-xy+y^2-xz-yz+z^2 & (1)\\
 & x+y+z & (2)\\
\hline
 & x^3-x^2y+xy^2-x^2z-xyz+xz^2 & \\
 & \quad +x^2y-xy^2 \qquad -xyz \qquad +y^3-y^2z+yz^2 & \\
 & \qquad +x^2z-xyz-xz^2 \qquad +y^2z-yz^2+z^3 & \\
\hline
(1)\cdot(2), & x^3 \qquad -3xyz \qquad +y^3 \qquad +z^3 &
\end{array}$$

$$\begin{array}{lll}
\textbf{29.} & a+b+c & (1)\\
 & a+b+c & (2)\\
\hline
 & a^2+ab \qquad +ac & \\
 & \quad +ab+b^2 \qquad +bc & \\
 & \qquad +ac+bc+c^2 & \\
\hline
(1)\cdot(2), & a^2+2ab+b^2+2ac+2bc+c^2 &
\end{array}$$

30.

$$\begin{array}{lll}
 & c + d - \frac{1}{2} & (1) \\
 & c + d - \frac{1}{2} & (2) \\ \hline
 & c^2 + cd - \frac{1}{2}c & \\
 & + cd + d^2 - \frac{1}{2}d & \\
 & - \frac{1}{2}c - \frac{1}{2}d + \frac{1}{4} & \\ \hline
(1)\cdot(2), & c^2 + 2cd + d^2 - c - d + \frac{1}{4} &
\end{array}$$

31.

$$\begin{array}{lll}
 & a - 2b + 3c - 4d & (1) \\
 & a - 2b + 3c - 4d & (2) \\ \hline
 & a^2 - 2ab + 3ac - 4ad & \\
 & - 2ab + 4b^2 - 6bc + 8bd & \\
 & + 3ac - 6bc + 9c^2 - 12cd & \\
 & - 4ad + 8bd - 12cd + 16d^2 & \\ \hline
(1)\cdot(2), & a^2 - 4ab + 6ac - 8ad + 4b^2 - 12bc + 16bd + 9c^2 - 24cd + 16d^2 &
\end{array}$$

32.

$$\begin{array}{lll}
 & x + y + z & (1) \\
 & x + y + z & (2) \\ \hline
 & x^2 + xy + xz & \\
 & + xy + y^2 + yz & \\
 & + xz + yz + z^2 & \\ \hline
(1)\cdot(2), & x^2 + 2xy + 2xz + y^2 + 2yz + z^2 & (3) \\
 & x + y + z & (4) \\ \hline
 & x^3 + 2x^2y + 2x^2z + xy^2 + 2xyz + xz^2 & \\
 & + x^2y + 2xy^2 + 2xyz + y^3 + 2y^2z + yz^2 & \\
 & + x^2z + 2xyz + 2xz^2 + y^2z + 2yz^2 + z^3 & \\ \hline
(3)\cdot(4), & x^3 + 3x^2y + 3x^2z + 3xy^2 + 6xyz + 3xz^2 + y^3 + 3y^2z + 3yz^2 + z^3 &
\end{array}$$

33.

$$\begin{array}{llllll}
 & x + y & (1) & & x - y & (6) \\
 & x + y & (2) & & x - y & (7) \\ \hline
 & x^2 + xy & & & x^2 - xy & \\
 & xy + y^2 & & & - xy + y^2 & \\ \hline
(1)\cdot(2), & x^2 + 2xy + y^2 & (3) & (6)\cdot(7), & x^2 - 2xy + y^2 & (8) \\
 & x + y & (4) & & x - y & (9) \\ \hline
 & x^3 + 2x^2y + xy^2 & & & x^3 - 2x^2y + xy^2 & \\
 & x^2y + 2xy^2 + y^3 & & & - x^2y + 2xy^2 - y^3 & \\ \hline
(3)\cdot(4), & x^3 + 3x^2y + 3xy^2 + y^3 & (5) & (8)\cdot(9), & x^3 - 3x^2y + 3xy^2 - y^3 & (10) \\
 & x^3 - 3x^2y + 3xy^2 - y^3 & (10) & & & \\ \hline
(5)+(10), & 2x^3 + 6xy^2 & & & &
\end{array}$$

34.

$$\begin{array}{lll}
 & 2a - b + 3 & (1) \\
 & 2a - b + 3 & (2) \\ \hline
 & 4a^2 - 2ab + 6a & \\
 & - 2ab + b^2 - 3b & \\
 & + 6a - 3b + 9 & \\ \hline
(1)\cdot(2), & 4a^2 - 4ab + b^2 + 12a - 6b + 9 &
\end{array}$$

35.

$$\begin{array}{lll}
 & 2x - 3ay & (1)\\
 & \underline{2x - 3ay} & (2)\\
 & 4x^2 - 6axy & \\
 & \underline{\quad - 6axy + 9a^2y^2} & \\
(1)\cdot(2), & 4x^2 - 12axy + 9a^2y^2 & (3)\\
 & \underline{2x - 3ay} & (4)\\
 & 8x^3 - 24ax^2y + 18a^2xy^2 & \\
 & \underline{\quad - 12ax^2y + 36a^2xy^2 - 27a^3y^3} & \\
(3)\cdot(4), & 8x^3 - 36ax^2y + 54a^2xy^2 - 27a^3y^3 &
\end{array}$$

36.

$$\begin{array}{lll}
 & x + 2y & (1)\\
 & \underline{x + 2y} & (2)\\
 & x^2 + 2xy & \\
 & \underline{\quad + 2xy + 4y^2} & \\
(1)\cdot(2) & x^2 + 4xy + 4y^2 & (3)\\
 & \underline{x^2 - 4xy + 4y^2} & (6)\\
(3)-(6), & \quad 8xy &
\end{array}
\qquad
\begin{array}{lll}
 & x - 2y & (4)\\
 & \underline{x - 2y} & (5)\\
 & x^2 - 2xy & \\
 & \underline{\quad - 2xy + 4y^2} & \\
(4)\cdot(5), & x^2 - 4xy + 4y^2 & (6)
\end{array}$$

37.

$$\begin{array}{lll}
 & 4x - 3y & (1)\\
 & \underline{4x - 3y} & (2)\\
 & 16x^2 - 12xy & \\
 & \underline{\quad - 12xy + 9y^2} & \\
(1)\cdot(2), & 16x^2 - 24xy + 9y^2 & (3)\\
 & \underline{9x^2 + 24xy + 16y^2} & (6)\\
(3)-(6), & 7x^2 - 48xy - 7y^2 &
\end{array}
\qquad
\begin{array}{lll}
 & 3x + 4y & (4)\\
 & \underline{3x + 4y} & (5)\\
 & 9x^2 + 12xy & \\
 & \underline{\quad 12xy + 16y^2} & \\
(4)\cdot(5), & 9x^2 + 24xy + 16y^2 & (6)
\end{array}$$

38.

$$\begin{array}{lll}
 & x - 3 & (a),\\
 & \underline{x - 3} & \\
 & x^2 - 3x & \\
 & \underline{\quad - 3x + 9} & \\
 & x^2 - 6x + 9 & \\
 & \underline{x - 3} & \\
 & x^3 - 6x^2 + 9x & \\
 & \underline{\quad - 3x^2 + 18x - 27} & \\
(a)^3, & x^3 - 9x^2 + 27x - 27 & (1)\\
(b)^2, & \underline{\quad + 4x^2 - 4x + 1} & (2)\\
(1)-(2), & x^3 - 13x^2 + 31x - 28 &
\end{array}
\qquad
\begin{array}{lll}
 & 2x - 1 & (b)\\
 & \underline{2x - 1} & \\
 & 4x^2 - 2x & \\
 & \underline{\quad - 2x + 1} & \\
(b)^2, & 4x^2 - 4x + 1 & (2)
\end{array}$$

39.

$$\begin{array}{lll}
 & x^a - 3 & (1)\\
 & \underline{x^a + 4} & (2)\\
 & x^{2a} - 3x^a & \\
 & \underline{\quad + 4x^a - 12} & \\
(1)\cdot(2), & x^{2a} + x^a - 12 &
\end{array}$$

40.

$$\begin{array}{lll}
 & x^{2a} + 5 & (1)\\
 & \underline{x^{2a} + 5} & (2)\\
 & x^{4a} + 5x^{2a} & \\
 & \underline{\quad + 5x^{2a} + 25} & \\
(1)\cdot(2), & x^{4a} + 10x^{2a} + 25 &
\end{array}$$

41.
$$\begin{array}{rl} & 2x^a - 3 \quad (a) \\ & \underline{2x^a - 3} \\ & 4x^{2a} - 6x^a \\ & \underline{\quad\quad - 6x^a + 9} \\ & 4x^{2a} - 12x^a + 9 \\ & \underline{2x^a - 3} \\ & 8x^{3a} - 24x^{2a} + 18x^a \\ & \underline{\quad\quad - 12x^{2a} + 36x^a - 27} \\ (a)^3, & 8x^{3a} - 36x^{2a} + 54x^a - 27 \end{array}$$

42.
$$\begin{array}{rl} & 2x^{2a} - 3x \quad (1) \\ & \underline{2x^{2a} - 3x} \quad (2) \\ & 4x^{4a} - 6x^{2a+1} \\ & \underline{\quad\quad - 6x^{2a+1} + 9x^2} \\ (1)\cdot(2), & 4x^{4a} - 12x^{2a+1} + 9x^2 \end{array}$$

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1. $5(x-1) = 30.$
Removing parentheses,
$5x - 5 = 30.$
Whence $x = 7.$

2. $3 + 2(x-3) = 1.$
$3 + 2x - 6 = 1.$
Whence $x = 2.$

3. $7(3x-2) + 11 = 60.$
$21x - 14 = 49.$
Whence $x = 3.$

4. $4(2x-5) + 15 = 3(x+10).$
$8x - 20 + 15 = 3x + 30.$
Whence $x = 7.$

5. $12y - 2(4y-7) - 16 = 0.$
$12y - 8y + 14 - 16 = 0.$
Whence $y = \frac{1}{2}.$

6. $9y - 3(2y-4) = 2(5-4y) + 2.$
$9y - 6y + 12 = 10 - 8y + 2.$
Whence $y = 0$

7. $4 - 2(4y-3) = 3(y-5).$
Simplifying,
$4 - 8y + 6 = 3y - 15.$
Whence $y = 2\frac{3}{11}.$

8.
$7(y-3) - 2(4+y) = 9.$
$7y - 21 - 8 - 2y = 9.$
Whence $y = 7\frac{3}{5}.$

9.
$5(n-7) + 24 + 4n = 0.$
$5n - 35 + 24 + 4n = 0.$
Whence $n = 1\frac{2}{9}.$

10. $5n - 9(2n+4) = 2(n-9).$
$5n - 18n - 36 = 2n - 18.$
Whence $n = -1\frac{1}{5}.$

11. $7n - 12 - 2(n-5) = n - 19.$
$6n - 2n + 10 = -7.$
Whence $n = -4\frac{1}{4}.$

12. $4(2n-7) - 3(4n-8) + 4 = 2n - 3.$
$8n - 28 - 12n + 24 + 4 = 2n - 3.$
Whence $n = \frac{1}{2}.$

13. $3h - 2(4h+8) = 3h - 24.$
$-8h - 16 = -24.$
Whence $h = 1.$

14. $5(3h+1) - 7h = 3(h-7) + 4.$
$15h + 5 - 7h = 3h - 21 + 4.$
Whence $h = -4\frac{2}{5}.$

15. $(h-2)(h-5) = (h+3)(h+2).$
Simplifying, $h^2 - 7h + 10 = h^2 + 5h + 6.$
Whence $h = \frac{1}{3}.$

16.
$$(h+4)(h+3)-(h+2)(h+1)-42=0.$$
$$h^2+7h+12-h^2-3h-2-42=0.$$
Whence $h=8.$

17.
$$(x+4)(x+6)=(x+18)(x+13).$$
$$x^2+10x+24=x^2+31x+234.$$
Whence $x=-10.$

18.
$$(k-7)(5+k)-(k-5)(k+7)+5=0.$$
$$(k^2-2k-35)-(k^2+2k-35)+5=0.$$
Simplifying, $-2k-2k+5=0.$
Whence $k=1\frac{1}{4}.$

19.
$$(2x-5)(4x-7)=8x^2+52.$$
Simplifying, $8x^2-34x+35=8x^2+52.$
Whence $x=-\frac{1}{2}.$

20.
$$(3y+5)(4y+7)-(2y+3)(6y+11)-2=0.$$
Simplifying, $12y^2+41y+35-(12y^2+40y+33)-2=0.$
Whence $y=0.$

21.
$$(n+3)(6n+5)-(2n+4)(3n-8)=38.$$
Simplifying, $6n^2+23n+15-(6n^2-4n-32)=38.$
Whence $n=-\frac{1}{3}.$

22.
$$(x+3)^2-(x+5)^2=-40.$$
Expanding, $x^2+6x+9-x^2-10x-25=-40.$
Whence $x=6.$

23.
$$(x+2)^2-(x-4)^2+48=0.$$
Expanding, $x^2+4x+4-x^2+8x-16+48=0.$
Whence $x=-3.$

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1. Area, $a\cdot b=ab$; perimeter, $2a+2b$.
2. Area, $3(x-4)=3x-12$; perimeter, $2(x-4)+2\cdot 3=2x-2$.
3. Area, $(2x-4)(x+2)=2x^2-8$;
   perimeter, $2(2x-4)+2(x+2)=6x-4$.
4. $4\cdot \$100=\$400$.
5. $n\cdot \$80=\$80n$.
6. $a\cdot b$ cents $=ab$ cents.
7. $(xa+yb)$ cents.
8. $x(b+10)$ dollars $=(bx+10x)$ dollars.
9. $x(b+2)$ dollars $+\,y(c-3)$ dollars $=(bx+2x+cy-3y)$ dollars.
10. 5% of 16 = .8; of $x=.05x$.

11. 3% of $(x+120) = .03x + 3.6$; of $(12x - 300a) = .36x - 9a$.

12. $3(n+4)$ years $= (3n+12)$ years.

13. (*a*) $.04x = 180$.

(*b*) $.03x = .05(1000 - x)$.

(*c*) $.05x = .04(1000 - x) - 20$.

14. Width, $10 + 2 + 2 = 14$ inches; length, $12 + 2 + 2 = 16$ inches.

15. Width, $10 + x + x = (10 + 2x)$ inches;

length, $12 + x + x = (12 + 2x)$ inches;

area, $(10 + 2x)(12 + 2x) = (120 + 44x + 4x^2)$ square inches;

area of picture $= 12 \cdot 10 = 120$ square inches;

area of frame $= 120 + 44x + 4x^2 - 120$

$= (44x + 4x^2)$ square inches.

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2. Let $n =$ the less number.
Then $49 - n =$ the greater number.
Therefore $2(49 - n) - 13 = 5n$.
Whence $n = 12\frac{1}{7}$,
and $49 - n = 36\frac{6}{7}$.

3. Let $n =$ the less number.
Then $143 - n =$ the greater number.
Therefore $10n + 5(143 - n) = 950$.
Whence $n = 47$,
and $143 - n = 96$.

4. Let $n =$ the greater number.
Then $45 - n =$ the less number.
Therefore $5n + 4(45 - n) = 207$.
Whence $n = 27$,
and $45 - n = 18$.

5. Let $n =$ the less number.
Then $88 - n =$ the greater number.
Therefore $3(88 - n) = 5n + 29$.
Whence $n = 29\frac{3}{8}$,
and $88 - n = 58\frac{5}{8}$.

6. Let $n =$ the greater number.
Then $93 - n =$ the less number.
Therefore $7(93 - n) - 7 = 6n$.
Whence $n = 49\frac{7}{13}$,
and $93 - n = 43\frac{6}{13}$.

7. Let $n =$ the less number.
Then $48 - n =$ the greater number.
Therefore $2(48 - n) - 7 = 3n - 5.$
Whence $n = 18\frac{4}{5},$
and $48 - n = 29\frac{1}{5}.$

8. Let $n =$ one number.
Then $12\frac{1}{2} - n =$ the other number.
Therefore $7n - 10(12\frac{1}{2} - n) = 45.$
Whence $n = 10,$
and $12\frac{1}{2} - n = 2\frac{1}{2}.$

9. Let n and $121 - n =$ the two parts of 121.
Therefore $4n + 8 = 3(121 - n).$
Whence $n = 50\frac{5}{7}$, one part,
and $121 - n = 70\frac{2}{7}$, the other.

10. Let n and $15 - n =$ the numbers.
Therefore $2n - 5(15 - n) = 240.$
Whence $n = 45$, one number,
and $15 - n = -30$, the other number.

11. Let $n =$ one number.
Then $14 - n =$ the other.
Therefore $9n - 11(14 - n) = 0.$
Whence $n = 7\frac{7}{10},$
and $14 - n = 6\frac{3}{10}.$

12. Let $n =$ the first number.
Then $n + 1 =$ the next consecutive number.
Therefore $n^2 + n^2 + 2n + 1 = 17 + 2n^2.$
Whence $n = 8,$
and $n + 1 = 9.$

13. Let $n =$ one number.
Then $n + 1 =$ the next number.
Therefore $(n^2 + 2n + 1) - n^2 = 75.$
Whence $n = 37,$
and $n + 1 = 38.$

14. Let n and $n + 1 =$ the numbers.
Therefore $(n^2 + 2n + 1) - n^2 = 23.$
Whence $n = 11$, the first number,
and $n + 1 = 12$, the next number.

15. Let $n =$ one odd number.
Then $n + 2 =$ the next odd number.
Therefore $(n^2 + 4n + 4) - n^2 = 104.$
Whence $n = 25,$
and $n + 2 = 27.$

16. Let $n =$ one odd number.
Then $n + 2 =$ the next odd number.
Therefore $(n^2 + 4n + 4) - n^2 = 40.$
Whence $n = 9,$
and $n + 2 = 11.$

17. Let $n =$ one even number.
Then $n + 2 =$ the next even number.
Therefore $n(n + 2) = (n^2 + 4n + 4) - 56.$
Whence $n = 26,$
and $n + 2 = 28.$

18. Let n and $n + 2 =$ two consecutive odd numbers.
Therefore $n(n + 2) = n^2 + 46.$
Whence $n = 23,$
and $n + 2 = 25.$

20. Let $s =$ the side of the square field in rods.
Then $s + 30 =$ the length of the rectangular field in rods,
and $s - 20 =$ the breadth of the rectangular field in rods.
Therefore $s^2 = (s + 30)(s - 20).$
Whence $s = 60,$
$s + 30 = 90,$
and $s - 20 = 40.$
Area of square $= 60 \cdot 60 = 3600$ square rods, or $22\frac{1}{2}$ acres.
Area of rectangle $= 90 \cdot 40 = 3600$ square rods, or $22\frac{1}{2}$ acres.

21. Let $x =$ the width of the court in feet.
Then $2x + 24 =$ the length of the court in feet.
Therefore $2x + 2(2x + 24) = 210.$
Whence $x = 27,$
and $2x + 24 = 78.$

22. Let $x =$ the width of the court in feet.
Then $2x + 6 =$ the length of the court in feet.
Therefore $2x + 2(2x + 6) = 228.$
Whence $x = 36,$
and $2x + 6 = 78.$

23. Let $l =$ the length of the court in feet.
Then $l - 20 =$ the breadth of the court in feet.
Therefore $2l + 2(l - 20) = 80 \cdot 3.$
Whence $l = 70,$
and $l - 20 = 50.$

24. Let $n =$ the breadth of the field in yards.
Then $3n - 50 =$ the length of the field in yards.
Therefore $2n + 2(3n - 50) = \frac{780}{3}.$
Whence $n = 45,$
and $3n - 50 = 85.$

26. Let $d =$ the number of dimes.
Then $38 - d =$ the number of quarters,
and $10d =$ the value of the dimes in cents.
Also $(38 - d)25 =$ the value of the quarters in cents.
Therefore $10d + (38 - d)25 = 530.$
Solving, $d = 28,$
and $38 - d = 10.$

27. Let $d =$ the number of dimes.
Then $d + 5 =$ the number of nickels,
and $2d + 5 =$ the number of quarters.
Also $10d =$ the value of the dimes in cents,
$5(d + 5) =$ the value of the nickels in cents,
and $25(2d + 5) =$ the value of the quarters in cents.
Therefore

$$10d + 5(d + 5) + 25(2d + 5) = 605.$$

Solving, $d = 7,$
$d + 5 = 12,$
and $2d + 5 = 19.$

28. Let $n =$ the number of nickels.
Then $40 - n =$ the number of dimes.
Also $5n =$ the value of the nickels in cents,
and $10(40 - n) =$ the value of the dimes in cents.
Therefore $5n + 10(40 - n) = 290.$
Solving, $n = 22,$
and $40 - n = 18.$

29. Let $b =$ B's age now in years.
Then $b + 20 =$ A's age now in years.
Therefore $b + 20 + 10 = 2(b + 10).$
Whence $b = 10,$
and $b + 20 = 30.$

30. Let $n =$ B's age now in years.
Then $4n =$ A's age now in years.
Therefore $4n + 20 = 2(n + 20)$.
Whence $n = 10$,
and $4n = 40$.

31. Let $n =$ B's age now in years.
Then $2n + 8 =$ A's age now in years.
Therefore $2n + 8 - 16 = 4(n - 16)$.
Solving, $n = 28$,
and $2n + 8 = 64$.

33. Let $x =$ the number of dollars invested at 5%.
Then $1400 - x =$ the number of dollars invested at 6%.
Hence $.05x =$ the income from the 5% investment,
and $.06(1400 - x) =$ the income from the 6% investment.
Therefore $.05x + .06(1400 - x) = 76$.
Multiplying by 100,

$$5x + 6(1400 - x) = 7600.$$

Solving, $x = 800$,
and $1400 - x = 600$.
Hence \$800 was invested at 5%, and \$600 at 6%.

34. Let $x =$ the number of dollars invested at 6%.
Then $x - 125 =$ the number of dollars invested at 8%.
Hence $.06x =$ the yearly income from the 6% investment,
and $.08(x - 125) =$ the yearly income from the 8% investment.
Therefore $.06x + .08(x - 125) = 53$.
Solving, $x = 450$,
and $x - 125 = 325$.
Hence \$450 is invested at 6%, and \$325 at 8%.

35. Let $x =$ the number of dollars invested at 5%.
Then $1240 - x =$ the number of dollars invested at 6%.
Hence $.05x =$ the income from the 5% investment,
and $.06(1240 - x) =$ the income from the 6% investment.
Therefore $.06(1240 - x) = 15 + .05x$.
Solving, $x = 540$,
and $1240 - x = 700$.
Hence \$540 is invested at 5%, and \$700 at 6%.

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1. $-10 \div 2 = -5.$
2. $12 \div (-3) = -4.$
3. $-16 \div (-4) = 4.$
4. $8a^3 \div a^2 = 8a.$
5. $-4a^6 \div 2a^2 = -2a^4.$
6. $6x^2 \div (-3x) = -2x.$
7. $-18x^7 \div (-6x^4) = 3x^3.$
8. $-25ax^3 \div 5ax = -5x^2.$
9. $12ax^3 \div (-3bx^3) = -\dfrac{4a}{b}.$
10. $-28ay^4 \div (-7cy^3) = \dfrac{4ay}{c}.$
11. $70x^4y^9 \div (-10x^2y^8) = -7x^2y.$
12. $48ax^5 \div (-16bx^4) = -\dfrac{3ax}{b}.$
13. $-36x^6y^6 \div (-6x^2y^3) = 6x^4y^3.$
14. $63c^3d^5 \div (-9bcd^3) = \dfrac{-7c^2d^2}{b}.$
15. $64a^5b^7 \div (-16ab^7) = -4a^4.$
16. $-28a^6b^{12} \div (-7a^4b^8) = 4a^2b^4.$
17. $\dfrac{15x^2y^4z^5}{75xy^4z} = \dfrac{xz^4}{5}.$
18. $\dfrac{42x^{24}y^{56}}{-6x^8y^7} = -7x^{16}y^{49}.$
19. $\dfrac{x^a}{x^2} = x^{a-2}.$
20. $\dfrac{-x^{3a}}{x^a} = -x^{2a}.$
21. $\dfrac{-17a^9b^8c^7}{51a^7b^8c} = \dfrac{-a^2c^6}{3}.$
22. $\dfrac{39x^{13}y^{26}z^{39}}{-13x^{13}y^{13}z^{13}} = -3y^{13}z^{26}.$
23. $\dfrac{-11a^2b^{11}c^{13}}{66ac^{12}} = \dfrac{-ab^{11}c}{6}.$
24. $\dfrac{-121a^{11}b^{22}c^{33}}{-11a^{11}b^{11}c^{11}} = 11b^{11}c^{22}.$
25. $\dfrac{x^{a+b}}{x^b} = x^a.$
26. $\dfrac{x^c}{x} = x^{c-1}.$
27. $\dfrac{3a^3x^{3a}}{a^2x^a} = 3ax^{2a}.$
28. $\dfrac{6x^{2a+3}}{-2x^4} = -3x^{2a-1}.$

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1. $\dfrac{6x^2-4x}{2x} = 3x-2.$
2. $\dfrac{9x-18x^4}{-3x} = \dfrac{9x}{-3x} + \dfrac{-18x^4}{-3x} = -3+6x^3.$
3. $\dfrac{4xy-12x^2}{-4x} = \dfrac{4xy}{-4x} + \dfrac{-12x^2}{-4x} = -y+3x.$
4. $\dfrac{9ax^3-12x^5}{-3ax^2} = \dfrac{9ax^3}{-3ax^2} + \dfrac{-12x^5}{-3ax^2} = -3x+\dfrac{4x^3}{a}.$
5. $\dfrac{25x^2y+30xy^5}{-5xy} = -5x-6y^4.$
6. $\dfrac{16bx^4-36x^2}{4bx^2} = 4x^2-\dfrac{9}{b}.$
7. $\dfrac{14x^3y^4-28x^5y^6}{7x^2y^3} = 2xy-4x^3y^3.$

8. $\dfrac{4\,x^4y - 8\,x^6y^2 + 12\,x^8y^4}{4\,x^4y} = 1 - 2\,x^2y + 3\,x^4y^3.$

9. $\dfrac{a^3cd^2 - a^2c^3}{a^2cd} = ad - \dfrac{c^2}{d}.$

10. $\dfrac{ax^4 - bx^3 + cx^2}{-\,x^2} = -\,ax^2 + bx - c.$

11. $\dfrac{15\,a^2b^2 + 9\,a^4b^2 - 30\,a^6b^2}{-\,3\,a^2b^2} = -\,5 - 3\,a^2 + 10\,a^4.$

12. $\dfrac{16\,a^4b^5c^6 - 24\,a^5b^6c^7 - 48\,a^6b^7c^8}{8\,a^3b^2c} = 2\,ab^3c^5 - 3\,a^2b^4c^6 - 6\,a^3b^5c^7.$

13. $\dfrac{85\,xyz - 51\,x^2yz^2 + 102\,x^3yz^3 - 170\,x^5y^5z}{-\,17\,xyz} = -\,5 + 3\,xz - 6\,x^2z^2 + 10\,x^4y^4.$

14. $\dfrac{4\,(x-3) + a\,(x-3)}{x-3} = 4 + a.$

15. $\dfrac{3\,x\,(3\,x+4) - 4\,y\,(3\,x+4)}{3\,x+4} = 3\,x - 4\,y.$

16. $\dfrac{5\,a\,(2\,x^2 - y) - 3\,b\,(2\,x^2 - y)}{2\,x^2 - y} = 5\,a - 3\,b.$

17. $\dfrac{(a+b)^4 - 3\,(a+b)^3}{(a+b)^2} = (a+b)^2 - 3\,(a+b).$

18. $\dfrac{21\,(x-y)^7 - 35\,(x-y)^5}{-\,7\,(x-y)^5} = -\,3\,(x-y)^2 + 5.$

19. $\dfrac{16\,(3\,x-4)^4 - 24\,(3\,x-4)^5 - 48\,(3\,x-4)^7}{-\,8\,(3\,x-4)^4} = -\,2 + 3\,(3\,x-4) + 6\,(3\,x-4)^3.$

20. $\dfrac{-\,5\,(ac^2 - 2\,d)^3 + x\,(ac^2 - 2\,d)}{5\,(ac^2 - 2\,d)} = -\,(ac^2 - 2\,d)^2 + \dfrac{x}{5}.$

21. $\dfrac{4\,x^4 - 8\,x^{3a} - 6\,x^{2a-2}}{2\,x^3} = 2\,x - 4\,x^{3a-3} - 3\,x^{2a-5}.$

22. $\dfrac{3\,x^a - 2\,x^{a+1} - x^{a+2} + x^2}{x^2} = 3\,x^{a-2} - 2\,x^{a-1} - x^a + 1.$

23. $\dfrac{6\,x^{2a-3} - 12\,x^{4a+4} - 18\,x^{3a+5}}{-\,3\,x^{2a}} = -\,2\,x^{-3} + 4\,x^{2a+4} + 6\,x^{a+5}.$

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1. $$\begin{array}{l|l} x^2 + 7\,x + 12 & \underline{x+3} \\ \underline{x^2 + 3\,x} & x + 4 \\ \quad\;\; 4\,x + 12 & \\ \quad\;\; \underline{4\,x + 12} & \end{array}$$

2. $$\begin{array}{l|l} x^2 - 2\,x - 15 & \underline{x-5} \\ \underline{x^2 - 5\,x} & x + 3 \\ \quad\;\; 3\,x - 15 & \\ \quad\;\; \underline{3\,x - 15} & \end{array}$$

3.
$$\begin{array}{l|l} x^2 + 5x + 6 & x + 3 \\ \cline{2-2} \underline{x^2 + 3x} & x + 2 \\ \cline{1-2} \phantom{x^2+{}}2x + 6 & \\ \phantom{x^2+{}}\underline{2x + 6} & \end{array}$$

4.
$$\begin{array}{l|l} x^2 - 7x + 6 & x - 1 \\ \cline{2-2} x^2 - x & x - 6 \\ \cline{1-2} - 6x + 6 & \\ \underline{- 6x + 6} & \end{array}$$

5.
$$\begin{array}{l|l} 6x^2 - 13x + 6 & 2x - 3 \\ \cline{2-2} 6x^2 - 9x & 3x - 2 \\ \cline{1-2} - 4x + 6 & \\ \underline{- 4x + 6} & \end{array}$$

6.
$$\begin{array}{l|l} 25x^4 + 30x^2 - 7 & 5x^2 + 7 \\ \cline{2-2} 25x^4 + 35x^2 & 5x^2 - 1 \\ \cline{1-2} - 5x^2 - 7 & \\ \underline{- 5x^2 - 7} & \end{array}$$

7.
$$\begin{array}{l|l} 12a^2 + 19a - 21 & 4a - 3 \\ \cline{2-2} 12a^2 - 9a & 3a + 7 \\ \cline{1-2} 28a - 21 & \\ \underline{28a - 21} & \end{array}$$

8.
$$\begin{array}{l|l} x^3 - 2x^2 + 4x - 8 & x - 2 \\ \cline{2-2} x^3 - 2x^2 & x^2 + 4 \\ \cline{1-2} \phantom{x^3 - 2x^2 +{}}4x - 8 & \\ \phantom{x^3 - 2x^2 +{}}\underline{4x - 8} & \end{array}$$

9.
$$\begin{array}{l|l} a^3 + 3a^2b + 3ab^2 + b^3 & a + b \\ \cline{2-2} \underline{a^3 + a^2b} & a^2 + 2ab + b^2 \\ \cline{2-2} \phantom{a^3+{}}2a^2b + 3ab^2 & \\ \phantom{a^3+{}}\underline{2a^2b + 2ab^2} & \\ \phantom{a^3+2a^2b+3{}}ab^2 + b^3 & \\ \phantom{a^3+2a^2b+3{}}\underline{ab^2 + b^3} & \end{array}$$

10.
$$\begin{array}{l|l} x^3 - 15x^2 + 65x - 63 & x - 7 \\ \cline{2-2} \underline{x^3 - 7x^2} & x^2 - 8x + 9 \\ \cline{2-2} - 8x^2 + 65x & \\ \underline{- 8x^2 + 56x} & \\ 9x - 63 & \\ \underline{9x - 63} & \end{array}$$

11.
$$\begin{array}{l|l} -25x^3+5x^2+5x-1 & 5x^2-1 \\ \cline{2-2} \underline{-25x^3 \qquad\quad +5x} & -5x+1 \\ \qquad 5x^2 \qquad -1 & \\ \qquad \underline{5x^2 \qquad -1} & \end{array}$$

12.
$$\begin{array}{l|l} 2x^3-14x^2+14x+12 & 2x-4 \\ \cline{2-2} \underline{2x^3-\ \ 4x^2} & x^2-5x-3 \\ \quad -10x^2+14x & \\ \quad \underline{-10x^2+20x} & \\ \qquad\quad -\ 6x+12 & \\ \qquad\quad \underline{-\ 6x+12} & \end{array}$$

13.
$$\begin{array}{l|l} 3a^3+28a^2+\ 89a-140 & 3a-5 \\ \cline{2-2} \underline{3a^3-\ \ 5a^2} & a^2+11a+48+\dfrac{100}{3a-5} \\ \quad 33a^2+\ 89a & \\ \quad \underline{33a^2-\ 55a} & \\ \qquad\quad 144a-140 & \\ \qquad\quad \underline{144a-240} & \\ \qquad\qquad\quad 100 & \end{array}$$

14.
$$\begin{array}{l|l} 6x^3-23x^2+37x-24 & 2x-3 \\ \cline{2-2} \underline{6x^3-\ \ 9x^2} & 3x^2-7x+8 \\ \quad -14x^2+37x & \\ \quad \underline{-14x^2+21x} & \\ \qquad\quad 16x-24 & \\ \qquad\quad \underline{16x-24} & \end{array}$$

15.
$$\begin{array}{l|l} 12a^3-53a^2+53a+8 & 4a^2-7a-1 \\ \cline{2-2} \underline{12a^3-21a^2-\ \ 3a} & 3a-8 \\ \quad -32a^2+56a+8 & \\ \quad \underline{-32a^2+56a+8} & \end{array}$$

16.
$$\begin{array}{l|l} 15a^3-56a^2+99a-70 & 3a^2-7a+10 \\ \cline{2-2} \underline{15a^3-35a^2+50a} & 5a-7 \\ \quad -21a^2+49a-70 & \\ \quad \underline{-21a^2+49a-70} & \end{array}$$

17.
$$\begin{array}{l|l} a^4+11a^3+23a^2-55a-140 & a^2-5 \\ \cline{2-2} \underline{a^4 \qquad\quad -\ 5a^2} & a^2+11a+28 \\ \quad 11a^3+28a^2 & \\ \quad \underline{11a^3 \qquad\quad -55a} & \\ \qquad\quad 28a^2 \qquad\quad -140 & \\ \qquad\quad \underline{28a^2 \qquad\quad -140} & \end{array}$$

18.
$$\begin{array}{r|l}
a^4 + 4a^3 + 6a^2 + 4a + 1 & a^2 + 2a + 1 \\ \cline{2-2}
\underline{a^4 + 2a^3 + a^2 \qquad\qquad} & a^2 + 2a + 1 \\
2a^3 + 5a^2 + 4a \qquad & \\
\underline{2a^3 + 4a^2 + 2a \qquad} & \\
a^2 + 2a + 1 & \\
\underline{a^2 + 2a + 1} &
\end{array}$$

19.
$$\begin{array}{r|l}
a^4 - 8a^3 + 24a^2 - 32a + 16 & a^2 - 4a + 4 \\ \cline{2-2}
\underline{a^4 - 4a^3 + 4a^2 \qquad\qquad\qquad} & a^2 - 4a + 4 \\
-4a^3 + 20a^2 - 32a \qquad & \\
\underline{-4a^3 + 16a^2 - 16a \qquad} & \\
4a^2 - 16a + 16 & \\
\underline{4a^2 - 16a + 16} &
\end{array}$$

20.
$$\begin{array}{r|l}
x^4 + 4x^3 - 31x^2 + 40x + 21 & x^2 - 7x - 3 \\ \cline{2-2}
\underline{x^4 - 7x^3 - 3x^2 \qquad\qquad\qquad} & x^2 + 11x + 49 + \dfrac{416x + 168}{x^2 - 7x - 3} \\
11x^3 - 28x^2 + 40x \qquad & \\
\underline{11x^3 - 77x^2 - 33x \qquad} & \\
49x^2 + 73x + 21 & \\
\underline{49x^2 - 343x - 147} & \\
416x + 168 &
\end{array}$$

21.
$$\begin{array}{r|l}
10x^4 - 27x^3 - 42x^2 + 11x - 36 & 2x^2 - 5x + 9 \\ \cline{2-2}
\underline{10x^4 - 25x^3 + 45x^2 \qquad\qquad\qquad} & 5x^2 - x - 46 + \dfrac{378 - 210x}{2x^2 - 5x + 9} \\
-2x^3 - 87x^2 + 11x \qquad & \\
\underline{-2x^3 + 5x^2 - 9x \qquad} & \\
-92x^2 + 20x - 36 & \\
\underline{-92x^2 + 230x - 414} & \\
-210x + 378 &
\end{array}$$

22.
$$\begin{array}{r|l}
x^4 - 3a^2x^2 + a^4 \qquad\qquad & x^2 - ax + a^2 \\ \cline{2-2}
\underline{x^4 - ax^3 + a^2x^2 \qquad\qquad} & x^2 + ax - 3a^2 + \dfrac{4a^4 - 4a^3x}{x^2 - ax + a^2} \\
ax^3 - 4a^2x^2 + a^4 \qquad & \\
\underline{ax^3 - a^2x^2 + a^3x \qquad} & \\
-3a^2x^2 - a^3x + a^4 & \\
\underline{-3a^2x^2 + 3a^3x - 3a^4} & \\
-4a^3x + 4a^4 &
\end{array}$$

23.
$$\begin{array}{r|l}
a^4 + 3a^2b^2 + 4b^4 \qquad\qquad & a^2 - ab + 2b^2 \\ \cline{2-2}
\underline{a^4 - a^3b + 2a^2b^2 \qquad\qquad} & a^2 + ab + 2b^2 \\
a^3b + a^2b^2 + 4b^4 \qquad & \\
\underline{a^3b - a^2b^2 + 2ab^3 \qquad} & \\
2a^2b^2 - 2ab^3 + 4b^4 & \\
\underline{2a^2b^2 - 2ab^3 + 4b^4} &
\end{array}$$

24. $$\begin{array}{l|l} 16x^4-60x^2y^2+25y^4 & 4x^2-10xy-5y^2 \\ \cline{2-2} \underline{16x^4-40x^3y-20x^2y^2} & 4x^2+10xy+15y^2+\dfrac{200xy^3+100y^4}{4x^2-10xy-5y^2} \\ \quad 40x^3y-40x^2y^2+25y^4 & \\ \quad \underline{40x^3y-100x^2y^2-50xy^3} & \\ \qquad 60x^2y^2+50xy^3+25y^4 & \\ \qquad \underline{60x^2y^2-150xy^3-75y^4} & \\ \qquad\qquad 200xy^3+100y^4 & \end{array}$$

25. $$\begin{array}{l|l} 9a^4+26a^2b^2+49b^4 & 3a^2+4ab+7b^2 \\ \cline{2-2} \underline{9a^4+12a^3b+21a^2b^2} & 3a^2-4ab+7b^2 \\ \quad -12a^3b+5a^2b^2+49b^4 & \\ \quad \underline{-12a^3b-16a^2b^2-28ab^3} & \\ \qquad 21a^2b^2+28ab^3+49b^4 & \\ \qquad \underline{21a^2b^2+28ab^3+49b^4} & \end{array}$$

26. $$\begin{array}{l|l} 4a^8-44a^4b^4+100b^8 & 2a^4+2a^2b^2-10b^4 \\ \cline{2-2} \underline{4a^8+4a^6b^2-20a^4b^4} & 2a^4-2a^2b^2-10b^4 \\ \quad -4a^6b^2-24a^4b^4+100b^8 & \\ \quad \underline{-4a^6b^2-4a^4b^4+20a^2b^6} & \\ \qquad -20a^4b^4-20a^2b^6+100b^8 & \\ \qquad \underline{-20a^4b^4-20a^2b^6+100b^8} & \end{array}$$

27. $$\begin{array}{l|l} 25x^3-10x^2+40x-18 & 5x-6 \\ \cline{2-2} \underline{25x^3-30x^2} & 5x^2+4x+12\tfrac{4}{5}+\dfrac{58\frac{4}{5}}{5x-6} \\ \quad 20x^2+40x & \\ \quad \underline{20x^2-24x} & \\ \qquad 64x-18 & \\ \qquad \underline{64x-76\tfrac{4}{5}} & \\ \qquad\quad 58\tfrac{4}{5} & \end{array}$$

28. $$\begin{array}{l|l} x^3-y^3 & x-y \\ \cline{2-2} \underline{x^3-x^2y} & x^2+xy+y^2 \\ \quad x^2y & \\ \quad \underline{x^2y-xy^2} & \\ \qquad xy^2-y^3 & \\ \qquad \underline{xy^2-y^3} & \end{array}$$

29. $$\begin{array}{l|l} a^3-125b^3 & a-5b \\ \cline{2-2} \underline{a^3-5a^2b} & a^2+5ab+25b^2 \\ \quad 5a^2b & \\ \quad \underline{5a^2b-25ab^2} & \\ \qquad 25ab^2-125b^3 & \\ \qquad \underline{25ab^2-125b^3} & \end{array}$$

30. $$\begin{array}{l|l} a^6+343b^3 & a^2+7b \\ \cline{2-2} \underline{a^6+7a^4b} & a^4-7a^2b+49b^2 \\ \quad -7a^4b & \\ \quad \underline{-7a^4b-49a^2b^2} & \\ \qquad 49a^2b^2+343b^3 & \\ \qquad \underline{49a^2b^2+343b^3} & \end{array}$$

31.
$$\begin{array}{l|l}
x^4 - 16 & x + 2 \\ \cline{2-2}
x^4 + 2x^3 & x^3 - 2x^2 + 4x - 8 \\ \cline{1-1}
\quad - 2x^3 & \\
\quad - 2x^3 - 4x^2 & \\ \cline{1-1}
\qquad\quad 4x^2 & \\
\qquad\quad 4x^2 + 8x & \\ \cline{1-1}
\qquad\qquad - 8x - 16 & \\
\qquad\qquad - 8x - 16 & \\ \cline{1-1}
\end{array}$$

32.
$$\begin{array}{l|l}
y^5 - 5y^2 - 3000 & y - 5 \\ \cline{2-2}
y^5 - 5y^4 & y^4 + 5y^3 + 25y^2 + 120y + 600 \\ \cline{1-1}
\quad 5y^4 & \\
\quad 5y^4 - 25y^3 & \\ \cline{1-1}
\qquad 25y^3 - 5y^2 & \\
\qquad 25y^3 - 125y^2 & \\ \cline{1-1}
\qquad\qquad 120y^2 & \\
\qquad\qquad 120y^2 - 600y & \\ \cline{1-1}
\qquad\qquad\qquad 600y - 3000 & \\
\qquad\qquad\qquad 600y - 3000 & \\ \cline{1-1}
\end{array}$$

33.
$$\begin{array}{l|l}
x^4 + y^4 & x - y \\ \cline{2-2}
x^4 - x^3y & x^3 + x^2y + xy^2 + y^3 + \dfrac{2y^4}{x - y} \\ \cline{1-1}
\qquad x^3y & \\
\qquad x^3y - x^2y^2 & \\ \cline{1-1}
\qquad\qquad x^2y^2 & \\
\qquad\qquad x^2y^2 - xy^3 & \\ \cline{1-1}
\qquad\qquad\qquad xy^3 + y^4 & \\
\qquad\qquad\qquad xy^3 - y^4 & \\ \cline{1-1}
\qquad\qquad\qquad\qquad 2y^4 & \\
\end{array}$$

34.
$$\begin{array}{l|l}
x^4 + y^4 & x + y \\ \cline{2-2}
x^4 + x^3y & x^3 - x^2y + xy^2 - y^3 + \dfrac{2y^4}{x + y} \\ \cline{1-1}
\qquad - x^3y & \\
\qquad - x^3y - x^2y^2 & \\ \cline{1-1}
\qquad\qquad x^2y^2 & \\
\qquad\qquad x^2y^2 + xy^3 & \\ \cline{1-1}
\qquad\qquad\qquad - xy^3 + y^4 & \\
\qquad\qquad\qquad - xy^3 - y^4 & \\ \cline{1-1}
\qquad\qquad\qquad\qquad 2y^4 & \\
\end{array}$$

35.
$$\begin{array}{l|l}
x^4 - y^4 & x + y \\ \cline{2-2}
\underline{x^4 + x^3y} & \underline{x^3 - x^2y + xy^2 - y^3} \\
\quad -x^3y & \\
\quad \underline{-x^3y - x^2y^2} & \\
\qquad\qquad x^2y^2 & \\
\qquad\qquad \underline{x^2y^2 + xy^3} & \\
\qquad\qquad\qquad -xy^3 - y^4 & \\
\qquad\qquad\qquad \underline{-xy^3 - y^4} &
\end{array}$$

36.
$$\begin{array}{l|l}
x^4 - y^4 & x - y \\ \cline{2-2}
\underline{x^4 - x^3y} & \underline{x^3 + x^2y + xy^2 + y^3} \\
\quad x^3y & \\
\quad \underline{x^3y - x^2y^2} & \\
\qquad\qquad x^2y^2 & \\
\qquad\qquad \underline{x^2y^2 - xy^3} & \\
\qquad\qquad\qquad xy^3 - y^4 & \\
\qquad\qquad\qquad \underline{xy^3 - y^4} &
\end{array}$$

37.
$$\begin{array}{l|l}
x^5 + y^5 & x + y \\ \cline{2-2}
\underline{x^5 + x^4y} & \underline{x^4 - x^3y + x^2y^2 - xy^3 + y^4} \\
\quad -x^4y & \\
\quad \underline{-x^4y - x^3y^2} & \\
\qquad\qquad x^3y^2 & \\
\qquad\qquad \underline{x^3y^2 + x^2y^3} & \\
\qquad\qquad\qquad -x^2y^3 & \\
\qquad\qquad\qquad \underline{-x^2y^3 - xy^4} & \\
\qquad\qquad\qquad\qquad xy^4 + y^5 & \\
\qquad\qquad\qquad\qquad \underline{xy^4 + y^5} &
\end{array}$$

38.
$$\begin{array}{l|l}
x^5 + y^5 & x - y \\ \cline{2-2}
\underline{x^5 - x^4y} & \\
\quad x^4y & \underline{x^4 + x^3y + x^2y^2 + xy^3 + y^4 + \dfrac{2\,y^5}{x - y}} \\
\quad \underline{x^4y - x^3y^2} & \\
\qquad\qquad x^3y^2 & \\
\qquad\qquad \underline{x^3y^2 - x^2y^3} & \\
\qquad\qquad\qquad x^2y^3 & \\
\qquad\qquad\qquad \underline{x^2y^3 - xy^4} & \\
\qquad\qquad\qquad\qquad xy^4 + y^5 & \\
\qquad\qquad\qquad\qquad \underline{xy^4 - y^5} & \\
\qquad\qquad\qquad\qquad\qquad 2\,y^5 &
\end{array}$$

39.
$$\begin{array}{l|l}
x^5 - y^5 & x + y \\ \cline{2-2}
\underline{x^5 + x^4y} & \\
\quad -x^4y & \underline{x^4 - x^3y + x^2y^2 - xy^3 + y^4 - \dfrac{2\,y^5}{x + y}} \\
\quad \underline{-x^4y - x^3y^2} & \\
\qquad\qquad x^3y^2 & \\
\qquad\qquad \underline{x^3y^2 + x^2y^3} & \\
\qquad\qquad\qquad -x^2y^3 & \\
\qquad\qquad\qquad \underline{-x^2y^3 - xy^4} & \\
\qquad\qquad\qquad\qquad xy^4 - y^5 & \\
\qquad\qquad\qquad\qquad \underline{xy^4 + y^5} & \\
\qquad\qquad\qquad\qquad\qquad -2\,y^5 &
\end{array}$$

40.
$$\begin{array}{l|l}
x^5 - y^5 & x - y \\ \hline
\underline{x^5 - x^4y} & x^4 + x^3y + x^2y^2 + xy^3 + y^4 \\
\quad x^4y & \\
\quad \underline{x^4y - x^3y^2} & \\
\qquad x^3y^2 & \\
\qquad \underline{x^3y^2 - x^2y^3} & \\
\qquad\quad x^2y^3 & \\
\qquad\quad \underline{x^2y^3 - xy^4} & \\
\qquad\qquad xy^4 - y^5 & \\
\qquad\qquad \underline{xy^4 - y^5} &
\end{array}$$

41.
$$\begin{array}{l|l}
x^{2a} - 5x^a + 6 & x^a - 3 \\ \hline
\underline{x^{2a} - 3x^a} & x^a - 2 \\
\quad -2x^a + 6 & \\
\quad \underline{-2x^a + 6} &
\end{array}$$

42.
$$\begin{array}{l|l}
x^{6a} - 7x^{3a} + 12 & x^{3a} - 4 \\ \hline
\underline{x^{6a} - 4x^{3a}} & x^{3a} - 3 \\
\quad -3x^{3a} + 12 & \\
\quad \underline{-3x^{3a} + 12} &
\end{array}$$

43.
$$\begin{array}{l|l}
x^{2a} + 3x^a + 2x^{a+1} + 3x & x^a + 2x \\ \hline
\underline{x^{2a} \qquad\quad + 2x^{a+1}} & x^a + 3 - \dfrac{3x}{x^a + 2x} \\
\quad 3x^a \qquad\qquad + 3x & \\
\quad \underline{3x^a \qquad\qquad + 6x} & \\
\qquad\qquad\qquad -3x &
\end{array}$$

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1. $x + 2a = 6a.$
Whence $x = 4a.$

2. $x + a = b.$
Whence $x = b - a.$

3. $bx + b = 4b.$
Dividing by b,
$x + 1 = 4.$
Whence $x = 3.$

4. $cx + c^2 = 6c^3.$
$x + c = 6c^2.$
Whence $x = 6c^2 - c.$

5. $5(b - x) = 10b.$
$b - x = 2b.$
Whence $x = -b.$

6. $bx - (b + c) = 5b - c.$
$bx - b - c = 5b - c.$
Whence $x = 6.$

7. $3ax - ab = 2ax - ac.$
Dividing by a,
$3x - b = 2x - c.$
Whence $x = b - c.$

8. $4bx - 7a^2b = 6ab^2 + 3bx.$
Dividing by b,
$x - 7a^2 = 6ab.$
Whence $x = 7a^2 + 6ab.$

9. $ax + bx = ac + bc.$
Grouping,
$x(a + b) = c(a + b).$
Whence $x = c.$

10. $a^2x + 1 - a^4 - x = 0.$
Grouping, $x(a^2 - 1) = a^4 - 1.$
Whence $x = a^2 + 1.$

11. $ax + 2ab = 2a^2 + bx.$
Grouping, $ax - bx = 2a^2 - 2ab.$
Whence $x = 2a.$

12. $ax - a^3 - 4 = 3a - x.$
Arranging, $ax + x = a^3 + 3a + 4.$
Dividing by $a + 1$, $x = a^2 - a + 4.$

13. $4b^2c^2 + (a + bx)c = (a - bx)c.$
$4b^2c + a + bx = a - bx.$
Whence $x = -2bc.$

14. $ax - ac + bc = 2ac - 5bc + 2bx.$
Arranging, $ax - 2bx = 3ac - 6bc.$
Whence $x = 3c.$

15. $(x + a)(x + b) = x^2 + 2a^2 + 3ab.$
$x^2 + ax + bx + ab = x^2 + 2a^2 + 3ab.$
Whence $x = 2a.$

16. $15(x - a) - 6(x + a) = 3(5a - 3x).$
Simplifying, $5x - 5a - 2x - 2a = 5a - 3x.$
Whence $x = 2a.$

17. $4x - cx - 8 + 2a + 6c = 6a - 3ac + 2cx.$
Arranging, $4x - 3cx = 4a - 3ac + 8 - 6c.$
Whence $x = a + 2.$

18. $9ab + (x - 3a)(x - 3b) = (x + 3a)(x - 3a) - 9a^2.$
Expanding,
$9ab + x^2 - 3ax - 3bx + 9ab = x^2 - 9a^2 - 9a^2.$
Whence $x = 6a.$

19. $(5a - 4b)x - 5(b^2 + 4a^2 + 6ab) = 10b^2 - 3(2a^2 + 3bx) - a(2x - b).$
Expanding,
$5ax - 4bx - 5b^2 - 20a^2 - 30ab = 10b^2 - 6a^2 - 9bx - 2ax + ab.$
$7ax + 5bx = 14a^2 + 31ab + 15b^2.$
Whence $x = 2a + 3b.$

20. $a^2x + 3ax + 10a = a^3 + x + 3.$
Arranging, $a^2x + 3ax - x = a^3 - 10a + 3.$
Whence $x = a - 3.$

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1. Let x = the number of hours which must elapse before A and B are 120 miles apart.
Then $8x + 10x = 120.$
Whence $x = 6\frac{2}{3}.$

Let $y =$ the number of hours which must elapse before A and B are 180 miles apart.

Then $8y + 10y = 180.$

Whence $y = 10.$

2. Let $r =$ the rate of B in miles per hour.

Then $2r =$ the rate of A in miles per hour.

Therefore $5 \cdot 2r + 5r = 135.$

Whence $r = 9,$

and $2r = 18.$

3. Let $r =$ the rate of B in miles per hour.

Then $r + 2 =$ the rate of A in miles per hour.

Therefore $8(r + 2) + 8r = 96.$

Whence $r = 5,$

and $r + 2 = 7.$

4. Let $r =$ the rate of B in miles per hour.

Then $r + 4 =$ the rate of A in miles per hour.

Therefore $6(r + 4) + 6r = 168.$

Whence $r = 12,$

and $r + 4 = 16.$

5. Let $r =$ the rate of A in miles per hour.

Then $r - 3 =$ the rate of B in miles per hour.

Therefore $r - 3 = \frac{3r}{4}.$

Whence $r = 12,$

and $r - 3 = 9.$

Let $h =$ the required number of hours.

Therefore $12h + 9h = 168.$

Whence $h = 8.$

6. Let $r =$ the rate of B in miles per hour.

Then $2r =$ the rate of A in miles per hour.

Therefore $3 \cdot 2r + 5r = 77.$

Whence $r = 7,$

and $2r = 14.$

A travels $3 \cdot 14$, or 42 miles, and B travels $5 \cdot 7$, or 35 miles.

7. Let $r =$ the rate of A in miles per hour.

Then $r - 4 =$ the rate of B in miles per hour.

Therefore $6 \cdot r = 9(r - 4).$

Whence $r = 12,$

and $r - 4 = 8.$

$6 \cdot 12 + 9 \cdot 8 = 144$ miles, the total distance traveled.

8. Let $x =$ the rate of each in miles per hour.
Therefore $8x + 8x = 144.$
Whence $x = 9.$

9. Let $x =$ the rate of A in miles per hour.
Then $x + 2 =$ the rate of B in miles per hour.
Therefore $9x + 9(x + 2) = 144.$
Whence $x = 7,$
and $x + 2 = 9.$

10. Let $r =$ the rate of A in miles per hour.
Then $3r =$ the rate of B in miles per hour.
Therefore $12 \cdot r + 12 \cdot 3r = 144.$
Whence $r = 3,$
and $3r = 9.$

11. Let $r =$ the rate of A in miles per hour.
Then $r + 4 =$ the rate of B in miles per hour.
Therefore $6 \cdot r + 6(r + 4) = 144.$
Whence $r = 10,$
and $r + 4 = 14.$

12. Let $r =$ the rate of A in miles per hour.
Then $r + 2 =$ the rate of B in miles per hour.
Therefore $8r + 8(r + 2) = 144.$
Whence $r = 8,$
and $r + 2 = 10.$

13. Let $r =$ the rate of B in miles per hour.
Then $r + 4 =$ the rate of A in miles per hour.
Therefore $9(r + 4) + 9r = 144.$
Whence $r = 6,$
and $r + 4 = 10.$

14. Let $r =$ the rate of A in miles per hour.
Then $r + 6 =$ the rate of B in miles per hour.
Therefore $r + 6 = 2r.$
Whence $r = 6,$
and $r + 6 = 12.$
Let $t =$ the required time in hours.
Therefore $t \cdot 6 + t \cdot 12 = 144.$
Whence $t = 8.$

15. Let $h =$ the required number of hours.
Therefore $6 \cdot h + 9(h - 4) = 144.$
Whence $h = 12.$

16. Let $h =$ the required number of hours.
Therefore $7(h-3)+9(h-5)=144.$
Whence $h = 13\frac{1}{8}$ hours.

17. Let $t =$ the required number of hours.
Therefore $3t+4t=42.$
Whence $t=6.$

18. Let $t =$ the required number of hours.
Therefore $8t+10t=72.$
Whence $t=4.$

19. Let $t =$ the required number of hours.
Therefore $37t+38t=285.$
Whence $t=3\frac{4}{5}.$

20. Let $x =$ the number of hours that A travels.
Then $x-4 =$ the number of hours that B travels.
Therefore $8x=10(x-4).$
Whence $x=20.$

21. Let $h =$ the number of hours traveled by the first.
Then $h-2 =$ the number of hours traveled by the second.
Therefore $10h+12(h-2)=108.$
Whence $h=6.$
A has traveled $10\cdot 6$, or 60 miles, and B $12\cdot 4$, or 48 miles.

22. Let $x =$ the time in hours of the passenger train.
Then $x+2 =$ the time in hours of the freight train.
Therefore $42x+24(x+2)=246.$
Whence $x=3.$

23. Let $x =$ the rate of the second messenger in miles per hour.
Therefore $8\cdot 8=6\cdot x.$
Whence $x=10\frac{2}{3}.$

24. Let $h =$ the required number of hours.
Therefore $45h-18h=144.$
Whence $h=5\frac{1}{3}.$

25. $\dfrac{216}{9+15}=9$ hours. $9\cdot 9=81$ miles, A's distance; $9\cdot 15=135$ miles, B's distance.

26. $\frac{5}{6}\cdot 216=180$ miles, B's distance; $\frac{216}{6}=36$ miles, A's distance.

27. $\dfrac{216-56}{2}=80$ miles, B's distance; $80+56=136$ miles, A's distance.

28. $108-24=84$ miles, A's distance; $108+24=132$ miles, B's distance.

29. $\frac{108 + 54}{6} = 27$ miles per hour, A's rate; $\frac{54}{9} = 6$ miles per hour, B's rate.

30. $\frac{108 + 36}{6} = 24$ miles per hour, A's rate; $\frac{72}{6} = 12$ miles per hour, B's rate.

31. $\frac{108 + 12}{6} = 20$ miles per hour, A's rate; $\frac{108 - 12}{6} = 16$ miles per hour, B's rate.

32. $\frac{216 - 96}{6} = 20$ miles per hour, A's rate; $\frac{96}{6} = 16$ miles per hour, B's rate.

33. Let $x =$ the rate of B in miles per hour.
Then $x + 4 =$ the rate of A in miles per hour.
Therefore $12 \cdot (x + 4) + 12 \cdot x = 216.$
Whence $x = 7,$
and $x + 4 = 11.$

34. Let $r =$ the rate of B in miles per hour.
Then $r + 2 =$ the rate of A in miles per hour.
Therefore $6(r + 2) + 6r = 216.$
Whence $r = 17,$
and $r + 2 = 19.$
Then $6 \cdot 17 = 102$ miles, B's distance,
and $6 \cdot 19 = 114$ miles, A's distance.

35. Let $r =$ the rate of B in miles per hour.
Then $r + 4 =$ the rate of A in miles per hour.
Therefore $9(r + 4) + 9r = 216.$
Whence $r = 10,$
and $r + 4 = 14.$
Then $9 \cdot 10 = 90$ miles, B's distance,
and $9 \cdot 14 = 126$ miles, A's distance.

36. Let $v =$ the velocity of the bullet in feet per second.
Therefore $\frac{1650}{v} + \frac{1650}{1100} = \frac{5}{2}.$
Whence $v = 1650.$

37. Let $v =$ the velocity of the bullet in feet per second.
Therefore $\frac{825}{v} + \frac{825}{1100} = \frac{7}{4}.$
Whence $v = 825.$

38. Let $x =$ the distance in feet from the boy to the target.
Therefore $\frac{3000 - x}{1100} = \frac{3000}{1650} + \frac{x}{1100}.$
Whence $x = 500.$

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1. $5a - 7x + 3b - 10c - 14a + 12y - 8x + 12a - 11x + 9b$
$= 3a - 26x + 12b - 10c + 12y.$

2. $2cd - a^2b + 7cd^2 - 12ab + 17ad - cd + 4c^2d$
$= cd + 7cd^2 + 4c^2d - a^2b - 12ab + 17ad.$

3.
$$\begin{array}{rrrr}
10x - & 9 + & 4ax - & 2cd \\
-15x & - & 6ax + & 12cd \\
 & 10 - & ax + & 5cd \\
11x - & 19 - & 7ax - & 6cd \\
\hline
6x - & 18 - & 10ax + & 9cd
\end{array}$$

4.
$$\begin{array}{rrrr}
a^3 + & 4a^2 - & a - & 3 \\
-5a^3 + & 3a^2 - & a & \\
6a^3 - & 3a^2 - & 2a + & 8 \\
-2a^3 - & 5a^2 - & a + & 5 \\
2a^3 - & 3a^2 + & 4a - & 5 \\
\hline
2a^3 - & 4a^2 - & a + & 5
\end{array}$$

5.
$$\begin{array}{l}
Ax^3 + \quad Bx^2 \quad + \quad Cx \\
\qquad\quad\;\; Ax^2 \quad + \quad Bx \quad + C \\
\hline
Ax^3 + (A + B)x^2 + (B + C)x + C
\end{array}$$

6.
$$\begin{array}{rrr}
7(a-b) - & 10(c-d) + & 8(x-y) \\
-4(a-b) - & 6(c-d) - & 7(x-y) \\
7(a-b) - & 15(c-d) + & 12(x-y) \\
\hline
10(a-b) - & 31(c-d) + & 13(x-y)
\end{array}$$

7.
$$\begin{array}{rrr}
-4(a+b) - & 2(x+y) + & 3(x-y) \\
4(a+b) - & 9(x+y) + & 8(x-y) \\
-(a+b) - & 10(x+y) + & (x-y) \\
\hline
-(a+b) - & 21(x+y) + & 12(x-y)
\end{array}$$

8.
$$\begin{array}{rrrr}
\frac{1}{3}ax + & \frac{2}{3}ax^2 - & \frac{5}{6}a^2x^2 + & a \\
-ax + & \frac{1}{6}ax^2 + & 2a^2x^2 - & 9a \\
 & \frac{1}{2}ax^2 + & a^2x^2 - & \frac{1}{4}a \\
\hline
-\frac{2}{3}ax + & \frac{4}{3}ax^2 + & 2\frac{1}{6}a^2x^2 - & 8\frac{1}{4}a
\end{array}$$

9.
$$\begin{array}{rrrr}
x^2 + & 10xy - & 5y^2 - & 9 \\
2x^2 - & 3xy + & y^2 - & 1 \\
\hline
-x^2 + & 13xy - & 6y^2 - & 8
\end{array}$$

10.
$$\begin{array}{rrrrr}
-a^2y + & 12ay^2 - & ay - & 15 & \\
3a^2y - & 7ay^2 - & 5ay & & +z \\
\hline
-4a^2y + & 19ay^2 + & 4ay - & 15 & -z
\end{array}$$

11.
$$\begin{array}{rrrrr}
3ab^2c^2d^3 - 12 & & & & \\
-4ab^2c^2d^3 & -17a^2bcd^2 & + 3abcd & + ax & \\
\hline
7ab^2c^2d^3 - 12 & + 17a^2bcd^2 & - 3abcd & - ax &
\end{array}$$

12.

$$\begin{array}{r} 0 \\ \underline{27\,xy + 13\,z - 12 + ab} \\ -27\,xy - 13\,z + 12 - ab \end{array}$$

13. $(a+b)-(a-b)+4\,a-(a+3\,b)+c$
$= a+b-a+b+4\,a-a-3\,b+c$
$= 3\,a-b+c.$

14. $15\,x-2\,(12\,x-16\,y)+xy+3\,(3\,x-4\,xy)-x$
$= 14\,x-24\,x+32\,y+xy+9\,x-12\,xy$
$= -x-11\,xy+32\,y.$

15. $ab-(7\,x+4\,y-2\,z)+42+5\,(5\,ab-x-y)$
$= ab-7\,x-4\,y+2\,z+42+25\,ab-5\,x-5\,y$
$= 26\,ab-12\,x-9\,y+2\,z+42.$

16. $[x-(-4\,x+8\,y+2\,z)+b-4\,(3\,x-z)]-5\,b$
$= x+4\,x-8\,y-2\,z+b-12\,x+4\,z-5\,b$
$= -7\,x-8\,y+2\,z-4\,b.$

17. $[ab-\{-5\,ac+7\,(d-a)+4\,ca+7\,d\}-6\,ba]+13\,a$
$= ab-\{-ac+14\,d-7\,a\}-6\,ba+13\,a$
$= -5\,ab+ac-14\,d+7\,a+13\,a$
$= 20\,a-5\,ab+ac-14\,d.$

18. $\{5\,x^2-2\,[-4-(2\,x^2-3\,x)]+5\}-(x^2+4\,x-2)$
$= 5\,x^2-2\,(-4-2\,x^2+3\,x)+5-x^2-4\,x+2$
$= 4\,x^2+8+4\,x^2-6\,x+7-4\,x$
$= 8\,x^2-10\,x+15.$

19.

$$\begin{array}{lrrrrrl} & x^4 & -4\,x^2y^2 & +4\,y^4 & & & (1) \\ & x^4 & +4\,x^2y^2 & +4\,y^4 & & & (2) \\ \hline & x^8 & -4\,x^6y^2 & +4\,x^4y^4 & & & \\ & & +4\,x^6y^2 & -16\,x^4y^4 & +16\,x^2y^6 & & \\ & & & +4\,x^4y^4 & -16\,x^2y^6 & +16\,y^8 & \\ \hline (1)\cdot(2), & x^8 & & -8\,x^4y^4 & & +16\,y^8 & \end{array}$$

20.

$$\begin{array}{lrrrrrrrl} & 4\,x^4 & -x^2 & +5 & & & & & (1) \\ & 3\,x^3 & -x & +7 & & & & & (2) \\ \hline & 12\,x^7 & -3\,x^5 & & +15\,x^3 & & & & \\ & & -4\,x^5 & & +x^3 & & -5\,x & & \\ & & & +28\,x^4 & & -7\,x^2 & & +35 & \\ \hline (1)\cdot(2), & 12\,x^7 & -7\,x^5 & +28\,x^4 & +16\,x^3 & -7\,x^2 & -5\,x & +35 & \end{array}$$

$$
\begin{array}{ll}
\textbf{21.} & a^2 + b^2 + c^2 - ab - ac - bc \qquad (1)\\
 & a + b + c \qquad (2)\\
\hline
 & a^3 + ab^2 + ac^2 - a^2b - a^2c - abc\\
 & \quad - ab^2 + a^2b - abc + b^3 + bc^2 - b^2c\\
 & \quad - ac^2 + a^2c - abc - bc^2 + b^2c + c^3\\
\hline
(1)\cdot(2), & a^3 - 3abc + b^3 + c^3
\end{array}
$$

$$
\begin{array}{ll}
\textbf{22.} & x^6 - x^5y + x^4y^2 - x^3y^3 + x^2y^4 - xy^5 + y^6 \qquad (1)\\
 & x + y \qquad (2)\\
\hline
 & x^7 - x^6y + x^5y^2 - x^4y^3 + x^3y^4 - x^2y^5 + xy^6\\
 & \quad + x^6y - x^5y^2 + x^4y^3 - x^3y^4 + x^2y^5 - xy^6 + y^7\\
\hline
(1)\cdot(2), & x^7 + y^7
\end{array}
$$

$$
\begin{array}{ll}
\textbf{23.} & -9x^5 - 4x^4 - 3x^3 + x^2 + 5x + 10 \qquad (1)\\
 & 2x^4 - x^3 - x^2 + 3x \qquad (2)\\
\hline
 & -18x^9 - 8x^8 - 6x^7 + 2x^6 + 10x^5 + 20x^4\\
 & \quad + 9x^8 + 4x^7 + 3x^6 - x^5 - 5x^4 - 10x^3\\
 & \quad + 9x^7 + 4x^6 + 3x^5 - x^4 - 5x^3 - 10x^2\\
 & \quad - 27x^6 - 12x^5 - 9x^4 + 3x^3 + 15x^2 + 30x\\
\hline
(1)\cdot(2), & -18x^9 + x^8 + 7x^7 - 18x^6 + 5x^4 - 12x^3 + 5x^2 + 30x
\end{array}
$$

$$
\begin{array}{lll}
\textbf{24.} & 3x^2 - 4y \quad (1) & 9x^4 - 16y^2 \quad (3)\\
 & 3x^2 + 4y \quad (2) & 9x^4 + 16y^2 \quad (4)\\
\hline
 & 9x^4 - 12x^2y & 81x^8 - 144x^4y^2\\
 & \quad + 12x^2y - 16y^2 & \quad + 144x^4y^2 - 256y^4\\
\hline
(1)\cdot(2), & 9x^4 - 16y^2 \quad (3) & (3)\cdot(4)\ \ 81x^8 - 256y^4
\end{array}
$$

$$
\begin{array}{ll}
\textbf{25.} & a - 3b + 2c \qquad (1)\\
 & a - 3b + 2c \qquad (2)\\
\hline
 & a^2 - 3ab + 2ac\\
 & \quad - 3ab + 9b^2 - 6bc\\
 & \quad + 2ac - 6bc + 4c^2\\
\hline
(1)\cdot(2), & a^2 - 6ab + 4ac + 9b^2 - 12bc + 4c^2
\end{array}
$$

$$
\begin{array}{ll}
\textbf{26.} & x - y + z \qquad (1)\\
 & x - y + z \qquad (2)\\
\hline
 & x^2 - xy + xz\\
 & \quad - xy + y^2 - yz\\
 & \quad + xz - yz + z^2\\
\hline
(1)\cdot(2), & x^2 - 2xy + 2xz + y^2 - 2yz + z^2 \qquad (3)\\
 & x - y + z \qquad (4)\\
\hline
 & x^3 - 2x^2y + 2x^2z + xy^2 - 2xyz + xz^2\\
 & \quad - x^2y + 2xy^2 - 2xyz - y^3 + 2y^2z - yz^2\\
 & \quad + x^2z - 2xyz + 2xz^2 + y^2z - 2yz^2 + z^3\\
\hline
(3)\cdot(4), & x^3 - 3x^2y + 3x^2z + 3xy^2 - 6xyz + 3xz^2 - y^3 + 3y^2z - 3yz^2 + z^3
\end{array}
$$

27.
$$\begin{array}{lllr}
 & x^a - y^b & & (1)\\
 & \underline{x^a + y^b} & & (2)\\
 & x^{2a} - x^a y^b & & \\
 & \underline{\qquad + x^a y^b - y^{2b}} & & \\
(1)\cdot(2), & x^{2a} \qquad\qquad - y^{2b} & &
\end{array}$$

28.
$$\begin{array}{llr}
 & x^a + 2\,y^b + 3\,z^c & (1)\\
 & \underline{x^a - 3\,y^b} & (2)\\
 & x^{2a} + 2\,x^a y^b + 3\,x^a z^c & \\
 & \underline{\qquad - 3\,x^a y^b \qquad\qquad - 6\,y^{2b} - 9\,y^b z^c} & \\
(1)\cdot(2), & x^{2a} - \quad x^a y^b + 3\,x^a z^c - 6\,y^{2b} - 9\,y^b z^c &
\end{array}$$

29.
$$\begin{array}{r|l}
6\,a^5 - 13\,a^4 + 4\,a^3 + 3\,a^2 & \underline{-a^2 + a}\\
\underline{6\,a^5 - \ \ 6\,a^4 \qquad\qquad} & -6\,a^3 + 7\,a^2 + 3\,a\\
-\ \ 7\,a^4 + 4\,a^3 \qquad & \\
\underline{-\ \ 7\,a^4 + 7\,a^3 \qquad} & \\
-3\,a^3 + 3\,a^2 & \\
\underline{-3\,a^3 + 3\,a^2} &
\end{array}$$

30.
$$\begin{array}{r|l}
-30\,a^4 - 11\,a^3 + 82\,a^2 + 12\,a - 48 & \underline{3\,a^2 + 2\,a - 4}\\
\underline{-30\,a^4 - 20\,a^3 + 40\,a^2 \qquad\qquad} & -10\,a^2 + 3\,a + 12\\
9\,a^3 + 42\,a^2 + 12\,a \qquad & \\
\underline{9\,a^3 + \ \ 6\,a^2 - 12\,a \qquad} & \\
36\,a^2 + 24\,a - 48 & \\
\underline{36\,a^2 + 24\,a - 48} &
\end{array}$$

31.
$$\begin{array}{r|l}
10\,x^6 - 11\,x^5 - \ \ 3\,x^4 + 20\,x^3 + 10\,x^2 + 2 & \underline{5\,x^3 - 3\,x^2 + 2\,x - 2}\\
\underline{10\,x^6 - \ \ 6\,x^5 + \ \ 4\,x^4 - \ \ 4\,x^3 \qquad\qquad\qquad} & 2\,x^3 - x^2 - 2\,x + 4 + \dfrac{24\,x^2 - 12\,x + 10}{5\,x^3 - 3\,x^2 + 2\,x - 2}\\
-\ \ 5\,x^5 - \ \ 7\,x^4 + 24\,x^3 + 10\,x^2 \qquad & \\
\underline{-\ \ 5\,x^5 + \ \ 3\,x^4 - \ \ 2\,x^3 + \ \ 2\,x^2 \qquad} & \\
-10\,x^4 + 26\,x^3 + \ \ 8\,x^2 + 2 & \\
\underline{-10\,x^4 + \ \ 6\,x^3 - \ \ 4\,x^2 + \ \ 4\,x \qquad} & \\
20\,x^3 + 12\,x^2 - \ \ 4\,x + \ \ 2 & \\
\underline{20\,x^3 - 12\,x^2 + \ \ 8\,x - \ \ 8} & \\
24\,x^2 - 12\,x + 10 &
\end{array}$$

32.
$$\begin{array}{r|l}
a^8 - 3\,a^6 + \quad a^4 - \quad 7\,a^2 + 3 & \underline{a^4 - 2\,a^2 + 1}\\
\underline{a^8 - 2\,a^6 + \quad a^4 \qquad\qquad\qquad} & a^4 - a^2 - 2 + \dfrac{5 - 10\,a^2}{a^4 - 2\,a^2 + 1}\\
-\quad a^6 \qquad\qquad - \ \ 7\,a^2 + 3 & \\
\underline{-\quad a^6 + 2\,a^4 - \qquad a^2 \qquad} & \\
-2\,a^4 - \ \ 6\,a^2 + 3 & \\
\underline{-2\,a^4 + \ \ 4\,a^2 - 2} & \\
-10\,a^2 + 5 &
\end{array}$$

$$\begin{array}{l}
a^2 - 1\\
\underline{a^2 - 1}\\
a^4 - \quad a^2\\
\underline{\quad - \quad a^2 + 1}\\
a^4 - 2\,a^2 + 1
\end{array}$$

33.

$$\begin{array}{l|l}
a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 7ab^4 - b^5 & a^2 - 2ab + b^2 \\ \hline
\underline{a^5 - 2a^4b + a^3b^2} & a^3 - 3a^2b + 3ab^2 - b^3 + \dfrac{2ab^4}{a^2 - 2ab + b^2} \\
-3a^4b + 9a^3b^2 - 10a^2b^3 & \\
\underline{-3a^4b + 6a^3b^2 - 3a^2b^3} & \\
3a^3b^2 - 7a^2b^3 + 7ab^4 & \\
\underline{3a^3b^2 - 6a^2b^3 + 3ab^4} & \\
-a^2b^3 + 4ab^4 - b^5 & \\
\underline{-a^2b^3 + 2ab^4 - b^5} & \\
2ab^4 &
\end{array}$$

$$\begin{array}{l}
a - b \\
\underline{a - b} \\
a^2 - ab \\
\underline{- ab + b^2} \\
a^2 - 2ab + b^2
\end{array}$$

34.

$$\begin{array}{l}
6a^2 + 5a - 6 \\
\underline{2a^2 - 13a + 20} \\
12a^4 + 10a^3 - 12a^2 \\
\quad - 78a^3 - 65a^2 + 78a \\
\underline{\quad\quad + 120a^2 + 100a - 120} \\
12a^4 - 68a^3 + 43a^2 + 178a - 120
\end{array}
\qquad
\begin{array}{l}
3a - 2 \\
\underline{2a - 5} \\
6a^2 - 4a \\
\underline{\quad - 15a + 10} \\
6a^2 - 19a + 10
\end{array}$$

$$\begin{array}{l|l}
12a^4 - 68a^3 + 43a^2 + 178a - 120 & 6a^2 - 19a + 10 \\ \hline
\underline{12a^4 - 38a^3 + 20a^2} & 2a^2 - 5a - 12 \\
-30a^3 + 23a^2 + 178a & \\
\underline{-30a^3 + 95a^2 - 50a} & \\
-72a^2 + 228a - 120 & \\
\underline{-72a^2 + 228a - 120} &
\end{array}$$

35.

$$\begin{array}{l|l}
x^3 + 36xy - 27y^3 + 64 & x - 3y + 4 \\ \hline
\underline{x^3 - 3x^2y + 4x^2} & x^2 + 3xy - 4x + 9y^2 + 12y + 16 \\
3x^2y - 4x^2 + 36xy & \\
\underline{3x^2y - 9xy^2 + 12xy} & \\
-4x^2 + 9xy^2 + 24xy & \\
\underline{-4x^2 + 12xy - 16x} & \\
9xy^2 + 12xy + 16x - 27y^3 + 64 & \\
\underline{9xy^2 - 27y^3 + 36y^2} & \\
12xy + 16x + 64 - 36y^2 & \\
\underline{12xy - 36y^2 + 48y} & \\
16x + 64 - 48y & \\
\underline{16x + 64 - 48y} &
\end{array}$$

36. $\quad 6y - 20 + 5y - 18 = 36y - 4 - 40y + 9.$

Collecting, $\quad 11y - 38 = -4y + 5.$

Whence $\quad y = 2\frac{13}{15}.$

37. $\quad 4y - 3(6 - y) + 2 = 6(y - 2) - 13y + 24.$

Simplifying, $\quad 4y - 18 + 3y = 6y - 12 - 13y + 22.$

Whence $\quad y = 2.$

38. $$6y - 4(1 - 2y) + 10 = 4(2y - 5) - 4y + 1.$$
Simplifying, $$6y - 4 + 8y + 9 = 8y - 20 - 4y.$$
Whence $$y = -2\tfrac{1}{2}.$$

39. $$(y + 7)(y - 11) = (y + 8)(y - 5) - 2.$$
Expanding, $$y^2 - 4y - 77 = y^2 + 3y - 40 - 2.$$
Whence $$y = -5.$$

40. $$(y - 3)^3 = (y^2 - 6y + 9)(5 + y) - 8y^2.$$
Expanding, $$y^3 - 9y^2 + 27y - 27 = y^3 - y^2 - 21y + 45 - 8y^2.$$
Whence $$y = \tfrac{3}{2}.$$

41. $y^2 + a^2 - (y + 1)^2 - (a + 1)^2 = 0.$
Expanding,
$$y^2 + a^2 - y^2 - 2y - 1 - a^2 - 2a - 1 = 0.$$
Whence $$y = -a - 1.$$

42. $$3ay + 6ac + 4b^2 = 3ab + 4by + 8bc.$$
Arranging, $$3ay - 4by = 3ab - 4b^2 - 6ac + 8bc.$$
Whence $$y = b - 2c.$$

43. $x = \dfrac{-10 + \sqrt{100 + 44}}{2} = 1.$

44. $x = \dfrac{6 + \sqrt{36 + 160}}{10} = \dfrac{6 + 14}{10} = 2.$

45. $x = \dfrac{-5 - \sqrt{25 + 96}}{6} = \dfrac{-5 - 11}{6} = -2\tfrac{2}{3}.$

46. $x = \dfrac{-7 - \sqrt{49 + 176}}{4} = \dfrac{-7 - 15}{4} = -5\tfrac{1}{2}.$

47. $$12s = \tfrac{1}{2} \cdot 18(20)^2.$$
Whence $$s = 300.$$

48. $e = \frac{1}{2} \cdot 7(32.2 \times 40)^2 = \frac{1}{2} \cdot 7(1288)^2 = \frac{1}{2} \cdot 7(1{,}658{,}944) = 5{,}806{,}304.$

49. $s = \dfrac{3 + 4 + 5}{2} = 6.$
$$\sqrt{s(s - a)(s - b)(s - c)} = \sqrt{6 \cdot 3 \cdot 2 \cdot 1} = 6.$$

50. Let $n =$ the number.
Then $n + 1$ and $n + 2 =$ the next two numbers.
Therefore $$n^2 + 38 = n^2 + 3n + 2.$$
Whence $$n = 12,$$
$$n + 1 = 13,$$
and $$n + 2 = 14.$$

51. Let $n =$ a certain odd number.
Then $n + 2$ and $n + 4 =$ the next two odd numbers.
Therefore $n^2 + 98 = n^2 + 6n + 8$.
Whence $n = 15$, the first odd number,
$n + 2 = 17$, the second odd number,
and $n + 4 = 19$, the third odd number.

52. Let $n =$ an odd number.
Then $n + 1$ and $n + 3 =$ the next two even numbers.
Therefore $n^2 + 47 = n^2 + 4n + 3$.
Whence $n = 11$, the odd number,
$n + 1 = 12$, the next even number,
and $n + 3 = 14$, the second even number.

53. Let $n =$ the first number.
Then $n + 1$, $n + 2$, and $n + 3 =$ the next three numbers.
Therefore $n^2 + n + 42 = n^2 + 5n + 6$.
Whence $n = 9$, the first number,
$n + 1 = 10$, the second number,
$n + 2 = 11$, the third number,
and $n + 3 = 12$, the fourth number.

54. Let $x =$ the number of years sought.
Therefore $42 + x = 2(15 + x)$.
Whence $x = 12$.

55. Let $x =$ the number of years sought.
Therefore $36 - x = 3(16 - x)$.
Whence $x = 6$.

56. Let $x =$ the number of years sought.
Therefore $52 + x = 2(21 + x)$.
Whence $x = 10$.

57. Let $x =$ the number of years sought.
Therefore $87 - x = 4(42 - x)$.
Whence $x = 27$.

58. Let $q =$ the number of quarters.
Then $2q =$ the number of nickels,
and $32 - 3q =$ the number of dimes.
Then $25q =$ the value of quarters in cents,
$10q =$ the value of nickels in cents,
and $320 - 30q =$ the value of dimes in cents.
Therefore $25q + 10x + 320 - 30q = 360$.

Whence $q = 8,$

$2q = 16,$

and $32 - 3q = 8.$

59. Let $d =$ the distance to the target in feet.

Therefore $$\frac{d}{1280} + \frac{d}{1120} = 6.$$

Whence $d = 3584.$

60. Let $x =$ age of each.

$(x + 12)3 - 36 + x =$ the result.

$4x =$ the result.

Therefore he divided each result by 4.

61. Let $x =$ the per cent over 60 years.

Then $9x =$ the per cent between 20 and 60,

and $9x + 5 =$ the per cent under 20.

Therefore $x + 9x + 9x + 5 = 100.$

Whence $x = 5,$

$9x = 45,$

and $9x + 5 = 50.$

62. Let $x =$ the length in feet of the Hoosac tunnel.

Then $2x - 1320 =$ the length in feet of the St. Gothard tunnel,

and $2x - 10{,}320 =$ the length in feet of the Mount Cenis tunnel.

Therefore $x + 2x - 1320 + 2x - 10{,}320 = 113{,}760.$

Whence $x = 25{,}080,$

$2x - 1320 = 48{,}840,$

and $2x - 10{,}320 = 39{,}840.$

63. Let $x =$ the height in feet of Niagara Falls.

Then $2x + 32 =$ the height in feet of falls of Zambesi,

$8x + 188 =$ the height in feet of falls of Yosemite,

and $8x - 332 =$ the height in feet of Staubbach Falls.

Therefore $x + 2x + 32 + 8x + 188 + 8x - 332 = 3004.$

Whence $x = 164,$

$2x + 32 = 360,$

$8x + 188 = 1500,$

and $8x - 332 = 980.$

64. Let $x =$ the weight in pounds of cubic foot of water.

Then $x - 11.5 =$ the weight in pounds of cubic foot of alcohol,

and $16x - 151.3 =$ the weight in pounds of cubic foot of mercury.

Therefore $x + x - 11.5 + 16x - 151.3 = 962.2.$

Whence $x = 62.5,$

$x - 11.5 = 51,$

and $16x - 151.3 = 848.7.$

65. Let $x =$ the rainfall of San Diego in inches.

Then $5x - 1 =$ the rainfall of Boston in inches,

$5x - 2 =$ the rainfall of St. Louis in inches,

and $15x - 12 =$ the rainfall of Neahbay in inches.

Therefore $x + 5x - 1 + 5x - 2 + 15x - 12 = 219.$

Whence $x = 9,$

$5x - 1 = 44,$

$5x - 2 = 43,$

and $15x - 12 = 123.$

66. Let $x =$ the number of years in the expectation of one at 65.

Then $2x - .94 =$ the number of years in the expectation of one at 45,

$2x + 14.37 =$ the number of years in the expectation of one at 10,

and $2x + 8.98 =$ the number of years in the expectation of one at 21.

Therefore $x + 2x - .94 + 2x + 14.37 + 2x + 8.98 = 109.42.$

Whence $x = 12.43,$

$2x - .94 = 23.92,$

$2x + 14.37 = 39.23,$

and $2x + 8.98 = 33.84.$

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3. $15^2 = (10 + 5)^2 = 100 + 25 + 2 \cdot 10 \cdot 5 = 225.$
4. $17^2 = (20 - 3)^2 = 400 + 9 - 2 \cdot 20 \cdot 3 = 289.$
5. $18^2 = (20 - 2)^2 = 400 + 4 - 2 \cdot 20 \cdot 2 = 324.$
6. $19^2 = (20 - 1)^2 = 400 + 1 - 2 \cdot 20 \cdot 1 = 361.$
7. $(x + y)^2 = x^2 + y^2 + 2xy.$
8. $(x - c)^2 = x^2 + c^2 - 2xc.$
9. $(x + 1)^2 = x^2 + 1 + 2x.$
10. $(a + 2)^2 = a^2 + 4a + 4.$
11. $(b^2 + 3)^2 = b^4 + 6b^2 + 9.$
12. $(c - 4)^2 = c^2 - 8c + 16.$
13. $(2c^2 + d)^2 = 4c^4 + 4c^2d + d^2.$
14. $(3x - y)^2 = 9x^2 - 6xy + y^2.$
15. $(x + 2y)^2 = x^2 + 4xy + 4y^2.$
16. $(a - 4b)^2 = a^2 - 8ab + 16b^2.$
17. $(y^5 - 2)^2 = y^{10} - 4y^5 + 4.$
18. $(2a + 1)^2 = 4a^2 + 4a + 1.$
19. $(3a^2 + 2)^2 = 9a^4 + 12a^2 + 4.$
20. $(4a + 3)^2 = 16a^2 + 24a + 9.$
21. $(2a^2 - 3b)^2 = 4a^4 - 12a^2b + 9b^2.$
22. $(3c^2 - 4a^2)^2 = 9c^4 - 24c^2a^2 + 16a^4.$
23. $(2x^3 - 6)^2 = 4x^6 - 24x^3 + 36.$
24. $(3xy + y^2)^2 = 9x^2y^2 + 6xy^3 + y^4.$
25. $(7x^2 - 3y^4)^2 = 49x^4 - 42x^2y^4 + 9y^8.$
26. $(8y^3 + 2bx)^2 = 64y^6 + 32y^3bx + 4b^2x^2.$
27. $(6xy - 2a^2)^2 = 36x^2y^2 - 24xya^2 + 4a^4.$

28. $(4xy - 2x^2)^2 = 16x^2y^2 - 16x^3y + 4x^4.$

29. $(3x^4 - 5x^6)^2 = 9x^8 - 30x^{10} + 25x^{12}.$

30. $(2x^2 - 6x^3y)^2 = 4x^4 - 24x^5y + 36x^6y^2.$

31. $41^2 = (40 + 1)^2 = 1681.$

32. $92^2 = (90 + 2)^2 = 8464.$

33. $101^2 = (100 + 1)^2 = 10{,}201.$

34. $202^2 = (200 + 2)^2 = 40{,}804.$

35. $1001^2 = (1000 + 1)^2 = 1{,}002{,}001.$

36. $\dfrac{a^2 + 2ab + b^2}{a + b} = a + b.$

37. $\dfrac{a^2 - 2ac + c^2}{a - c} = a - c.$

38. $\dfrac{a^2 - 4a + 4}{a - 2} = a - 2.$

39. $\dfrac{9x^2 - 6x + 1}{3x - 1} = 3x - 1.$

40. $\dfrac{y^2 + 40y + 400}{20 + y} = 20 + y.$

41. $\dfrac{9x^2 - 12xy + 4y^2}{2y - 3x} = 2y - 3x.$

42. $\dfrac{4a^2 - 20a + 25}{-5 + 2a} = -5 + 2a.$

43. $\dfrac{4x^2 - 12xy + 9y^2}{3y - 2x} = 3y - 2x.$

44. $(x^2 - 2xy + y^2) \div (x - y) = x - y.$

45. $(a^2 + 2a + 1) \div (a + 1) = a + 1.$

46. $(a^2 + 4a + 4) \div (a + 2) = a + 2.$

47. $(4x^2 + 4x + 1) \div (2x + 1) = 2x + 1.$

48. $(16x^2 - 8x + 1) \div (4x - 1) = 4x - 1.$

49. $(16x^2 - 8xy + y^2) \div (4x - y) = 4x - y.$

50. $(9x^4 - 6x^2 + 1) \div (3x^2 - 1) = 3x^2 - 1.$

51. $(16y^6 - 40y^3 + 25) \div (4y^3 - 5) = 4y^3 - 5.$

52. $c^2 + 2cd + d^2 = (c + d)(c + d).$

53. $a^2 - 2a + 1 = (a - 1)(a - 1).$

54. $x^2 - 10x + 25 = (x - 5)(x - 5).$

55. $4a^2 - 4a + 1 = (2a - 1)(2a - 1).$

56. $4x^2 + 12x + 9 = (2x + 3)(2x + 3).$

57. $9a^2b^2 - 6ab + 1 = (3ab - 1)(3ab - 1).$

58. $16x^4y^2 - 24x^2ya + 9a^2 = (4x^2y - 3a)(4x^2y - 3a).$

59. $25a^4b^6 - 60a^2b^3c^2 + 36c^4 = (5a^2b^3 - 6c^2)(5a^2b^3 - 6c^2).$

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1.
$$\begin{aligned}(a + b + c)^2 &= [(a + b) + c][(a + b) + c]\\ &= (a + b)^2 + 2c(a + b) + c^2\\ &= a^2 + 2ab + b^2 + 2ac + 2bc + c^2.\end{aligned}$$

2.
$$\begin{aligned}(a + b - c)^2 &= [(a + b) - c]^2\\ &= (a + b)^2 - 2c(a + b) + c^2\\ &= a^2 + 2ab + b^2 - 2ac - 2bc + c^2.\end{aligned}$$

3.
$$\begin{aligned}(a + b - 1)^2 &= [(a + b) - 1]^2\\ &= (a + b)^2 - 2(a + b) + 1\\ &= a^2 + 2ab + b^2 - 2a - 2b + 1.\end{aligned}$$

4. $(a+b+1)^2 = [(a+b)+1]^2$
$= (a+b)^2 + 2(a+b) + 1$
$= a^2 + 2ab + b^2 + 2a + 2b + 1.$

5. $(a-b+c)^2 = [(a-b)+c]^2$
$= (a-b)^2 + 2c(a-b) + c^2$
$= a^2 - 2ab + b^2 + 2ac - 2bc + c^2.$

6. $(a-b-c)^2 = [(a-b)-c]^2$
$= (a-b)^2 - 2c(a-b) + c^2$
$= a^2 - 2ab + b^2 - 2ac + 2bc + c^2.$

7. $(a+2b-3)^2 = [(a+2b)-3]^2$
$= (a+2b)^2 - 2 \cdot 3(a+2b) + 9$
$= a^2 + 4ab + 4b^2 - 6a - 12b + 9.$

8. $(a-2b+c)^2 = [(a-2b)+c]^2$
$= (a-2b)^2 + 2c(a-2b) + c^2$
$= a^2 - 4ab + 4b^2 + 2ac - 4bc + c^2.$

9. $(2a+b-4c)^2 = [(2a+b)-4c]^2$
$= (2a+b)^2 - 8c(2a+b) + 16c^2$
$= 4a^2 + 4ab + b^2 - 16ac - 8bc + 16c^2.$

10. $(3a-2b+5c)^2 = [(3a-2b)+5c]^2$
$= (3a-2b)^2 + 10c(3a-2b) + 25c^2$
$= 9a^2 - 12ab + 4b^2 + 30ac - 20bc + 25c^2.$

11. $(2a-3b-4c)^2 = [(2a-3b)-4c]^2$
$= (2a-3b)^2 - 2 \cdot 4c(2a-3b) + 16c^2$
$= 4a^2 - 12ab + 9b^2 - 16ac + 24bc + 16c^2.$

12. $(4c-1+2ab)^2 = [(4c-1)+2ab]^2$
$= (4c-1)^2 + 2 \cdot 2ab(4c-1) + 4a^2b^2$
$= 16c^2 - 8c + 1 + 16abc - 4ab + 4a^2b^2.$

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1. $(6+3)(6-3) = 36 - 9 = 27.$
2. $(8+4)(8-4) = 64 - 16 = 48.$
3. $(9-2)(9+2) = 81 - 4 = 77.$
4. $(10-4)(10+4) = 100 - 16 = 84.$
5. $(7+2)(7-2) = 49 - 4 = 45.$
6. $(12-3)(12+3) = 144 - 9 = 135.$
8. $35 \cdot 25 = (30+5)(30-5) = 900 - 25 = 875.$
9. $33 \cdot 27 = (30+3)(30-3) = 900 - 9 = 891.$
10. $36 \cdot 44 = (40-4)(40+4) = 1600 - 16 = 1584.$
11. $35 \cdot 45 = (40-5)(40+5) = 1600 - 25 = 1575.$
12. $52 \cdot 48 = (50+2)(50-2) = 2500 - 4 = 2496.$
13. $65 \cdot 75 = (70-5)(70+5) = 4900 - 25 = 4875.$
14. $72 \cdot 68 = (70+2)(70-2) = 4900 - 4 = 4896.$
15. $75 \cdot 85 = (80-5)(80+5) = 6400 - 25 = 6375.$
16. $97 \cdot 103 = (100-3)(100+3) = 10{,}000 - 9 = 9991.$

17. $(x+3)(x-3)=x^2-9$.
18. $(a+5)(a-5)=a^2-25$.
19. $(2x+4)(2x-4)=4x^2-16$.
20. $(3n+5)(3n-5)=9n^2-25$.
21. $(x+y)(x-y)=x^2-y^2$.
22. $(x-a)(x+a)=x^2-a^2$.
23. $(x-1)(x+1)=x^2-1$.
24. $(a+2)(a-2)=a^2-4$.
25. $(a^2+3)(a^2-3)=a^4-9$.
26. $(4+y^2)(4-y^2)=16-y^4$.
27. $(4-x)(x+4)=16-x^2$.
28. $(2c+a)(2c-a)=4c^2-a^2$.
29. $(3a+b)(b-3a)=b^2-9a^2$.
30. $(4b+2c)(4b-2c)=16b^2-4c^2$.
31. $(3xy-2)(3xy+2)=9x^2y^2-4$.
32. $(4ab-3)(4ab+3)=16a^2b^2-9$.
33. $(a^4-b^3)(a^4+b^3)=a^8-b^6$.
34. $(a^6+a^3)(-a^3+a^6)=a^{12}-a^6$.
35. $(x^3-2y)(x^3+2y)=x^6-4y^2$.
36. $(4ab-a^2)(4ab+a^2)=16a^2b^2-a^4$.
37. $(3cd^2+2d)(-2d+3cd^2)=9c^2d^4-4d^2$.
38. $(6cd+3)(-3+6cd)=36c^2d^2-9$.
39. $(4xy+2y)(2y-4xy)=4y^2-16x^2y^2$.
40. $(3abc-2bc)(2bc+3abc)=9a^2b^2c^2-4b^2c^2$.
41. $(a^2-b^2)\div(a+b)=a-b$.
42. $(c^2-d^2)\div(c-d)=c+d$.
43. $(9-b^2)\div(3+b)=3-b$.
44. $(36-a^2)\div(6-a)=6+a$.
45. $(9x^2-16)\div(3x+4)=3x-4$.
46. $(x^4-1)\div(x^2-1)=x^2+1$.
47. $(x^2-y^2)\div(x-y)=x+y$.
48. $(x^2-1)\div(x+1)=x-1$.
49 $(4x^2-9)\div(2x-3)=2x+3$.
50. $(25-16x^2)\div(5+4x)=5-4x$.
51. $(16-x^8)\div(4-x^4)=4+x^4$.
52. $(y^6-4)\div(y^3-2)=y^3+2$.
53. $(c^2-d^2)=(c-d)(c+d)$.
54. $(n^2-4)=(n+2)(n-2)$.
55. $n^2-16=(n+4)(n-4)$.
56. $9-4a^2=(3-2a)(3+2a)$.
57. $36b^2-1=(6b+1)(6b-1)$.
58. $a^2-9=(a+3)(a-3)$.
59. $25-4n^2=(5-2n)(5+2n)$.
60. $100-9x^2=(10-3x)(10+3x)$.
61. $[(x+y)+1][(x+y)-1]=(x+y)^2-1=x^2+2xy+y^2-1$.
62. $(x+a+3)(x+a-3)=[(x+a)+3][(x+a)-3]$
$=x^2+2xa+a^2-9$.
63. $[(x-a)+3][(x-a)-3]=(x-a)^2-9$
$=x^2-2ax+a^2-9$.
64. $(x+4+c)(x+4-c)=[(x+4)+c][(x+4)-c]$
$=(x+4)^2-c^2$
$=x^2+8x+16-c^2$.
65. $(2a-b+c)(2a-b-c)=[(2a-b)+c][(2a-b)-c]$
$=(2a-b)^2-c^2$
$=4a^2-4ab+b^2-c^2$.
66. $[x+(b+c)][x-(b+c)]=x^2-(b+c)^2=x^2-b^2-2bc-c^2$.
67. $[x+(b-c)][x-(b-c)]=x^2-(b-c)^2=x^2-b^2+2bc-c^2$.
68. $[3+(x-y)][3-(x-y)]=9-(x-y)^2=9-x^2+2xy-y^2$.
69. $[4x+(2y-x)][4x-(2y-x)]=16x^2-(2y-x)^2$
$=16x^2-4y^2+4xy-x^2$
$=15x^2-4y^2+4xy$.

70. $[10-(a-5)][10+(a-5)]=100-(a-5)^2$
$=100-a^2+10a-25$
$=75+10a-a^2.$

71. $49x^2-1=(7x+1)(7x-1).$

72. $64x^2-25=(8x-5)(8x+5).$

73. $(a+b)^2-1=(a+b+1)(a+b-1).$

74. $(x-y)^2-4=(x-y-2)(x-y+2).$

75. $(2x-1)^2-a^2=(2x-1+a)(2x-1-a).$

76. $(3y-z)^2-b^2=(3y-z+b)(3y-z-b).$

77. $b^2-(x+y)^2=[b+(x+y)][b-(x+y)]=(b+x+y)(b-x-y).$

78. $b^2-(x-y)^2=[b+(x-y)][b-(x-y)].$

79. $c^2-(x+c)^2=[c+(x+c)][c-(x+c)].$

80. $4-(x-2)^2=[2+(x-2)][2-(x-2)].$

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1. $(x+1)(x+2)=x^2+3x+2.$
2. $(x+2)(x+3)=x^2+5x+6.$
3. $(x+3)(x+4)=x^2+7x+12.$
4. $(x+4)(x+5)=x^2+9x+20.$
5. $(x+5)(x+6)=x^2+11x+30.$
6. $(n+1)(n-2)=n^2-n-2.$
7. $(n-2)(n+3)=n^2+n-6.$
8. $(n-3)(n+4)=n^2+n-12.$
9. $(n-4)(n+5)=n^2+n-20.$
10. $(n-5)(n+6)=n^2+n-30.$
11. $(a-1)(a-2)=a^2-3a+2.$
12. $(a-2)(a-3)=a^2-5a+6.$
13. $(a-3)(a-4)=a^2-7a+12.$
14. $(a-4)(a-5)=a^2-9a+20.$
15. $(a-4)(a-6)=a^2-10a+24.$
16. $(2y+3)(2y+4)=4y^2+14y+12.$
17. $(2y+2)(2y+3)=4y^2+10y+6.$
18. $(3a+1)(3a+4)=9a^2+15a+4.$
19. $(2n+3)(2n+5)=4n^2+16n+15.$
20. $(2n+3)(2n-5)=4n^2-4n-15.$
21. $(2n+3)(2n-4)=4n^2-2n-12.$
22. $(2a+1)(2a-5)=4a^2-8a-5.$
23. $(3a-5)(3a+1)=9a^2-12a-5.$
24. $(4a+3)(4a-5)=16a^2-8a-15.$
25. $(4a-5)(4a-6)=16a^2-44a+30.$
26. $(4ab+1)(4ab-6)=16a^2b^2-20ab-6.$
27. $(3x-2)(3x+5)=9x^2+9x-10.$

28. $(4a - 3b)(4a + 5b) = 16a^2 + 8ab - 15b^2$.
29. $(5a - 6b)(5a - 7b) = 25a^2 - 65ab + 42b^2$.

30. $\dfrac{x^2 + 3x + 2}{x + 1} = x + 2$.
31. $\dfrac{x^2 + 5x + 6}{x + 2} = x + 3$.
32. $\dfrac{x^2 + 4x + 3}{x + 3} = x + 1$.
33. $\dfrac{x^2 + 6x + 5}{x + 1} = x + 5$.
34. $\dfrac{x^2 + 7x + 12}{x + 3} = x + 4$.
35. $\dfrac{x^2 - 7x + 12}{x - 3} = x - 4$.
36. $\dfrac{x^2 - 5x + 6}{x - 3} = x - 2$.
37. $\dfrac{x^2 - 5x + 4}{x - 1} = x - 4$.
38. $\dfrac{x^2 - 6x + 5}{x - 5} = x - 1$.
39. $\dfrac{x^2 - 7x + 6}{x - 6} = x - 1$.
40. $\dfrac{x^2 - 7x + 12}{x - 4} = x - 3$.

41. $x^2 + 3x + 2 = (x + 1)(x + 2)$.
42. $x^2 + 5x + 6 = (x + 3)(x + 2)$.
43. $x^2 + 7x + 12 = (x + 4)(x + 3)$.
44. $x^2 + 6x + 8 = (x + 4)(x + 2)$.
45. $x^2 + 7x + 10 = (x + 2)(x + 5)$.
46. $x^2 + 8x + 15 = (x + 3)(x + 5)$.
47. $x^2 - 3x + 2 = (x - 2)(x - 1)$.
48. $x^2 - 5x + 6 = (x - 3)(x - 2)$.
49. $x^2 - 6x + 8 = (x - 4)(x - 2)$.
50. $x^2 - 8x + 15 = (x - 5)(x - 3)$.
51. $x^2 - 8x + 12 = (x - 6)(x - 2)$.
52. $x^2 - 9x + 14 = (x - 7)(x - 2)$.
53. $x^2 + 8x + 7 = (x + 1)(x + 7)$.
54. $x^2 + 9x + 8 = (x + 1)(x + 8)$.
55. $x^2 - 10x + 9 = (x - 9)(x - 1)$.
56. $x^2 - 11x + 10 = (x - 10)(x - 1)$.
57. $x^2 - 10x + 16 = (x - 2)(x - 8)$.
58. $x^2 + 10x + 21 = (x + 3)(x + 7)$.
59. $x^2 + 9x + 20 = (x + 4)(x + 5)$.
60. $x^2 - 9x + 18 = (x - 6)(x - 3)$.
61. $x^2 + 9x + 14 = (x + 2)(x + 7)$.
62. $x^2 - 12x + 32 = (x - 4)(x - 8)$.
63. $x^2 + 11x + 10 = (x + 10)(x + 1)$.

Page 100 (First set)

1. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$.
2. $(a + b + 1)^2 = a^2 + b^2 + 1 + 2ab + 2a + 2b$.
3. $(a + b + 2)^2 = a^2 + b^2 + 4 + 2ab + 4a + 4b$.
4. $(2a + b - c)^2 = 4a^2 + b^2 + c^2 + 4ab - 4ac - 2bc$.
5. $(a - b - c)^2 = a^2 + b^2 + c^2 - 2ab - 2ac + 2bc$.
6. $(2x + y - 1)^2 = 4x^2 + y^2 + 1 + 4xy - 4x - 2y$.
7. $(4x + y - 2)^2 = 16x^2 + y^2 + 4 + 8xy - 16x - 4y$.
8. $(a - bc + d)^2 = a^2 + b^2c^2 + d^2 - 2abc + 2ad - 2bcd$.
9. $(3a - 3b + 1)^2 = 9a^2 + 9b^2 + 1 - 18ab + 6a - 6b$.
10. $(a - b + c - d)^2$
$= a^2 + b^2 + c^2 + d^2 - 2ab + 2ac - 2ad - 2bc + 2bd - 2cd$.
11. $(x + y - a + 1)^2$
$= x^2 + y^2 + a^2 + 1 + 2xy - 2ax + 2x - 2ay + 2y - 2a$.
12. $(a - y + b - 3)^2$
$= a^2 + y^2 + b^2 + 9 - 2ay + 2ab - 6a - 2by + 6y - 6b$.

Page 100 (Second set)

1. $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$.
2. $(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$.
3. $(x+1)^3 = x^3 + 3x^2 + 3x + 1$.
4. $(x-1)^3 = x^3 - 3x^2 + 3x - 1$.
5. $(x+2)^3 = x^3 + 6x^2 + 12x + 8$.
6. $(x-2)^3 = x^3 - 6x^2 + 12x - 8$.
7. $(x-3)^3 = x^3 - 9x^2 + 27x - 27$.
8. $(x+3)^3 = x^3 + 9x^2 + 27x + 27$.
9. $(2x+y)^3 = 8x^3 + 12x^2y + 6xy^2 + y^3$.
10. $(x-2y)^3 = x^3 - 6x^2y + 12xy^2 - 8y^3$.
11. $(2x-1)^3 = 8x^3 - 12x^2 + 6x - 1$.
12. $(3x+2)^3 = 27x^3 + 54x^2 + 36x + 8$.
13. $(3x-2y)^3 = 27x^3 - 54x^2y + 36xy^2 - 8y^3$.
14. $(4x+3y)^3 = 64x^3 + 144x^2y + 108xy^2 + 27y^3$.
15. $(5x-2y)^3 = 125x^3 - 150x^2y + 60xy^2 - 8y^3$.

16. The cube of the binomial $a+b$ is equal to the cube of a plus three times the square of a multiplied by b, plus three times the product of a and the square of b, plus the cube of b.

17. The cube of the binomial $a-b$ is equal to the cube of a minus three times the square of a multiplied by b, plus three times the product of a and the square of b, minus the cube of b.

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1. $\sqrt{4a^2} = 2a$.
2. $\sqrt{9a^4} = 3a^2$.
3. $\sqrt{25a^6x^4} = 5a^3x^2$.
4. $\sqrt{4x^4} = 2x^2$.
5. $\sqrt{9x^6} = 3x^3$.
6. $\sqrt{16x^{16}} = 4x^8$.
7. $\sqrt{49x^{14}y^{10}} = 7x^7y^5$.
8. $\sqrt{81x^{12}d^{20}} = 9x^6d^{10}$.
9. $\sqrt{121c^{22}d^6} = 11c^{11}d^3$.
10. $\sqrt{169x^6} = 13x^3$.
11. $\sqrt{196y^{10}} = 14y^5$.
12. $\sqrt{225y^4z^6} = 15y^2z^3$.
13. $\sqrt{256c^{12}} = 16c^6$.
14. $\sqrt{400a^2b^2c^4} = 20abc^2$.
15. $\sqrt{324b^8} = 18b^4$.
16. $\sqrt{625b^2y^{18}z^6} = 25by^9z^3$.
17. $\sqrt{x^{2n}} = x^n$.
18. $\sqrt{x^{4n}} = x^{2n}$.
19. $\sqrt{x^{6n}} = x^{3n}$.
20. $\sqrt{a^{4n}b^{8n}} = a^{2n}b^{4n}$.
21. $\sqrt[3]{8} = 2$.
22. $\sqrt[3]{-8a^3} = -2a$.
23. $\sqrt[3]{27} = 3$.
24. $\sqrt[3]{64} = 4$.
25. $\sqrt[3]{-125} = -5$.
26. $\sqrt[3]{216} = 6$.
27. $\sqrt[3]{-a^3} = -a$.
28. $\sqrt[3]{-a^6} = -a^2$.
29. $\sqrt[3]{a^9b^3} = a^3b$.
30. $\sqrt[3]{343y^3} = 7y$.
31. $\sqrt[3]{512x^{15}} = 8x^5$.
32. $\sqrt[3]{-729a^3b^9} = -9ab^3$.
33. $\sqrt[3]{1000a^6b^{12}} = 10a^2b^4$.
34. $\sqrt[3]{-27a^3b^6c^{12}} = -3ab^2c^4$.
35. $\sqrt[3]{8a^{12}b^3c^9} = 2a^4bc^3$.

36. $\sqrt[3]{-125\,a^6b^6z^6} = -5\,a^2b^2z^2.$

37. $\sqrt[3]{x^{3n}} = x^n.$

38. $\sqrt[3]{x^{6n}} = x^{2n}.$

39. $\sqrt[3]{-x^{9n}} = -x^{3n}.$

40. $\sqrt[3]{x^{12n}y^{6n}} = x^{4n}y^{2n}.$

41. $\sqrt[5]{x^{15}} = x^3.$

42. $\sqrt[5]{32\,x^{10}} = 2\,x^2.$

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1. $3\,x + 6 = 3\,(x + 2).$
2. $x^3 - x^2 = x^2\,(x - 1).$
3. $8\,x - 2\,x^4 = 2\,x\,(4 - x^3).$
4. $xy + y^2 = y\,(x + y).$
5. $5\,b^3 - 15\,b^6 = 5\,b^3\,(1 - 3\,b^3).$
6. $c^3 - c^2y = c^2\,(c - y).$
7. $x^3 - x + x^2 = x\,(x^2 - 1 + x).$
8. $c^5 - c^3 + c^2 = c^2\,(c^3 - c + 1).$
9. $3\,y - 9\,y^4 + 12 = 3\,(y - 3\,y^4 + 4).$
10. $c^2 + 2\,bc - c = c\,(c + 2\,b - 1).$
11. $3\,y^2 - 15\,y + 6\,y^3 = 3\,y\,(y - 5 + 2\,y^2).$
12. $10\,ab - 14\,bc - 8\,b^2 = 2\,b\,(5\,a - 7\,c - 4\,b).$
13. $y^5 + 3\,y^4z + 6\,y^3z^2 = y^3\,(y^2 + 3\,yz + 6\,z^2).$
14. $4\,a^2x^2 - 6\,a^3y^3 + 12\,a^4x^2y^2 = 2\,a^2\,(2\,x^2 - 3\,ay^3 + 6\,a^2x^2y^2).$
15. $12\,z^3 + 30\,a^4z^7 - 18\,c^3z^5 = 6\,z^3\,(2 + 5\,a^4z^4 - 3\,c^3z^2).$
16. $6\,c^6 + 10\,c^{10} - 20\,c^2 = 2\,c^2\,(3\,c^4 + 5\,c^8 - 10).$
17. $8\,y^8 - 4\,y^4 - 12\,y^{12} = 4\,y^4\,(2\,y^4 - 1 - 3\,y^8).$
18. $x^4 + x^3 - x^2 + x = x\,(x^3 + x^2 - x + 1).$
19. $-8\,y^6 - 4\,y^6 + 12\,y^3 + 6\,y^5 = 6\,y^3\,(-2\,y^3 + 2 + y^2).$
20. $14\,b^5 - 49\,b^7 + 21\,b^2 - 7\,b^3 = 7\,b^2\,(2\,b^3 - 7\,b^5 + 3 - b).$
21. $-a^2x^2y^3 - 3\,a^4x^3y^2 - 2\,a^3x^5y^6z^3 + 5\,a^4x^4y^4z$
    $= a^2x^2y^2\,(-y - 3\,a^2x - 2\,ax^3y^4z^3 + 5\,a^2x^2y^2z).$
22. $-18\,a^3b^3c^4 - 45\,a^4cy^2 - 36\,a^3c^2xz - 63\,a^4cxy$
    $= -9\,a^3c\,(2\,b^3c^3 + 5\,ay^2 + 4\,cxz + 7\,axy).$
23. $32\,c^7x^2 + 80\,c^{10}bx - 112\,c^6b + 48\,c^{12}x$
    $= 16\,c^6\,(2\,cx^2 + 5\,c^4bx - 7\,b + 3\,c^6x).$
24. $56\,a^4b^3x^2 - 28\,a^3x + 112\,a^5x^2y^2 - 196\,a^6x^3$
    $= 28\,a^3x\,(2\,ab^3x - 1 + 4\,a^2xy^2 - 7\,a^3x^2).$

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1. $2\,(x + y) + a\,(x + y) = (x + y)\,(2 + a).$
2. $b\,(x + 2\,y) + (x + 2\,y) = (x + 2\,y)\,(b + 1).$
3. $c\,(c - a) - a\,(c - a) = (c - a)\,(c - a).$
4. $2\,c\,(3\,b - d) + 3\,a\,(3\,b - d) = (3\,b - d)\,(2\,c + 3\,a).$
5. $6\,d\,(c + x) - (c + x) + 4\,k\,(c + x) = (c + x)\,(6\,d - 1 + 4\,k).$
6. $-5\,(2\,a - 3\,b) + 7\,a\,(2\,a - 3\,b) - 3\,b\,(2\,a - 3\,b)$
   $= (2\,a - 3\,b)\,(7\,a - 5 - 3\,b).$
7. $k\,(a - b) + 3\,(b - a) = k\,(a - b) - 3\,(a - b) = (a - b)\,(k - 3).$

8. $h(x-y)+k(y-x)=h(x-y)-k(x-y)$
$=(x-y)(h-k).$

9. $5x(c-2d)-6(2d-c)=5x(c-2d)+6(c-2d)$
$=(c-2d)(5x+6).$

10. $3y(k-4h)+(4h-k)=3y(k-4h)-(k-4h)$
$=(k-4h)(3y-1).$

11. $2c(4k-3c)-5h(3c-4k)=2c(4k-3c)+5h(4k-3c)$
$=(4k-3c)(2c+5h).$

12. $3y(5x-b)-7(b-5x)+6z(5x-b)$
$=3y(5x-b)+7(5x-b)+6z(5x-b)$
$=(5x-b)(3y+7+6z).$

16. $ax+bx+ay+by=(ax+bx)+(ay+by)$
$=x(a+b)+y(a+b)$
$=(a+b)(x+y).$

17. $ch+h^2+cx+hx=(ch+h^2)+(cx+hx)$
$=h(c+h)+x(c+h)$
$=(c+h)(h+x).$

18. $bh-ch+bk-ck=(bh-ch)+(bk-ck)$
$=h(b-c)+k(b-c)$
$=(b-c)(h+k).$

19. $bx+xz-ab-az=(bx+xz)+(-ab-az)$
$=x(b+z)-a(b+z)$
$=(b+z)(x-a).$

20. $ax+3a-bx-3b=(ax+3a)+(-bx-3b)$
$=a(x+3)-b(x+3)$
$=(x+3)(a-b).$

21. $2x^2+10xy-4x-20y=2(x^2+5xy-2x-10y)$
$=2[x(x+5y)-2(x+5y)]$
$=2(x-2)(x+5y).$

22. $6ab-2ac+3b-c=(6ab-2ac)+(3b-c)$
$=2a(3b-c)+(3b-c)$
$=(3b-c)(2a+1).$

23. $36ax+45ac-4x-5c=9a(4x+5c)-(4x+5c)$
$=(4x+5c)(9a-1).$

24. $6a^3+14ab-15a^2y-35by=2a(3a^2+7b)-5y(3a^2+7b)$
$=(3a^2+7b)(2a-5y).$

25. $3ax-ay-6bx+2by=a(3x-y)-2b(3x-y)$
$=(3x-y)(a-2b).$

26. $12ax-6ay-50cx+25cy=6a(2x-y)-25c(2x-y)$
$=(2x-y)(6a-25c).$

27. $2h^3-3h^2k-10h+15k=h^2(2h-3k)-5(2h-3k)$
$=(2h-3k)(h^2-5).$

28. $10\,dh - 45\,hk - 22\,cd + 99\,ck = 5\,h(2\,d - 9\,k) - 11\,c(2\,d - 9\,k)$
$= (2\,d - 9\,k)(5\,h - 11\,c).$

29. $28\,hx + 9\,ky - 21\,hy - 12\,kx = 28\,hx - 21\,hy - 12\,kx + 9\,ky$
$= 7\,h(4\,x - 3\,y) - 3\,k(4\,x - 3\,y)$
$= (4\,x - 3\,y)(7\,h - 3\,k).$

30. $-2\,ax + 7\,a^2 + 16\,bx - 56\,ab = -a(2\,x - 7\,a) + 8\,b(2\,x - 7\,a)$
$= (2\,x - 7\,a)(8\,b - a).$

31. $6\,hk + 15\,xy - 10\,ky - 9\,hx = 6\,hk - 9\,hx - 10\,ky + 15\,xy$
$= 3\,h(2\,k - 3\,x) - 5\,y(2\,k - 3\,x)$
$= (2\,k - 3\,x)(3\,h - 5\,y).$

32. $-21\,ax + 12\,cx - 4\,cd + 7\,ad = 12\,cx - 21\,ax - 4\,cd + 7\,ad$
$= 3\,x(4\,c - 7\,a) - d(4\,c - 7\,a)$
$= (4\,c - 7\,a)(3\,x - d).$

33. $7\,gy - 77\,hy - cg + 11\,ch = 7\,y(g - 11\,h) - c(g - 11\,h)$
$= (g - 11\,h)(7\,y - c).$

34. $5\,a^3 + 10\,a - 5\,a^2 - 10 = 5\,[a(a^2 + 2) - (a^2 + 2)]$
$= 5\,(a^2 + 2)(a - 1).$

35. $3\,a - 5\,ax^3 - 6\,ax + 10\,ax^4 = a[(3 - 5\,x^3) - 2\,x(3 - 5\,x^3)]$
$= a(3 - 5\,x^3)(1 - 2\,x).$

36. $4\,abxy - 24\,dxy - 3\,abgh + 18\,dgh = 4\,xy(ab - 6\,d) - 3\,gh(ab - 6\,d)$
$= (ab - 6\,d)(4\,xy - 3\,gh).$

37. $8\,acxy - 20\,bx^2y - 6\,abc^2 + 15\,b^2cx$
$= 4\,xy(2\,ac - 5\,bx) - 3\,bc(2\,ac - 5\,bx)$
$= (2\,ac - 5\,bx)(4\,xy - 3\,bc).$

38. $ax + bx + cx + ay + by + cy = x(a + b + c) + y(a + b + c)$
$= (a + b + c)(x + y).$

39. $cg + 2\,ch + ck - gx - 2\,hx - kx = c(g + 2\,h + k) - x(g + 2\,h + k)$
$= (g + 2\,h + k)(c - x).$

40. $3\,ab - bx + 2\,by - 3\,ac + cx - 2\,cy$
$= b(3\,a - x + 2\,y) - c(3\,a - x + 2\,y)$
$= (3\,a - x + 2\,y)(b - c).$

41. $4\,ax - ac + 2\,ay + 12\,bx - 3\,bc + 6\,by$
$= a(4\,x - c + 2\,y) + 3\,b(4\,x - c + 2\,y)$
$= (4\,x - c + 2\,y)(a + 3\,b).$

42. $ax - 6\,cy - 2\,bx + 4\,by + 3\,cx - 2\,ay$
$= ax - 2\,bx + 3\,cx - 2\,ay + 4\,by - 6\,cy$
$= x(a - 2\,b + 3\,c) - 2\,y(a - 2\,b + 3\,c)$
$= (a - 2\,b + 3\,c)(x - 2\,y).$

43. $-2\,xz + 6\,ay + cz + 4\,xy - 3\,az - 2\,cy$
$= 6\,ay + 4\,xy - 2\,cy - 3\,az - 2\,xz + cz$
$= 2\,y(3\,a + 2\,x - c) - z(3\,a + 2\,x - c)$
$= (3\,a + 2\,x - c)(2\,y - z).$

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1. $c^2 + 2cd + d^2$.
2. $a^2 + 4a + 4$.
3. $4a^2 + 4a + 1$.
4. $x^6 + 6x^3 + 9$.
5. $4x^2 + 20x + 25$.
6. $9x^6 + 12x^3y + 4y^2$.
7. $100x^2 + 120xy + 36y^2$.
8. $49 + 112y^4 + 64y^8$.
9. $x^2 + 2x + 1$.
10. $y^2 - 6y + 9$.
11. $4y^2 - 40y + 100$.
12. $9y^2 + 36y + 36$.
13. $16z^4 - 16xz^2 + 4x^2$.
14. $121y^6 - 88y^3z + 16z^2$.
15. $100a^8 + 240a^4x^5 + 144x^{10}$.
16. $y^4 - 6y^2 + 9$.
17. $y^8 - 10y^4 + 25$.
18. $16k^4 + 16k^2 + 4$.
19. $25k^4 - 80k^2h^2 + 64h^4$.
20. $16a^{16} + 104a^8k + 169k^2$.

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1. $c^2 + 2cd + d^2 = (c + d)^2$.
2. $x^2 + 2xy + y^2 = (x + y)^2$.
3. $x^2 + 2x + 1 = (x + 1)^2$.
4. $a^2 - 10a + 25 = (a - 5)^2$.
5. $a^2 + 8a + 16 = (a + 4)^2$.
6. $4h^2 + 4h + 1 = (2h + 1)^2$.
7. $9 - 6x + x^2 = (3 - x)^2$.
8. $16k^2 - 8k + 1 = (4k - 1)^2$.
9. $4h^2 - 12h + 9 = (2h - 3)^2$.
10. $x^4 + 1 + 2x^2 = (x^2 + 1)^2$.
11. $x^6 - 6x^3 + 9 = (x^3 - 3)^2$.
12. $25h^2 + 4k^2 - 20hk = (5h - 2k)^2$.
13. $9h^2 - 24hk + 16k^2 = (3h - 4k)^2$.
14. $x^2 - 24xz^2 + 144z^4 = (x - 12z^2)^2$.
15. $12x^4z^3 + 9z^6 + 4x^8 = (3z^3 + 2x^4)^2$.
16. $c^6d^{10} + 2c^3d^5 + 1 = (c^3d^5 + 1)^2$.
17. $24x^2y^2 + 16y^4 + 9x^4 = (4y^2 + 3x^2)^2$.
18. $x^4 + 4y^4 + 4x^2y^2 = (x^2 + 2y^2)^2$.
19. $4x^2y^8 - 4xy^4 + 1 = (2xy^4 - 1)^2$.
20. $25h^6k^2 - 30h^3kz + 9z^2 = (5h^3k - 3z)^2$.
21. $121c^6d^4 - 220c^3d^2g^2 + 100g^4 = (11c^3d^2 - 10g^2)^2$.
22. $169a^4 - 156a^2x^4y^3 + 36x^8y^6 = (13a^2 - 6x^4y^3)^2$.

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1. $x^3 + 2x^2 + x = x(x^2 + 2x + 1) = x(x + 1)(x + 1)$.
2. $x^5 - 4x^4 + 4x^3 = x^3(x^2 - 4x + 4) = x^3(x - 2)(x - 2)$.
3. $2a^4 + 12a^2 + 18 = 2(a^4 + 6a^2 + 9) = 2(a^2 + 3)(a^2 + 3)$.
4. $3a^2 + 18a + 27 = 3(a^2 + 6a + 9) = 3(a + 3)(a + 3)$.
5. $x^6 - 20x^4 + 100x^2 = x^2(x^4 - 20x^2 + 100) = x^2(x^2 - 10)(x^2 - 10)$.
6. $5cx^6 - 70cx^3 + 245c = 5c(x^6 - 14x^3 + 49) = 5c(x^3 - 7)(x^3 - 7)$.
7. $16x^8 - 24x^6 + 9x^4 = x^4(16x^4 - 24x^2 + 9) = x^4(4x^2 - 3)(4x^2 - 3)$.
8. $28x^2 - 28x^3 + 7x^4 = 7x^2(4 - 4x + x^2) = 7x^2(2 - x)(2 - x)$.
9. $3ax + 3ay + 3bx + 3by = 3[a(x + y) + b(x + y)] = 3(x + y)(a + b)$.

10. $4cx - 4cy + 4dx - 4dy = 4[c(x-y) + d(x-y)] = 4(x-y)(c+d).$

11. $ax - a + x - 1 = a(x-1) + (x-1) = (x-1)(a+1).$

12. $63xy - 84y^2 + 98yz = 7y(9x - 12y + 14z).$

13. $30ax - 34bx - 15a + 17b = 2x(15a - 17b) - (15a - 17b)$
$= (15a - 17b)(2x - 1).$

14. $ax - bx + cx + ay - by + cy = x(a - b + c) + y(a - b + c)$
$= (a - b + c)(x + y).$

15. $14anx - 21bny - 7n = 7n(2ax - 3by - 1).$

16. $3ax - 5by - 5ay + 3bx = 3x(a+b) - 5y(a+b)$
$= (a+b)(3x - 5y).$

17. $56a^2 - 40ab + 63ac - 45bc = 8a(7a - 5b) + 9c(7a - 5b)$
$= (7a - 5b)(8a + 9c).$

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1. $x^2 + 5x + 6 = (x+2)(x+3).$

2. $x^2 + 9x + 20 = (x+4)(x+5).$

3. $x^2 - 7x + 12 = (x-3)(x-4).$

4. $x^2 - 9x + 14 = (x-2)(x-7).$

5. $x^2 + 19x + 18 = (x+1)(x+18).$

6. $x^2 - 18x + 17 = (x-1)(x-17).$

7. $x^2 - x - 6 = (x+2)(x-3).$

8. $x^2 - 3x - 40 = (x-8)(x+5).$

9. $x^2 + x - 30 = (x+6)(x-5).$

10. $x^2 - 2x - 35 = (x-7)(x+5).$

11. $x^2 + 11x - 42 = (x+14)(x-3).$

12. $6 - 5x - x^2 = (6+x)(1-x).$

13. $14 - 5x - x^2 = -1(x^2 + 5x - 14) = -(x+7)(x-2) = (x+7)(2-x).$

14. $21 + 4x - x^2 = -1(x^2 - 4x - 21) = -1(x-7)(x+3) = (7-x)(x+3).$

15. $24 + 10a - a^2 = -1(a^2 - 10a - 24) = -1(a-12)(a+2) = (12-a)(a+2).$

16. $10 + 3x - x^2 = -1(x^2 - 3x - 10) = -1(x-5)(x+2) = (5-x)(x+2).$

17. $72 + c - c^2 = -1(c^2 - c - 72) = -1(c-9)(c+8) = (9-c)(c+8).$

18. $x^2 + x - 90 = (x+10)(x-9).$

19. $28 + x^2 - 11x = (x-4)(x-7).$

20. $27 - 12x + x^2 = (-9 + x)(-3 + x) = (x-9)(x-3).$

21. $8x + x^2 - 48 = (x+12)(x-4).$

22. $x^2 - x - 90 = (x-10)(x+9).$

23. $x^2 - 5ax + 6a^2 = (x - 2a)(x - 3a).$

24. $x^2 + 7ax + 12a^2 = (x + 3a)(x + 4a).$

25. $a^2x^2 - 2ax - 15 = (ax - 5)(ax + 3).$

26. $h^2k^2 - 9hk + 20 = (hk - 4)(hk - 5).$

27. $x^4 - 5x^2 - 14 = (x^2 - 7)(x^2 + 2).$

28. $x^2 + x^4 - 110 = (x^2 + 11)(x^2 - 10).$

29. $x^6 - 11x^3 + 18 = (x^3 - 9)(x^3 - 2).$

30. $b^2x^2 - 3bxy - 28y^2 = (bx - 7y)(bx + 4y).$

31. $h^2k^2 - 5hkx - 36x^2 = (hk - 9x)(hk + 4x).$

32. $a^6b^2 - 5a^3bx - 24x^2 = (a^3b - 8x)(a^3b + 3x).$

33. $h^4d^2 + 10h^2dg^3 - 24g^6 = (h^2d + 12g^3)(h^2d - 2g^3).$

34. $9k^2x^4y^6 + k^4x^8 - 22y^{12} = (k^2x^4 + 11y^6)(k^2x^4 - 2y^6).$

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1. $x^4 + 6x^3 + 9x^2 = x^2(x^2 + 6x + 9) = x^2(x + 3)(x + 3)$.
2. $x^3 - 5x^2 + 6x = x(x^2 - 5x + 6) = x(x - 3)(x - 2)$.
3. $3b^2 - 9b - 30 = 3(b^2 - 3b - 10) = 3(b - 5)(b + 2)$.
4. $3ax^3 - 3ax^2 - 6ax = 3ax(x^2 - x - 2) = 3ax(x - 2)(x + 1)$.
5. $4x^5 - 4x^3 - 360x = 4x(x^4 - x^2 - 90) = 4x(x^2 - 10)(x^2 + 9)$.
6. $3x^4 - 21x^3 - 54x^2 = 3x^2(x^2 - 7x - 18) = 3x^2(x - 9)(x + 2)$.
7. $4a^6y^4 - 64a^4y^2 + 256a^2 = 4a^2(a^4y^4 - 16a^2y^2 + 64)$
$= 4a^2(a^2y^2 - 8)(a^2y^2 - 8)$.
8. $3x - 3ax - 18a^2x = -3x(6a^2 + a - 1)$
$= -3x(2a + 1)(3a - 1)$
$= 3x(1 + 2a)(1 - 3a)$.
9. $56x + x^2 - x^3 = -x(x^2 - x - 56)$
$= -x(x - 8)(x + 7)$
$= x(8 - x)(x + 7)$.
10. $140h^2 + 23h^3 - h^4 = -h^2(h^2 - 23h - 140)$
$= -h^2(h - 28)(h + 5)$
$= h^2(28 - h)(h + 5)$.
11. $3x^3 - 3x^4 + 60x^2 = -3x^2(x^2 - x - 20)$
$= -3x^2(x - 5)(x + 4)$
$= 3x^2(5 - x)(x + 4)$.
12. $32a^2x^2 + 2a^2 - 16a^2x = 2a^2(16x^2 - 8x + 1) = 2a^2(4x - 1)^2$.
13. $4ay^2x^2 - 40ay^2x + 100ay^2 = 4ay^2(x^2 - 10x + 25)$
$= 4ay^2(x - 5)(x - 5)$.
14. $3x^4 - 3x^3 + 3x^2 - 3x = 3x(x^3 - x^2 + x - 1)$
$= 3x[x^2(x - 1) + (x - 1)]$
$= 3x(x - 1)(x^2 + 1)$.
15. $2ax^3 - 4ax^2 - 12ax + 24a = 2a(x^3 - 2x^2 - 6x + 12)$
$= 2a[x^2(x - 2) - 6(x - 2)]$
$= 2a(x - 2)(x^2 - 6)$.
16. $5xyz - 15yz + 10xz - 30z = 5z(xy - 3y + 2x - 6)$
$= 5z[y(x - 3) + 2(x - 3)]$
$= 5z(x - 3)(y + 2)$.

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1. $2x^2 + 5x + 3 = 2x^2 + 2x + 3x + 3$
$= 2x(x + 1) + 3(x + 1)$
$= (2x + 3)(x + 1)$.
2. $3x^2 + 5x + 2 = 3x^2 + 3x + 2x + 2$
$= 3x(x + 1) + 2(x + 1)$
$= (3x + 2)(x + 1)$.

3. $2x^2 - 7x + 3 = 2x^2 - 6x - x + 3$
$= 2x(x-3) - (x-3)$
$= (2x-1)(x-3).$

4. $x^2 + 6x + 9 = x^2 + 3x + 3x + 9$
$= x(x+3) + 3(x+3)$
$= (x+3)(x+3).$

5. $3x^2 + 8x + 4 = 3x^2 + 6x + 2x + 4$
$= 3x(x+2) + 2(x+2)$
$= (3x+2)(x+2).$

6. $5x^2 - 8x + 3 = 5x^2 - 5x - 3x + 3$
$= 5x(x-1) - 3(x-1)$
$= (5x-3)(x-1).$

7. $9x^2 + 18x + 8 = 9x^2 + 12x + 6x + 8$
$= 3x(3x+4) + 2(3x+4)$
$= (3x+2)(3x+4).$

8. $7x^2 + 9ax + 2a^2 = 7x^2 + 7ax + 2ax + 2a^2$
$= 7x(x+a) + 2a(x+a)$
$= (7x+2a)(x+a).$

9. $4x^2 + 16x + 15 = 4x^2 + 6x + 10x + 15$
$= 2x(2x+3) + 5(2x+3)$
$= (2x+5)(2x+3).$

10. $2x^2 - 3x - 2 = 2x^2 + x - 4x - 2$
$= x(2x+1) - 2(2x+1)$
$= (x-2)(2x+1).$

11. $2x^2 - x - 3 = 2x^2 + 2x - 3x - 3$
$= 2x(x+1) - 3(x+1)$
$= (2x-3)(x+1).$

12. $4x^2 - 7x - 15 = 4x^2 - 12x + 5x - 15$
$= 4x(x-3) + 5(x-3)$
$= (x-3)(4x+5).$

13. $3x^4 + 7x^2 - 6 = 3x^4 - 2x^2 + 9x^2 - 6$
$= x^2(3x^2-2) + 3(3x^2-2)$
$= (x^2+3)(3x^2-2).$

14. $3x^2 + 5xy - 2y^2 = 3x^2 + 6xy - xy - 2y^2$
$= 3x(x+2y) - y(x+2y)$
$= (3x-y)(x+2y).$

15. $3x^2 - x - 2 = 3x^2 - 3x + 2x - 2$
$= 3x(x-1) + 2(x-1)$
$= (3x+2)(x-1).$

16. $2x^2 - 5x - 3 = 2x^2 - 6x + x - 3$
$= 2x(x-3) + (x-3)$
$= (2x+1)(x-3).$

17. $$\begin{aligned}2x^2 + xz - 3z^2 &= 2x^2 + 3xz - 2xz - 3z^2\\ &= x(2x+3z) - z(2x+3z)\\ &= (x-z)(2x+3z).\end{aligned}$$

18. $$\begin{aligned}6x^2 - x - 2 &= 6x^2 - 4x + 3x - 2\\ &= 2x(3x-2) + (3x-2)\\ &= (2x+1)(3x-2).\end{aligned}$$

19. $$\begin{aligned}25x^2 - 20x + 3 &= 25x^2 - 15x - 5x + 3\\ &= 5x(5x-3) - (5x-3)\\ &= (5x-3)(5x-1).\end{aligned}$$

20. $$\begin{aligned}49x^2 - 21x + 2 &= 49x^2 - 7x - 14x + 2\\ &= 7x(7x-1) - 2(7x-1)\\ &= (7x-2)(7x-1).\end{aligned}$$

21. $$\begin{aligned}36x^2 - 36x + 5 &= 36x^2 - 30x - 6x + 5\\ &= 6x(6x-5) - (6x-5)\\ &= (6x-5)(6x-1).\end{aligned}$$

22. $$\begin{aligned}6x^4 + 13x^2 + 6 &= 6x^4 + 9x^2 + 4x^2 + 6\\ &= 3x^2(2x^2+3) + 2(2x^2+3)\\ &= (3x^2+2)(2x^2+3).\end{aligned}$$

23. $$\begin{aligned}9x^6 - 6x^8 - 8 &= 9x^6 - 12x^3 + 6x^3 - 8\\ &= 3x^3(3x^3-4) + 2(3x^3-4)\\ &= (3x^3+2)(3x^3-4).\end{aligned}$$

24. $$\begin{aligned}5x^2 + 2x - 16 &= 5x^2 + 10x - 8x - 16\\ &= 5x(x+2) - 8(x+2)\\ &= (5x-8)(x+2).\end{aligned}$$

25. $$\begin{aligned}3x^2 - 11x - 20 &= 3x^2 - 15x + 4x - 20\\ &= 3x(x-5) + 4(x-5)\\ &= (3x+4)(x-5).\end{aligned}$$

26. $$\begin{aligned}5x^4 + 18x^2y + 16y^2 &= 5x^4 + 10x^2y + 8x^2y + 16y^2\\ &= 5x^2(x^2+2y) + 8y(x^2+2y)\\ &= (5x^2+8y)(x^2+2y).\end{aligned}$$

27. $$\begin{aligned}6x^2 + 23x - 55 &= 6x^2 - 10x + 33x - 55\\ &= 2x(3x-5) + 11(3x-5)\\ &= (2x+11)(3x-5).\end{aligned}$$

28. $$\begin{aligned}6x^6 + 23x^3 + 21 &= 6x^6 + 9x^3 + 14x^3 + 21\\ &= 3x^3(2x^3+3) + 7(2x^3+3)\\ &= (3x^3+7)(2x^3+3).\end{aligned}$$

29. $$\begin{aligned}10x^2 - 7x - 12 &= 10x^2 - 15x + 8x - 12\\ &= 5x(2x-3) + 4(2x-3)\\ &= (5x+4)(2x-3).\end{aligned}$$

30. $$\begin{aligned}4c^2 + 11cg - 3g^2 &= 4c^2 + 12cg - cg - 3g^2\\ &= 4c(c+3g) - g(c+3g)\\ &= (4c-g)(c+3g).\end{aligned}$$

31. $$\begin{aligned}8x^2 + 26xy + 15y^2 &= 8x^2 + 6xy + 20xy + 15y^2\\ &= 2x(4x+3y) + 5y(4x+3y)\\ &= (2x+5y)(4x+3y).\end{aligned}$$

32. $$\begin{aligned}35x^2 + 22x + 3 &= 35x^2 + 15x + 7x + 3\\ &= 5x(7x+3) + (7x+3)\\ &= (7x+3)(5x+1).\end{aligned}$$

33. $$\begin{aligned}14x^2 - x - 3 &= 14x^2 - 7x + 6x - 3\\ &= 7x(2x-1) + 3(2x-1)\\ &= (7x+3)(2x-1).\end{aligned}$$

34. $$\begin{aligned}21x^4 - x^2 - 2 &= 21x^4 - 7x^2 + 6x^2 - 2\\ &= 7x^2(3x^2-1) + 2(3x^2-1)\\ &= (7x^2+2)(3x^2-1).\end{aligned}$$

35. $$\begin{aligned}22x^2 - 3x - 7 &= 22x^2 + 11x - 14x - 7\\ &= 11x(2x+1) - 7(2x+1)\\ &= (11x-7)(2x+1).\end{aligned}$$

36. $$\begin{aligned}18x^2 + 65x + 7 &= 18x^2 + 2x + 63x + 7\\ &= 2x(9x+1) + 7(9x+1)\\ &= (2x+7)(9x+1).\end{aligned}$$

37. $$\begin{aligned}26x^2 + 9x - 2 &= 26x^2 + 13x - 4x - 2\\ &= 13x(2x+1) - 2(2x+1)\\ &= (13x-2)(2x+1).\end{aligned}$$

38. $$\begin{aligned}14x^2 - 39x + 10 &= 14x^2 - 35x - 4x + 10\\ &= 7x(2x-5) - 2(2x-5)\\ &= (7x-2)(2x-5).\end{aligned}$$

39. $$\begin{aligned}35x^2 - 39x - 36 &= 35x^2 - 60x + 21x - 36\\ &= 5x(7x-12) + 3(7x-12)\\ &= (5x+3)(7x-12).\end{aligned}$$

40. $$\begin{aligned}42x^2 - 9x - 6 &= 3(14x^2 - 3x - 2)\\ &= 3(14x^2 - 7x + 4x - 2)\\ &= 3[7x(2x-1) + 2(2x-1)]\\ &= 3(2x-1)(7x+2).\end{aligned}$$

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1. $$\begin{aligned}4ax^2 + 8ax + 3a &= a(4x^2 + 8x + 3)\\ &= a(4x^2 + 6x + 2x + 3)\\ &= a[2x(2x+3) + (2x+3)]\\ &= a(2x+1)(2x+3).\end{aligned}$$

2. $$\begin{aligned}2c^2x^2 + c^2x - 3c^2 &= c^2(2x^2 + x - 3)\\ &= c^2(2x^2 + 3x - 2x - 3)\\ &= c^2[x(2x+3) - (2x+3)]\\ &= c^2(x-1)(2x+3).\end{aligned}$$

3. $48ax^4 - 120ax^2 + 75a = 3a(16x^4 - 40x^2 + 25)$
$= 3a(4x^2 - 5)(4x^2 - 5).$

4. $18c^2d - 30cd - 28d = 2d(9c^2 - 15c - 14)$
$= 2d(9c^2 - 21c + 6c - 14)$
$= 2d[3c(3c - 7) + 2(3c - 7)]$
$= 2d(3c + 2)(3c - 7).$

5. $x^4 + x^3 - 110x^2 = x^2(x^2 + x - 110) = x^2(x + 11)(x - 10).$

6. $75x^3 + 60x^2 + 12x = 3x(25x^2 + 20x + 4) = 3x(5x + 2)(5x + 2).$

7. $4x^2y^2 - 4xy^2 - 120y^2 = 4y^2(x^2 - x - 30) = 4y^2(x - 6)(x + 5).$

8. $70x^3 - 85x^2y - 30xy^2 = 5x(14x^2 - 17xy - 6y^2)$
$= 5x(14x^2 - 21xy + 4xy - 6y^2)$
$= 5x[7x(2x - 3y) + 2y(2x - 3y)]$
$= 5x(7x + 2y)(2x - 3y).$

9. $18x^3 - 39x^2 + 18x = 3x(6x^2 - 13x + 6)$
$= 3x(6x^2 - 9x - 4x + 6)$
$= 3x[3x(2x - 3) - 2(2x - 3)]$
$= 3x(3x - 2)(2x - 3).$

10. $36x^4 - 6x^3 - 12x^2 = 6x^2(6x^2 - x - 2)$
$= 6x^2(6x^2 - 4x + 3x - 2)$
$= 6x^2[2x(3x - 2) + (3x - 2)]$
$= 6x^2(3x - 2)(2x + 1).$

11. $4x^4 - 20x^3 - 24x^2 = 4x^2(x^2 - 5x - 6) = 4x^2(x - 6)(x + 1).$

12. $5ax^2 + 5ax + 5a = 5a(x^2 + x + 1).$

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1. $k^2 - h^2 = (k + h)(k - h).$
2. $h^2 - 1 = (h + 1)(h - 1).$
3. $k^2 - 4 = (k + 2)(k - 2).$
4. $a^4 - x^2 = (a^2 + x)(a^2 - x).$
5. $1 - a^2 = (1 + a)(1 - a).$
6. $1 - 4a^2 = (1 + 2a)(1 - 2a).$
7. $4a^2 - 9 = (2a + 3)(2a - 3).$
8. $25 - x^2 = (5 - x)(5 + x).$
9. $h^4 - 81 = (h^2 + 9)(h^2 - 9)$
$= (h^2 + 9)(h + 3)(h - 3).$
10. $16x^4 - 25 = (4x^2 + 5)(4x^2 - 5).$
11. $36c^8 - 49d^8 = (6c^4 + 7d^4)(6c^4 - 7d^4).$
12. $a^2b^4 - 121 = (ab^2 + 11)(ab^2 - 11).$
13. $100 - z^8 = (10 + z^4)(10 - z^4).$
14. $x^4 - y^4 = (x^2 + y^2)(x^2 - y^2) = (x^2 + y^2)(x + x)(x - y)$
15. $64h^6 - 169 = (8h^3 + 13)(8h^3 - 13).$
16. $a^8 - b^8 = (a^4 + b^4)(a^4 - b^4)$
$= (a^4 + b^4)(a^2 + b^2)(a^2 - b^2)$
$= (a^4 + b^4)(a^2 + b^2)(a + b)(a - b).$
17. $1 - 16x^4 = (1 + 4x^2)(1 - 4x^2) = (1 + 4x^2)(1 + 2x)(1 - 2x).$
18. $16x^4 - y^8 = (4x^2 + y^4)(4x^2 - y^4) = (4x^2 + y^4)(2x + y^2)(2x - y^2).$

19. $x^{16} - y^{16} = (x^8 + y^8)(x^8 - y^8)$
$= (x^8 + y^8)(x^4 + y^4)(x^4 - y^4)$
$= (x^8 + y^8)(x^4 + y^4)(x^2 + y^2)(x^2 - y^2)$
$= (x^8 + y^8)(x^4 + y^4)(x^2 + y^2)(x + y)(x - y).$

20. $x^{16} - 4y^{16} = (x^8 + 2y^8)(x^8 - 2y^8).$

21. $144a^2b^4 - 9 = 9(4ab^2 + 1)(4ab^2 - 1).$

22. $196 - x^4y^6 = (14 + x^2y^3)(14 - x^2y^3).$

23. $225a^4 - 16b^8c^{12} = (15a^2 + 4b^4c^6)(15a^2 - 4b^4c^6).$

24. $x^{2n} - y^2 = (x^n + y)(x^n - y).$

25. $x^2 - y^{4n} = (x + y^{2n})(x - y^{2n}).$

26. $a^{4n} - b^2 = (a^{2n} + b)(a^{2n} - b).$

27. $a^{6n} - b^{2n} = (a^{3n} + b^n)(a^{3n} - b^n).$

28. $a^{4n} - b^{6n} = (a^{2n} + b^{3n})(a^{2n} - b^{3n}).$

29. $4x^{2n} - y^{4n} = (2x^n + y^{2n})(2x^n - y^{2n}).$

30. $9x^{2n} - 4y^{6n} = (3x^n + 2y^{3n})(3x^n - 2y^{3n}).$

31. $(a + b)^2 - 4 = (a + b + 2)(a + b - 2).$

32. $(x - y)^2 - 4c^2 = (x - y + 2c)(x - y - 2c).$

33. $(a - 2b)^2 - 9x^2 = (a - 2b + 3x)(a - 2b - 3x).$

34. $4(x + y)^2 - 1 = (2x + 2y + 1)(2x + 2y - 1).$

35. $25(3b - c)^2 - 64 = (15b - 5c + 8)(15b - 5c - 8).$

37. $81 - (x - y)^2 = [9 + (x - y)][9 - (x - y)]$
$= (9 + x - y)(9 - x + y).$

38. $4a^2 - (2x + y)^2 = [2a + (2x + y)][2a - (2x + y)]$
$= (2a + 2x + y)(2a - 2x - y).$

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1. $(a - b)^2 - 9x^2 = (a - b + 3x)(a - b - 3x).$

2. $16y^2 - (h + 2k)^2 = [4y + (h + 2k)][4y - (h + 2k)]$
$= (4y + h + 2k)(4y - h - 2k).$

3. $25a^4 - (2b - 3c)^2 = [5a^2 + (2b - 3c)][5a^2 - (2b - 3c)]$
$= (5a^2 + 2b - 3c)(5a^2 - 2b + 3c).$

4. $49x^6 - (7c - 2d)^2 = [7x^3 + (7c - 2d)][7x^3 - (7c - 2d)]$
$= (7x^3 + 7c - 2d)(7x^3 - 7c + 2d).$

5. $1 - (5h - 3k)^2 = [1 + (5h - 3k)][1 - (5h - 3k)]$
$= (1 + 5h - 3k)(1 - 5h + 3k).$

7. $(a - b)^2 - (a + b)^2 = [(a - b) + (a + b)][(a - b) - (a + b)]$
$= (a - b + a + b)(a - b - a - b)$
$= 2a(-2b)$
$= -4ab.$

8. $(a - b)^2 - (x + y)^2 = [(a - b) + (x + y)][(a - b) - (x + y)]$
$= (a - b + x + y)(a - b - x - y).$

9. $(a+2b)^2-(a-3b)^2=[(a+2b)+(a-3b)][(a+2b)-(a-3b)]$
$=(a+2b+a-3b)(a+2b-a+3b)$
$=(2a-b)5b.$

10. $(2a-3b)^2-(3a-2b)^2$
$=[(2a-3b)+(3a-2b)][(2a-3b)-(3a-2b)]$
$=(2a-3b+3a-2b)(2a-3b-3a+2b)$
$=(5a-5b)(-a-b)$
$=5(b-a)(a+b).$

11. $(5h+3k)^2-(2c-9d)^2$
$=[(5h+3k)+(2c-9d)][(5h+3k)-(2c-9d)]$
$=(5h+3k+2c-9d)(5h+3k-2c+9d).$

12. $(4b-a)^2-(7a-6b)^2$
$=[(4b-a)+(7a-6b)][(4b-a)-(7a-6b)]$
$=(4b-a+7a-6b)(4b-a-7a+6b)$
$=(6a-2b)(10b-8a)$
$=4(3a-b)(5b-4a).$

13. $4(a-2b)^2-(2x-y)^2$
$=[2(a-2b)+(2x-y)][2(a-2b)-(2x-y)]$
$=(2a-4b+2x-y)(2a-4b-2x+y).$

14. $9(a+b)^2-(2a-5b)^2$
$=[3(a+b)+(2a-5b)][3(a+b)-(2a-5b)]$
$=(3a+3b+2a-5b)(3a+3b-2a+5b)$
$=(5a-2b)(a+8b).$

15. $(a-2b)^2-4(a+b)^2$
$=[(a-2b)+2(a+b)][(a-2b)-2(a+b)]$
$=(a-2b+2a+2b)(a-2b-2a-2b)$
$=3a(-a-4b)$
$=-3a(a+4b).$

17. $x^2+2ax+a^2-y^2=(x+a)^2-y^2=(x+a-y)(x+a+y).$

18. $x^2+2x+1-4z^2=(x+1)^2-4z^2=(x+1+2z)(x+1-2z).$

19. $c^2-2cd+d^2-16a^2=(c-d)^2-16a^2=(c-d+4a)(c-d-4a).$

20. $4x^2-12cx+9c^2-25y^2=(2x-3c)^2-25y^2$
$=(2x-3c+5y)(2x-3c-5y).$

21. $x^2-y^2-2ax+a^2=x^2-2ax+a^2-y^2$
$=(x-a)^2-y^2$
$=(x-a+y)(x-a-y).$

22. $k^2-g^4-4kh+4h^2=k^2-4kh+4h^2-g^4$
$=(k-2h)^2-g^4$
$=(k-2h+g^2)(k-2h-g^2).$

23. $6xy-c^2+9x^2+y^2=9x^2+6xy+y^2-c^2$
$=(3x+y)^2-c^2$
$=(3x+y-c)(3x+y+c).$

24. $12\,ab - 4\,h^2 + 4\,b^2 + 9\,a^2 = 9\,a^2 + 12\,ab + 4\,b^2 - 4\,h^2$
$= (3\,a + 2\,b)^2 - 4\,h^2$
$= (3\,a + 2\,b - 2\,h)(3\,a + 2\,b + 2\,h).$

25. $1 - 4\,ax + 4\,a^2x^2 - x^2 = (1 - 2\,ax)^2 - x^2$
$= (1 - 2\,ax + x)(1 - 2\,ax - x).$

26. $4\,c^2 - 20\,cd + 25\,d^2 - 9\,d^4 = (2\,c - 5\,d)^2 - (3\,d^2)^2$
$= (2\,c - 5\,d + 3\,d^2)(2\,c - 5\,d - 3\,d^2).$

27. $9\,d^2 - 25\,a^2 - 6\,cd + c^2 = 9\,d^2 - 6\,cd + c^2 - 25\,a^2$
$= (3\,d - c)^2 - 25\,a^2$
$= (3\,d - c + 5\,a)(3\,d - c - 5\,a).$

29. $a^2 - b^2 - 2\,bc - c^2 = a^2 - (b^2 + 2\,bc + c^2)$
$= a^2 - (b + c)^2$
$= (a + b + c)(a - b - c).$

30. $x^2 - a^2 - 2\,ac - c^2 = x^2 - (a^2 + 2\,ac + c^2)$
$= x^2 - (a + c)^2$
$= (x + a + c)(x - a - c).$

31. $y^2 - b^2 + 4\,bc - 4\,c^2 = y^2 - (b^2 - 4\,bc + 4\,c^2)$
$= y^2 - (b - 2\,c)^2$
$= (y - b + 2\,c)(y + b - 2\,c).$

32. $2\,bc - c^2 - b^2 + a^4 = a^4 - (b^2 - 2\,bc + c^2)$
$= a^4 - (b - c)^2$
$= (a^2 - b + c)(a^2 + b - c).$

33. $9\,x^2 - 4\,y^2 - a^2 - 4\,ay = 9\,x^2 - (4\,y^2 + 4\,ay + a^2)$
$= 9\,x^2 - (2\,y + a)^2$
$= (3\,x - 2\,y - a)(3\,x + 2\,y + a).$

34. $6\,x + 9\,y^2 - 9 - x^2 = 9\,y^2 - (x^2 - 6\,x + 9)$
$= 9\,y^2 - (x - 3)^2$
$= (3\,y - x + 3)(3\,y + x - 3).$

35. $4\,bc + 1 - 4\,c^2 - b^2 = 1 - (b^2 - 4\,bc + 4\,c^2)$
$= 1 - (b - 2\,c)^2$
$= (1 - b + 2\,c)(1 + b - 2\,c).$

36. $4\,bc - 4\,b^2 + 4\,x^6 - c^2 = 4\,x^6 - (4\,b^2 - 4\,bc + c^2)$
$= 4\,x^6 - (2\,b - c)^2$
$= (2\,x^3 - 2\,b + c)(2\,x^3 + 2\,b - c).$

38. $a^2 + 2\,ab + b^2 - c^2 - 2\,cd - d^2$
$= (a^2 + 2\,ab + b^2) - (c^2 + 2\,cd + d^2)$
$= (a + b)^2 - (c + d)^2$
$= (a + b + c + d)(a + b - c - d).$

39. $9\,k^2 - 6\,hk + h^2 - 4\,c^2 - 4\,cd - d^2$
$= (9\,k^2 - 6\,hk + h^2) - (4\,c^2 + 4\,cd + d^2)$
$= (3\,k - h)^2 - (2\,c + d)^2$
$= (3\,k - h - 2\,c - d)(3\,k - h + 2\,c + d).$

40. $x^2 - 1 + y^2 - a^2 + 2xy + 2a = (x^2 + 2xy + y^2) - (a^2 - 2a + 1)$
$= (x + y)^2 - (a - 1)^2$
$= (x + y - a + 1)(x + y + a - 1).$

41. $1 + 2bc + 2a - c^2 - b^2 + a^2 = (a^2 + 2a + 1) - (b^2 - 2bc + c^2)$
$= (a + 1)^2 - (b - c)^2$
$= (a + 1 + b - c)(a + 1 - b + c).$

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1. $x^3 - x = x(x^2 - 1) = x(x + 1)(x - 1).$

2. $x^4 - 2x^2 + 1 = (x^2 - 1)^2 = (x + 1)(x + 1)(x - 1)(x - 1).$

3. $x^8 - 2x^4 + 1 = (x^4 - 1)^2$
$= (x^2 + 1)(x^2 - 1)(x^2 + 1)(x^2 - 1)$
$= (x^2 + 1)(x^2 + 1)(x + 1)(x + 1)(x - 1)(x - 1).$

4. $x^5 - 8x^3 + 16x = x(x^4 - 8x^2 + 16)$
$= x(x^2 - 4)(x^2 - 4)$
$= x(x + 2)(x - 2)(x + 2)(x - 2).$

5. $x^5 - x = x(x^4 - 1)$
$= x(x^2 + 1)(x^2 - 1)$
$= x(x^2 + 1)(x + 1)(x - 1).$

6. $x^{10} - x^2 = x^2(x^8 - 1)$
$= x^2(x^4 + 1)(x^4 - 1)$
$= x^2(x^4 + 1)(x^2 + 1)(x^2 - 1)$
$= x^2(x^4 + 1)(x^2 + 1)(x + 1)(x - 1).$

7. $x^4 - 10x^2 + 9 = (x^2 - 1)(x^2 - 9) = (x + 1)(x - 1)(x + 3)(x - 3).$

8. $x^4 - 13x^2 + 36 = (x^2 - 9)(x^2 - 4) = (x + 3)(x - 3)(x + 2)(x - 2).$

9. $3a^2x^4 - 12a^4x^2 + 12a^6 = 3a^2(x^4 - 4a^2x^2 + 4a^4)$
$= 3a^2(x^2 - 2a^2)(x^2 - 2a^2).$

10. $18a^2x^2 - 24a^2x - 10a^2 = 2a^2(9x^2 - 12x - 5)$
$= 2a^2(9x^2 - 15x + 3x - 5)$
$= 2a^2[3x(3x - 5) + (3x - 5)]$
$= 2a^2(3x + 1)(3x - 5).$

11. $3x^4 - 15x^2 + 12 = 3(x^4 - 5x^2 + 4)$
$= 3(x^2 - 1)(x^2 - 4)$
$= 3(x + 1)(x - 1)(x + 2)(x - 2).$

12. $12a - 39ay - 51ay^2 = 3a(4 - 13y - 17y^2)$
$= 3a(4 - 17y + 4y - 17y^2)$
$= 3a[(4 - 17y) + y(4 - 17y)]$
$= 3a(4 - 17y)(1 + y).$

13. $4n^6 + 48n^2 - 28n^4 = 4n^2(n^4 - 7n^2 + 12)$
$= 4n^2(n^2 - 3)(n^2 - 4)$
$= 4n^2(n^2 - 3)(n + 2)(n - 2).$

14. $16x^4 + 8x^2 - 3 = 16x^4 + 12x^2 - 4x^2 - 3$
$= 4x^2(4x^2+3) - (4x^2+3)$
$= (4x^2-1)(4x^2+3)$
$= (2x+1)(2x-1)(4x^2+3).$

15. $a^3 - a + a^2b - b = a(a^2-1) + b(a^2-1)$
$= (a+b)(a^2-1)$
$= (a+b)(a+1)(a-1).$

16. $x^3 - x^2 - 4x + 4 = x^2(x-1) - 4(x-1)$
$= (x-1)(x^2-4)$
$= (x-1)(x+2)(x-2).$

17. $3a^3 + 3a^2 - 27a - 27 = 3(a^3 + a^2 - 9a - 9)$
$= 3[a^2(a+1) - 9(a+1)]$
$= 3(a+1)(a^2-9)$
$= 3(a+1)(a+3)(a-3).$

18. $2a^3b + 3a^2b - 8ab - 12b = b(2a^3 + 3a^2 - 8a - 12)$
$= b[a^2(2a+3) - 4(2a+3)]$
$= b(2a+3)(a^2-4)$
$= b(2a+3)(a+2)(a-2).$

19. $4a^2 - a^4 + 81 + 10a^2x - 36a - 25x^2$
$= (4a^2 - 36a + 81) - (a^4 - 10a^2x + 25x^2)$
$= (2a-9)^2 - (a^2-5x)^2$
$= (2a - 9 - a^2 + 5x)(2a - 9 + a^2 - 5x).$

20. $12cd^3 - 6a^3x - a^6 + 4c^2 + 9d^6 - 9x^2$
$= (4c^2 + 12cd^3 + 9d^6) - (a^6 + 6a^3x + 9x^2)$
$= (2c + 3d^3)^2 - (a^3 + 3x)^2$
$= (2c + 3d^3 - a^3 - 3x)(2c + 3d^3 + a^3 + 3x).$

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1. $x^3 + 1 = (x+1)(x^2 - x + 1).$
2. $x^3 - 1 = (x-1)(x^2 + x + 1).$
3. $a^3 - 8 = (a-2)(a^2 + 2a + 4).$
4. $27 + b^3 = (3+b)(9 - 3b + b^2).$
5. $64x^3 - 1 = (4x-1)(16x^2 + 4x + 1).$
6. $x^3 - y^6 = (x - y^2)(x^2 + xy^2 + y^4).$
7. $x^3 + 8y^3 = (x + 2y)(x^2 - 2xy + 4y^2).$
8. $x^3 + y^9 = (x + y^3)(x^2 - xy^3 + y^6).$
9. $8x^3 - 27y^3 = (2x - 3y)[(2x)^2 + (2x)(3y) + (3y)^2]$
$= (2x - 3y)(4x^2 + 6xy + 9y^2).$
10. $27a^3 + 64b^3 = (3a + 4b)(9a^2 - 12ab + 16b^2).$
11. $125y^3 + 8x^3 = (5y + 2x)(25y^2 - 10xy + 4x^2).$
12. $216 - 27x^3 = 27(8 - x^3) = 27(2 - x)(4 + 2x + x^2).$
13. $x^6 + y^9 = (x^2 + y^3)(x^4 - x^2y^3 + y^6).$
14. $a^6 + b^6 = (a^2 + b^2)(a^4 - a^2b^2 + b^4).$
15. $a^6 - b^6 = (a^3 - b^3)(a^3 + b^3)$
$= (a-b)(a^2 + ab + b^2)(a+b)(a^2 - ab + b^2).$

16. $x^6 - 64 = (x^3 + 8)(x^3 - 8)$
$= (x + 2)(x^2 - 2x + 4)(x - 2)(x^2 + 2x + 4).$

17. $64x^6 + 1 = (4x^2 + 1)(16x^4 - 4x^2 + 1).$

18. $1 - 64x^6 = (1 - 8x^3)(1 + 8x^3)$
$= (1 - 2x)(1 + 2x + 4x^2)(1 + 2x)(1 - 2x + 4x^2).$

19. $a^6b^6 - 64 = (a^3b^3 - 8)(a^3b^3 + 8)$
$= (ab - 2)(a^2b^2 + 2ab + 4)(ab + 2)(a^2b^2 - 2ab + 4).$

20. $x^{12} + y^{12} = (x^4)^3 + (y^4)^3 = (x^4 + y^4)(x^8 - x^4y^4 + y^8).$

21. $27x^{27} - 1 = (3x^9)^3 - 1^3 = (3x^9 - 1)(9x^{18} + 3x^9 + 1).$

22. $k^{12} - y^6 = (k^6 + y^3)(k^6 - y^3)$
$= [(k^2)^3 + y^3][(k^2)^3 - y^3]$
$= (k^2 + y)(k^4 - k^2y + y^2)(k^2 - y)(k^4 + k^2y + y^2).$

23. $x^{3a} + 1 = (x^{2a} - x^a + 1)(x^a + 1).$

24. $x^{3a} - y^{3a} = (x^a - y^a)(x^{2a} + x^ay^a + y^{2a}).$

25. $x^{3a} + y^{6a} = (x^a + y^{2a})(x^{2a} - x^ay^{2a} + y^{4a}).$

26. $8x^{6a} - y^{9a} = (2x^{2a})^3 - (y^{3a})^3 = (2x^{2a} - y^{3a})(4x^{4a} + 2x^{2a}y^{3a} + y^{6a}).$

27. $27x^{3m} - 64 = (3x^m - 4)(9x^{2m} + 12x^m + 16).$

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1. $x^6 + 2x^3 + 1 = (x^3 + 1)(x^3 + 1)$
$= (x + 1)(x^2 - x + 1)(x + 1)(x^2 - x + 1).$

2. $x^6 - 2x^3y^3 + y^6 = (x^3 - y^3)(x^3 - y^3)$
$= (x - y)(x^2 + xy + y^2)(x - y)(x^2 + xy + y^2).$

3. $x^4 + x = x(x^3 + 1) = x(x + 1)(x^2 - x + 1).$

4. $x^5 - x^2 = x^2(x^3 - 1) = x^2(x - 1)(x^2 + x + 1).$

5. $8x^3 - 64 = 8(x^3 - 8) = 8(x - 2)(x^2 + 2x + 4).$

6. $8x^6 + 216x^3 = 8x^3(x^3 + 27) = 8x^3(x + 3)(x^2 - 3x + 9).$

7. $x^5 - x^3y^2 - x^2y^3 + y^5 = x^3(x^2 - y^2) - y^3(x^2 - y^2)$
$= (x^3 - y^3)(x^2 - y^2)$
$= (x - y)(x^2 + xy + y^2)(x + y)(x - y).$

8. $x^5 + x^3 + 8x^2 + 8 = x^3(x^2 + 1) + 8(x^2 + 1)$
$= (x^3 + 8)(x^2 + 1)$
$= (x + 2)(x^2 - 2x + 4)(x^2 + 1).$

9. $x^7 - x^4 - 16x^3 + 16 = x^4(x^3 - 1) - 16(x^3 - 1)$
$= (x^4 - 16)(x^3 - 1)$
$= (x^2 + 4)(x^2 - 4)(x^3 - 1)$
$= (x^2 + 4)(x + 2)(x - 2)(x - 1)(x^2 + x + 1).$

10. $x^6 - 7x^3 - 8 = (x^3 + 1)(x^3 - 8)$
$= (x + 1)(x^2 - x + 1)(x - 2)(x^2 + 2x + 4).$

11. $8x^6 - 3 - 23x^3 = 8x^6 - 24x^3 + x^3 - 3$
$= (x^3 - 3)(2x + 1)(4x^2 - 2x + 1).$

12. $x^4 + x^3 - x - 1 = x^3(x + 1) - (x + 1) = (x + 1)(x - 1)(x^2 + x + 1).$

13. $x^6 - x^2y^2 = x^2(x^2 + y)(x^2 - y).$

14. $x^8 + 64x^2 = x^2(x^6 + 64)$
$$= x^2[(x^2)^3 + 4^3]$$
$$= x^2(x^2 + 4)(x^4 - 4x^2 + 16).$$

16. $1 - 3x + 3x^2 - x^3 = (1 - x^3) - (3x - 3x^2)$
$$= (1 - x)(1 + x + x^2) - 3x(1 - x)$$
$$= (1 - x)(1 + x + x^2 - 3x)$$
$$= (1 - x)(1 - 2x + x^2)$$
$$= (1 - x)(1 - x)(1 - x).$$

17. $x^3 + 3x^2y + 3xy^2 + y^3 = (x^3 + y^3) + (3x^2y + 3xy^2)$
$$= (x + y)(x^2 - xy + y^2) + 3xy(x + y)$$
$$= (x + y)(x^2 - xy + y^2 + 3xy)$$
$$= (x + y)(x^2 + 2xy + y^2)$$
$$= (x + y)(x + y)(x + y).$$

18. $x^3 - 3x^2y + 3xy^2 - y^3 = (x^3 - y^3) - (3x^2y - 3xy^2)$
$$= (x - y)(x^2 + xy + y^2) - 3xy(x - y)$$
$$= (x - y)(x^2 + xy + y^2 - 3xy)$$
$$= (x - y)(x^2 - 2xy + y^2)$$
$$= (x - y)(x - y)(x - y).$$

19. $x^3 + 2x^2 + 4x + 8 = x^2(x + 2) + 4(x + 2)$
$$= (x + 2)(x^2 + 4).$$

20. $8 - 4x + 2x^2 - x^3 = 4(2 - x) + x^2(2 - x)$
$$= (2 - x)(4 + x^2).$$

21. $x^3 + 9x^2 + 27x + 27 = x^3 + 27 + 9x^2 + 27x$
$$= (x + 3)(x^2 - 3x + 9) + 9x(x + 3)$$
$$= (x + 3)(x^2 - 3x + 9 + 9x)$$
$$= (x + 3)(x + 3)(x + 3).$$

22. $x^3 - 9x^2 + 27x - 27 = x^3 - 27 - 9x^2 + 27x$
$$= (x - 3)(x^2 + 3x + 9) - 9x(x - 3)$$
$$= (x - 3)(x^2 + 3x + 9 - 9x)$$
$$= (x - 3)(x - 3)(x - 3).$$

23. $x^3 + 12x^2 + 48x + 64 = (x^3 + 64) + (12x^2 + 48x)$
$$= (x + 4)(x^2 - 4x + 16) + 12x(x + 4)$$
$$= (x + 4)(x^2 - 4x + 16 + 12x)$$
$$= (x + 4)(x^2 + 8x + 16)$$
$$= (x + 4)(x + 4)(x + 4).$$

24. $64 - 48x + 12x^2 - x^3 = (64 - x^3) - (48x - 12x^2)$
$$= (4 - x)(16 + 4x + x^2) - 12x(4 - x)$$
$$= (4 - x)(16 + 4x + x^2 - 12x)$$
$$= (4 - x)(16 - 8x + x^2)$$
$$= (4 - x)(4 - x)(4 - x).$$

25. $8x^3 - 12x^2 + 6x - 1 = (8x^3 - 1) - (12x^2 - 6x)$
$= (2x - 1)(4x^2 + 2x + 1) - 6x(2x - 1)$
$= (2x - 1)^3.$

26. $x^4 + x^3 + x^2 - 1 = x^3(x + 1) + (x - 1)(x + 1)$
$= (x + 1)(x^3 + x - 1).$

27. $x^3 + a^3 + x^2 - a^2 = (x + a)(x^2 - ax + a^2) + (x + a)(x - a)$
$= (x + a)(x^2 - ax + a^2 + x - a).$

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1. $x^4 + 16x^2y^2 + 64y^4 = (x^2 + 8y^2)(x^2 + 8y^2).$
2. $4a^2x^4 + 4a^2x^2 - 120a^2 = 4a^2(x^4 + x^2 - 30) = 4a^2(x^2 + 6)(x^2 - 5).$
3. $9x^2 + 27x + 14 = 9x^2 + 6x + 21x + 14$
$= 3x(3x + 2) + 7(3x + 2)$
$= (3x + 7)(3x + 2).$
4. $3x^4 + x^2 - 2 = 3x^4 + 3x^2 - 2x^2 - 2$
$= 3x^2(x^2 + 1) - 2(x^2 + 1)$
$= (3x^2 - 2)(x^2 + 1).$
5. $3x^3 - 75x = 3x(x^2 - 25) = 3x(x + 5)(x - 5).$
6. $x^2 + 2a^2x^2 + x^2a^4 = x^2(1 + 2a^2 + a^4) = x^2(1 + a^2)(1 + a^2).$
7. $x^3 - 27 = (x - 3)(x^2 + 3x + 9).$
8. $x^2 - xz + 2xy - 2yz = x(x - z) + 2y(x - z)$
$= (x + 2y)(x - z).$
9. $2ac - bc + 6ad - 3bd = c(2a - b) + 3d(2a - b)$
$= (c + 3d)(2a - b).$
10. $25x^2y^2 + 16x^4 - 40x^3y = x^2(25y^2 - 40xy + 16x^2)$
$= x^2(5y - 4x)(5y - 4x).$
11. $x^4 - 15x^3 + 56x^2 = x^2(x^2 - 15x + 56) = x^2(x - 8)(x - 7).$
12. $9x^6 - 6x^3 - 35 = 9x^6 - 21x^3 + 15x^3 - 35$
$= 3x^3(3x^3 - 7) + 5(3x^3 - 7)$
$= (3x^3 + 5)(3x^3 - 7).$
13. $a^6 - 4096 = (a^3 + 64)(a^3 - 64)$
$= (a + 4)(a^2 - 4a + 16)(a - 4)(a^2 + 4a + 16)$
14. $a^{2x+2} - 1 = (a^{x+1} + 1)(a^{x+1} - 1).$
15. $4x^4 - 13x^2y^2 + 9y^4 = 4x^4 - 4x^2y^2 - 9x^2y^2 + 9y^4$
$= 4x^2(x^2 - y^2) - 9y^2(x^2 - y^2)$
$= (4x^2 - 9y^2)(x^2 - y^2)$
$= (2x + 3y)(2x - 3y)(x + y)(x - y).$
16. $216 + x^3 = 6^3 + x^3 = (6 + x)(36 - 6x + x^2).$
17. $x^9 - 512 = (x^3)^3 - 8^3$
$= (x^3 - 8)[(x^3)^2 + (x^3)8 + 8^2]$
$= (x - 2)(x^2 + 2x + 4)(x^6 + 8x^3 + 64).$

18. $x^2 - 4y^2 + 9 - 6x = x^2 - 6x + 9 - 4y^2$
$= (x-3)^2 - 4y^2$
$= (x - 3 + 2y)(x - 3 - 2y).$

19. $ah - 2ak + 2bh - 4bk = a(h - 2k) + 2b(h - 2k)$
$= (h - 2k)(a + 2b).$

20. $4x^4 - 25y^8 = (2x^2 + 5y^4)(2x^2 - 5y^4).$

21. $a^2 - 25c^2 + 10ab + 25b^2 = (a^2 + 10ab + 25b^2) - 25c^2$
$= (a + 5b)^2 - 25c^2$
$= (a + 5b + 5c)(a + 5b - 5c).$

22. $y^6 - 343 = (y^2)^3 - 7^3 = (y^2 - 7)(y^4 + 7y^2 + 49).$

23. $9x^4 - 9x^2 - 28 = 9x^4 - 21x^2 + 12x^2 - 28$
$= 3x^2(3x^2 - 7) + 4(3x^2 - 7)$
$= (3x^2 + 4)(3x^2 - 7).$

24. $9x^2y^2 + 39xy^2 - 30y^2 = 3y^2(3x^2 + 13x - 10)$
$= 3y^2(3x^2 + 15x - 2x - 10)$
$= 3y^2[3x(x + 5) - 2(x + 5)]$
$= 3y^2(3x - 2)(x + 5).$

25. $3x^6y^2 + 63x^3y^2 - 300y^2 = 3y^2(x^6 + 21x^3 - 100)$
$= 3y^2(x^3 + 25)(x^3 - 4).$

26. $12 - 15a + 16x - 20ax = 3(4 - 5a) + 4x(4 - 5a)$
$= (4 - 5a)(3 + 4x).$

27. $2cx + 3dx - 2cy - 3dy = x(2c + 3d) - y(2c + 3d)$
$= (x - y)(2c + 3d).$

28. $y^4 - 2y^2z^4 + z^8 = (y^2 - z^4)(y^2 - z^4) = (y + z^2)(y - z^2)(y + z^2)(y - z^2).$

29. $81c^{10} - 64d^{10} = (9c^5 - 8d^5)(9c^5 + 8d^5).$

30. $100x^8 - 220x^4y^2 + 121y^4 = (10x^4 - 11y^2)(10x^4 - 11y^2).$

31. $27y^3 - 512 = (3y)^3 - 8^3 = (3y - 8)(9y^2 + 24y + 64).$

32. $81 - 16x^8 = (9 + 4x^4)(9 - 4x^4) = (9 + 4x^4)(3 + 2x^2)(3 - 2x^2).$

33. $25x^4 - 3025x^8 = 25x^4(1 - 121x^4) = 25x^4(1 + 11x^2)(1 - 11x^2).$

34. $16c^7 - 4c^5 - 72c^3 = 4c^3(4c^4 - c^2 - 18)$
$= 4c^3(4c^4 + 8c^2 - 9c^2 - 18)$
$= 4c^3[4c^2(c^2 + 2) - 9(c^2 + 2)]$
$= 4c^3(c^2 + 2)(4c^2 - 9)$
$= 4c^3(c^2 + 2)(2c + 3)(2c - 3).$

35. $c^3 + c^2d + 3cd^2 + 3d^3 = c^2(c + d) + 3d^2(c + d) = (c^2 + 3d^2)(c + d).$

36. $1 - 18x^4 + 81x^8 = (1 - 9x^4)(1 - 9x^4)$
$= (1 + 3x^2)(1 - 3x^2)(1 + 3x^2)(1 - 3x^2).$

37. $x^4 + 12x^2 - 64 = (x^2 + 16)(x^2 - 4) = (x^2 + 16)(x + 2)(x - 2).$

38. $c^9 - 512c^{12} = c^9(1 - 512c^3)$
$= c^9[1 - (8c)^3]$
$= c^9(1 - 8c)(1 + 8c + 64c^2).$

39. $h^{6a} - k^{8a} = (h^{3a} + k^{4a})(h^{3a} - k^{4a}).$

40. $a^2 - b^4 - 12\,ab + 36\,b^2 = a^2 - 12\,ab + 36\,b^2 - b^4$
$= (a - 6\,b)^2 - b^4$
$= (a - 6\,b - b^2)(a - 6\,b + b^2).$

41. $5\,h^2 + 21\,hk - 20\,k^2 = 5\,h^2 - 4\,hk + 25\,hk - 20\,k^2$
$= h(5\,h - 4\,k) + 5\,k(5\,h - 4\,k)$
$= (5\,h - 4\,k)(h + 5\,k).$

42. $x^6 - x^3 - 2 = (x^3 - 2)(x^3 + 1) = (x^3 - 2)(x + 1)(x^2 - x + 1).$

43. $2\,x^2y^2 - y^4 - x^4 = -1(x^4 - 2\,x^2y^2 + y^4)$
$= -1(x^2 - y^2)(x^2 - y^2)$
$= (y^2 - x^2)(x^2 - y^2)$
$= (y + x)(y - x)(x + y)(x - y).$

44. $5\,h^3 - 9\,h^2k - 9\,k^3 + 5\,hk^2 = (5\,h^3 + 5\,hk^2) - (9\,h^2k + 9\,k^3)$
$= 5\,h(h^2 + k^2) - 9\,k(h^2 + k^2)$
$= (h^2 + k^2)(5\,h - 9\,k).$

45. $3\,x^2 - 17\,x + 10 = 3\,x^2 - 15\,x - 2\,x + 10$
$= 3\,x(x - 5) - 2(x - 5)$
$= (x - 5)(3\,x - 2).$

46. $9\,x^{5n} - x^{3n} = x^{3n}(9\,x^{2n} - 1) = x^{3n}(3\,x^n + 1)(3\,x^n - 1).$

47. $27\,a - 18\,ab^2 - 3\,a^9 + 3\,ab^4 = 3\,a(9 - 6\,b^2 - a^8 + b^4)$
$= 3\,a[(9 - 6\,b^2 + b^4) - a^8]$
$= 3\,a[(3 - b^2)^2 - a^8]$
$= 3\,a(3 - b^2 - a^4)(3 - b^2 + a^4).$

48. $1 + 2\,x^2 - 3\,x^4 = -1(3\,x^4 - 2\,x^2 - 1)$
$= -1(3\,x^4 - 3\,x^2 + x^2 - 1)$
$= -1[3\,x^2(x^2 - 1) + (x^2 - 1)]$
$= -1(x^2 - 1)(3\,x^2 + 1)$
$= -1(x + 1)(x - 1)(3\,x^2 + 1)$
$= (x + 1)(1 - x)(3\,x^2 + 1).$

49. $x^4 - x^2 - 12 = (x^2 - 4)(x^2 + 3) = (x + 2)(x - 2)(x^2 + 3).$

50. $16\,x^4 + 8\,x^2 - 3 = 16\,x^4 - 4\,x^2 + 12\,x^2 - 3$
$= 4\,x^2(4\,x^2 - 1) + 3(4\,x^2 - 1)$
$= (4\,x^2 - 1)(4\,x^2 + 3)$
$= (2\,x + 1)(2\,x - 1)(4\,x^2 + 3).$

51. $1024 - 64\,h^3 + h^6 = (32 - h^3)(32 - h^3).$

52. $h^2k^2 - h^2 - k^2 + 1 = h^2(k^2 - 1) - (k^2 - 1)$
$= (k^2 - 1)(h^2 - 1)$
$= (k + 1)(k - 1)(h + 1)(h - 1).$

53. $12 - 2\,h - 4\,h^2 = -2(2\,h^2 + h - 6)$
$= -2(2\,h^2 - 3\,h + 4\,h - 6)$
$= -2[h(2\,h - 3) + 2(2\,h - 3)]$
$= -2(2\,h - 3)(h + 2)$
$= 2(3 - 2\,h)(h + 2).$

54. $3 - y^2 + 3y^3 - y^5 = (3 - y^2) + y^3(3 - y^2)$
$= (3 - y^2)(1 + y^3)$
$= (3 - y^2)(1 + y)(1 - y + y^2).$

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1. $x^2 - 9 = 0.$
Factoring the left member,
$(x + 3)(x - 3) = 0.$
Whence $x = -3$ or $+3.$

2. $x^2 = 16.$
Then
$(x + 4)(x - 4) = 0.$
Whence $x = -4$ or $+4.$

3. $x^2 - 3x = 0.$
Then $x(x - 3) = 0.$
Whence $x = 0$ or $3.$

4. $x^2 = 7x.$
Then $x(x - 7) = 0.$
Whence $x = 0$ or $7.$

5. $x^2 - 7x = -12.$
Then
$(x - 3)(x - 4) = 0.$
Whence $x = 3$ or $4.$

6. $x^2 - x = 20.$
Then
$(x - 5)(x + 4) = 0.$
Whence $x = 5$ or $-4.$

7. $3x^2 - 15x = 0.$
Then $3x(x - 5) = 0.$
Whence $x = 0$ or $5.$

8. $5x^2 + 35x = 0.$
Then $5x(x + 7) = 0.$
Whence $x = 0$ or $-7.$

9. $8 - 9x = -x^2.$
Then
$(x - 8)(x - 1) = 0.$
Whence $x = 8$ or $1.$

10. $12x - 28 = -x^2.$
Then
$(x + 14)(x - 2) = 0.$
Whence $x = -14$ or $2.$

11. $4x^2 = 16x.$
Then $4x(x - 4) = 0.$
Whence $x = 0$ or $4.$

12. $x^2 - a^2 = 0.$
Then
$(x + a)(x - a) = 0.$
Whence $x = -a$ or $a.$

13. $x^2 = 9b^2.$
Then
$(x + 3b)(x - 3b) = 0.$
Whence $x = -3b$ or $3b.$

14. $x^2 - 2ax + a^2 = 0.$
Then
$(x - a)(x - a) = 0.$
Whence $x = a.$

15. $x^2 + 4b^2 = 4bx.$
Then
$(x - 2b)(x - 2b) = 0.$
Whence $x = 2b.$

16.
$x^2 + ax + 3x + 3a = 0.$
Then
$(x + a)(x + 3) = 0.$
Whence $x = -a$ or $-3.$

17. $x^2 + bx = 4x + 4b.$
Then
$(x - 4)(x + b) = 0.$
Whence $x = 4$ or $-b.$

18. $x^2 + ax + bx + ab = 0.$

Then $(x + a)(x + b) = 0.$

Whence $x = -a$ or $-b.$

19. $4x^2 + 8x + 3 = 0.$

Then $(2x + 1)(2x + 3) = 0.$

Whence $x = -\frac{1}{2}$ or $-\frac{3}{2}.$

20. $9x^2 = 3x + 2.$

Then $(3x - 2)(3x + 1) = 0.$

Whence $x = \frac{2}{3}$ or $-\frac{1}{3}.$

21. $16x^2 - 12x = 10.$

Then $(4x - 5)(2x + 1) = 0.$

Whence $x = \frac{5}{4}$ or $-\frac{1}{2}.$

22. $50x + 24 = 25x^2.$

Then $(5x - 12)(5x + 2) = 0.$

Whence $x = \frac{12}{5}$ or $-\frac{2}{5}.$

23. $x^2 + bx = 0.$

Then $x(x + b) = 0.$

Whence $x = 0$ or $-b.$

24. $x^2 - ax - bx = 0.$

Then $x(x - a - b) = 0.$

Whence $x = 0$ or $a + b.$

25. $x^2 + 3x = ax.$

Then $x(x + 3 - a) = 0.$

Whence $x = 0$ or $a - 3.$

26. $4x = x^2 + 4.$

Then $(x - 2)(x - 2) = 0.$

Whence $x = 2.$

27. $x^3 - 9x = 0.$

Then $x(x + 3)(x - 3) = 0.$

Whence $x = 0, -3,$ or $3.$

28. $x^3 + x^2 = 4x + 4.$

Then $(x + 2)(x - 2)(x + 1) = 0.$

Whence $x = -2, 2,$ or $-1.$

29. $x^3 - 5x^2 + 6x = 0.$

Then $x(x - 3)(x - 2) = 0.$

Whence $x = 0, 3,$ or $2.$

30. $2x^3 - x^2 = 32x - 16.$

Then $(x - 4)(x + 4)(2x - 1) = 0.$

Whence $x = 4, -4,$ or $\frac{1}{2}.$

31. $5x^2 + x^3 = 45 + 9x.$

Then $(x + 3)(x - 3)(x + 5) = 0.$

Whence $x = -3, 3,$ or $-5.$

32. $x^4 - 5x^2 + 4 = 0.$

Then $(x + 2)(x - 2)(x + 1)(x - 1) = 0.$

Whence $x = -2, 2, -1,$ or $1.$

33. $9 + x^4 = 10x^2.$

Then $(x + 3)(x - 3)(x + 1)(x - 1) = 0.$

Whence $x = -3, 3, -1,$ or $1.$

34. $x^4 - 36x^2 = 0.$
Then
$x^2(x+6)(x-6) = 0.$
Whence $x = 0, -6, \text{ or } 6.$

35. $x^4 + 15x^2 = 8x^3.$
Then
$x^2(x-5)(x-3) = 0.$
Whence $x = 0, 5, \text{ or } 3.$

36. $x^4 - 2x^2 + 1 = 0.$
Then $(x+1)^2(x-1)^2 = 0.$
Whence $x = -1 \text{ or } 1.$

37. It does not follow from $(x+1)(x-1) = x - 1$ that $x + 1 = 1$, for by the condition of the problem $x - 1 = 0$, and it is not permissible to divide each member of an equation by zero.

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1. Let $n =$ the number.
Then $n^2 + n = 90,$
or $(n-9)(n+10) = 0.$
Whence $n = 9 \text{ or } -10.$

2. Let $n =$ the number.
Then $n^2 - 2n = 35,$
or $(n-7)(n+5) = 0.$
Whence $n = 7 \text{ or } -5.$

3. Let $n =$ the number.
Then $n^2 + 2n + 5 = 148,$
or $(n+13)(n-11) = 0.$
Whence $n = -13 \text{ or } 11.$

4. Let $n =$ the number.
Then $4n^2 = 7n,$
or $n(n - \frac{7}{4}) = 0.$
Whence $n = 0 \text{ or } \frac{7}{4}.$

5. Let $n =$ the number.
Then
$(n+20)(n+21) = 930,$
or $(n-10)(n+51) = 0.$
Whence $n = 10 \text{ or } -51.$

6. Let $n =$ the number
Then
$(17-n)(23-n) = 216,$
or $(n-35)(n-5) = 0.$
Whence $n = 35 \text{ or } 5.$

7. Let $n =$ the number
Then
$(27-n)(n+21) = 540,$
or $(n-9)(n+3) = 0.$
Whence $n = 9 \text{ or } -3.$

8. Let $n =$ the number.
Then $(n+15)(22-n) = 70 + 23n,$
or $(n-10)(n+26) = 0.$
Whence $n = 10 \text{ or } -26.$

9. Let $n =$ the one number.
Then $n + 6 =$ the other.
Therefore $(n+6)^2 - n^2 = 120.$
Whence $n = 7,$
and $n + 6 = 13.$
Also $n^2 - (n+6)^2 = 120.$
Whence $n = -13,$
and $n + 6 = -7.$
The negative numbers are rejected.

10. Let $n =$ the number.

Then $9n^2 - 5n = 8n^2$,

or $n(n-5) = 0$.

Whence $n = 0$ or 5.

11. Let $f =$ the frontage in feet.

Then $4f =$ the depth in feet.

Therefore $4f^2 = 2500$,

or $(f-25)(f+25) = 0$.

Whence $f = 25$ or -25,

and $4f = 100$ or -100.

The negative lengths, being meaningless, are rejected.

12. Let $b =$ the breadth in feet.

Then $b + 6 =$ the length in feet.

Therefore $b(b+6) = 216$,

or $(b+18)(b-12) = 0$.

Whence $b = -18$ or 12,

and $b + 6 = -12$ or 18.

The negative values are rejected.

13. Let $w =$ the width in rods.

Then $w + 6 =$ the length in rods.

Therefore $w(w+6) = 24 \cdot 9$,

or $(w+18)(w-12) = 0$.

Whence $w = -18$ or 12,

and $w + 6 = -12$ or 18.

The negative values are rejected.

14. Let $n =$ the first number.

Then $n + 1 =$ the second.

Hence $n^2 + (n+1)^2 = 145$,

or $(n+9)(n-8) = 0$.

Whence $n = -9$ or 8,

and $n + 1 = -8$ or 9.

Therefore the numbers are -9 and -8, or 8 and 9.

15. Let $n =$ the first odd number.

Then $n + 2 =$ the second.

Hence $n^2 + (n+2)^2 = 290$,

or $(n+13)(n-11) = 0$.

Whence $n = -13$ or 11,

and $n + 2 = -11$ or 13.

Therefore the numbers are -13 and -11, or 11 and 13.

16. Let n, $n+2$, and $n+4$ represent the odd numbers.

Then $$n^2 + (n+2)^2 + (n+4)^2 = 251,$$
or $$(n+11)(n-7) = 0.$$
Whence $$n = -11 \text{ or } 7.$$
Therefore the numbers are -11, -9, and -7; or 7, 9, and 11.

17. Let $n =$ the side of the square in inches.

Then $$n^2 + 4 \cdot 8 \cdot n = 185,$$
or $$(n+37)(n-5) = 0.$$
Whence $$n = -37 \text{ or } 5.$$
Therefore n is 5.

18. Let $x =$ the edge of the cube in inches.

Then $$6x^2 = 294,$$
or $$(x-7)(x+7) = 0.$$
Whence $$x = 7 \text{ or } -7.$$
Therefore the edge of the cube is 7 inches.

19. Let $d =$ the depth in feet,

$4d =$ the length in feet,

and $3d =$ the width in feet.

Then $$2 \cdot d \cdot 4d + 2 \cdot d \cdot 3d + 2 \cdot 4d \cdot 3d = 608,$$
or $$(d-4)(d+4) = 0.$$
Whence $$d = 4 \text{ or } -4.$$
Therefore the depth is 4 feet, the length is 16 feet, and the width is 12 feet.

20. Let $x =$ the depth in inches.

Then $x + 3 =$ the length in inches,

and $x + 1 =$ the width in inches.

$$2x(x+3) + 2x(x+1) + 2(x+3)(x+1) = 62,$$
or $$(3x+14)(x-2) = 0.$$
Whence $$x = -4\tfrac{2}{3} \text{ or } 2.$$
Therefore the depth is 2 inches, the length is 5 inches, and the width is 3 inches.

21. Let $b =$ the base of the triangle in feet.

Then $$\frac{6 \cdot b}{2} = 30.$$
Whence $$b = 10.$$

22. Let $b =$ the base of the triangle in feet.

Then $3b =$ the altitude in feet.

Therefore $$\frac{3b \cdot b}{2} = 54,$$

or $(b-6)(b+6)=0.$

Whence $b=6$ or $-6.$

Hence the base is 6 feet and the altitude is 18 feet.

23. Let $a=$ the altitude in feet.

Then $5a=$ the base in feet.

Therefore $\dfrac{5a\cdot a}{2}=40,$

or $(a-4)(a+4)=0.$

Whence $a=4$ or $-4.$

Hence the altitude is 4 feet and the base is 20 feet.

24. Let $a=$ the altitude in meters.

Then $6a=$ the base in meters.

Therefore $\dfrac{6a\cdot a}{2}=75,$

or $(a-5)(a+5)=0.$

Whence $a=5$ or $-5.$

Hence the altitude is 5 meters and the base is 30 meters.

25. If $x=$ the base in feet,

then $x+2=$ the altitude in feet.

Therefore $\dfrac{x(x+2)}{2}=24,$

or $(x-6)(x+8)=0.$

Whence $x=6$ or $-8.$

Therefore the base is 6 feet and the altitude is 8 feet.

26. Let $b=$ the base in feet.

Then $b+3=$ the altitude in feet.

Therefore $\dfrac{b(b+3)}{2}=54,$

or $(b-9)(b+12)=0.$

Whence $b=9$ or $-12.$

Therefore the base is 9 feet and the altitude is 12 feet.

27. If x and $x+2$ represent the legs in feet,

then $\dfrac{x(x+2)}{2}=24,$

or $(x-6)(x+8)=0.$

Whence $x=6$ or $-8.$

Therefore the legs are 6 feet and 8 feet.

28. If x and $x+7$ represent the legs in feet,

then $$\frac{x(x+7)}{2}=30,$$

or $$(x-5)(x+12)=0.$$

Whence $$x=5 \text{ or } -12.$$

Therefore the legs are 5 feet and 12 feet.

29. Let $a=$ the altitude in inches.

Then $6+2a=$ the base in inches.

Therefore $$\frac{a(6+2a)}{2}=2\tfrac{5}{8}\cdot 144,$$

or $$(a-18)(a+21)=0.$$

Whence $$a=18 \text{ or } -21.$$

Therefore the altitude is 18 inches and the base is 42 inches.

30. Let $b=$ the base in feet.

Then $6+3b=$ the altitude in feet.

Therefore $$\frac{b(6+3b)}{2}=4\cdot 9,$$

or $$(b-4)(b+6)=0.$$

Whence $$b=4 \text{ or } -6.$$

Therefore the base is 4 feet and the altitude is 18 feet.

31. Let $b=$ the base in centimeters.

Then $b-5=$ the altitude in centimeters.

Therefore $$\frac{b(b-5)}{2}=150,$$

or $$(b+15)(b-20)=0.$$

Whence $$b=-15 \text{ or } 20.$$

Therefore the base is 20 centimeters and the altitude is 15 centimeters.

32. $\dfrac{12(10+18)}{2}=168$, area.

33. If x and $x+4$ represent the bases in inches,

then $$8x+16=96.$$

Whence $$x=10,$$

and $$x+4=14.$$

34. If $12=$ one base in feet,

and $a=$ the altitude in feet,

then $2a=$ the other base in feet.

Therefore $$\frac{a(2a+12)}{2}=112,$$

or $$(a-8)(a+14)=0.$$

Whence $$a=8 \text{ or } -14.$$

Therefore the altitude is 8 feet and the other base is 16 feet.

35. If $b =$ the longer base in feet,

then $\dfrac{2b}{3} =$ the other base in feet,

and $\dfrac{b}{3} =$ the altitude in feet.

Hence $$\frac{b}{3}\cdot\frac{1}{2}\left(b+\frac{2b}{3}\right)=360,$$

or $$(b-36)(b+36)=0.$$

Whence $$b=36 \text{ or } -36.$$

Therefore the longer base is 36 feet, the other base is 24 feet, and the altitude is 12 feet.

36. If $a =$ the altitude in feet,

then $a+4 =$ the shorter base in feet,

and $a+6 =$ the longer base in feet.

Therefore $$\frac{a(a+4+a+6)}{2}=66,$$

or $$(a-6)(a+11)=0.$$

Whence $$a=6 \text{ or } -11.$$

Therefore the altitude is 6 feet, the shorter base is 10 feet, and the longer base is 12 feet.

37. If $a =$ the altitude in feet,

then $a+8 =$ the shorter base in feet,

and $a+12 =$ the longer base in feet.

Therefore $$\frac{a(a+8+a+12)}{2}=16\cdot 9,$$

or $$(a-8)(a+18)=0.$$

Whence $$a=8 \text{ or } -18.$$

Therefore the altitude is 8 feet, the shorter base is 16 feet, and the longer base is 20 feet.

38. If $b =$ the shorter base in feet,

then $b+4 =$ the longer base in feet,

and $b+2 =$ the altitude in feet.

Therefore $$\frac{(b+2)(b+b+4)}{2}=4\cdot 9,$$

or $$(b-4)(b+8)=0.$$

Whence $$b = 4 \text{ or } -8.$$

Therefore the shorter base is 4 feet, the longer base is 8 feet, and the altitude is 6 feet.

39. If $b =$ one base in feet,

then $b =$ the altitude in feet,

and $b + 2 =$ the other base in feet.

Therefore $$\frac{b(b + b + 2)}{2} = 10 \cdot 9,$$

or $$(b - 9)(b + 10) = 0.$$

Whence $$b = 9 \text{ or } -10.$$

Therefore one base is 9 feet, the altitude is 9 feet, and the other base is 11 feet.

40. If $b =$ the shorter base in feet,

then $b + 10 =$ the longer base in feet,

and $5b + 2 =$ the altitude in feet.

Therefore $$\frac{(5b + 2)(b + b + 10)}{2} = 22 \cdot 9,$$

or $$(b - 4)(5b + 47) = 0.$$

Whence $$b = 4 \text{ or } -9\tfrac{2}{5}.$$

Therefore the shorter base is 4 feet, the longer base is 14 feet, and the altitude is 22 feet.

41. If $s =$ the shorter base in centimeters,

then $2s =$ the longer base in centimeters,

and $2s - 10 =$ the altitude in centimeters.

Therefore $$(2s - 10)\left(\frac{s + 2s}{2}\right) = 900,$$

or $$(s - 20)(s + 15) = 0.$$

Whence $$s = 20 \text{ or } -15.$$

Therefore the shorter base is 20 centimeters, the longer base is 40 centimeters, and the altitude is 30 centimeters.

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1. $12 = 2^2 \cdot 3$ and $18 = 2 \cdot 3^2$. $\therefore$ H.C.F. $= 2 \cdot 3 = 6$.

2. $24 = 2^3 \cdot 3$ and $56 = 2^3 \cdot 7$. $\therefore$ H.C.F. $= 2^3 = 8$.

3. $96 = 2^5 \cdot 3$ and $144 = 2^4 \cdot 3^2$. $\therefore$ H.C.F. $= 2^4 \cdot 3 = 48$.

4. $84 = 2^2 \cdot 3 \cdot 7$ and $196 = 2^2 \cdot 7^2$. $\therefore$ H.C.F. $= 2^2 \cdot 7 = 28$.

5. $125 = 5^3$ and $225 = 5^2 \cdot 3^2$. $\therefore$ H.C.F. $= 5^2 = 25$.

6. $64 = 2^6$, $96 = 2^5 \cdot 3$, and $256 = 2^8$. $\therefore$ H.C.F. $= 2^5 = 32$.

7. $90 = 3^2 \cdot 2 \cdot 5$, $108 = 3^3 \cdot 2^2$, and $324 = 3^4 \cdot 2^2$. $\therefore$ H.C.F. $= 3^2 \cdot 2 = 18$.

8. $12x^4 = 2^2 \cdot 3 \cdot x^4$ and $18x^3 = 2 \cdot 3^2 \cdot x^3$. $\therefore$ H.C.F. $= 2 \cdot 3 \cdot x^3 = 6x^3$.

9. $16x^2y^4 = 2^4 \cdot x^2 \cdot y^4$ and $24x^3y^5 = 2^3 \cdot 3 \cdot x^3 \cdot y^5$. $\therefore$ H.C.F. $= 2^3x^2y^4 = 8x^2y^4$.
10. $27c^2d = 3^3 \cdot c^2 \cdot d$ and $21cd^4 = 3 \cdot 7 \cdot c \cdot d^4$. $\therefore$ H.C.F. $= 3cd$.
11. $32a^3bc^4 = 2^5 \cdot a^3 \cdot b \cdot c^4$ and $48a^2b^2d = 2^4 \cdot 3 \cdot a^2 \cdot b^2 \cdot d$.
$\therefore$ H.C.F. $= 2^4a^2b = 16a^2b$.
12. $125m^7n^5p = 5^3 \cdot m^7 \cdot n^5 \cdot p$ and $100ng^4 = 5^2 \cdot 2^2 \cdot n \cdot g^4$.
$\therefore$ H.C.F. $= 5^2 \cdot n = 25n$.
13. $18h^2k^2 = 2 \cdot 3^2 \cdot h^2 \cdot k^2$, $36h^4k = 2^2 \cdot 3^2 \cdot h^4 \cdot k$,
and $24h^3k^3 = 2^3 \cdot 3 \cdot h^3 \cdot k^3$. $\therefore$ H.C.F. $= 2 \cdot 3 \cdot h^2 \cdot k = 6h^2k$.
14. $9xy^4 = 3^2 \cdot x \cdot y^4$, $54x^5y = 2 \cdot 3^3 \cdot x^5 \cdot y$, and $15x^4y^5 = 3 \cdot 5 \cdot x^4 \cdot y^5$.
$\therefore$ H.C.F. $= 3xy$.
15. $27a^4b^5c^2 = 3^3 \cdot a^4 \cdot b^5 \cdot c^2$, $54a^3b^2d = 2 \cdot 3^3 \cdot a^3 \cdot b^2 \cdot d$,
and $81a^2b^2e^3 = 3^4 \cdot a^2 \cdot b^2 \cdot e^3$. $\therefore$ H.C.F. $= 3^3 \cdot a^2 \cdot b^2 = 27a^2b^2$.
16. $x^2 - 9 = (x+3)(x-3)$ and $x^2 - 5x + 6 = (x-2)(x-3)$.
$\therefore$ H.C.F. $= x - 3$.
17. $x^2 + 3x - 10 = (x+5)(x-2)$ and $x^2 + 6x + 5 = (x+5)(x+1)$.
$\therefore$ H.C.F. $= x + 5$.
18. $x^3 - 4x = x(x+2)(x-2)$ and $x^3 - 8x^2 + 12x = x(x-2)(x-6)$.
$\therefore$ H.C.F. $= x^2 - 2x$.
19. $2c^3 + 12c^2 + 18c = 2c(c+3)^2$ and $c^3 - 2c^2 - 15c = c(c+3)(c-5)$.
$\therefore$ H.C.F. $= c^2 + 3c$.
20. $8 + y^3 = (2+y)(4 - 2y + y^2)$ and $y^2 + 4y + 4 = (2+y)^2$.
$\therefore$ H.C.F. $= 2 + y$.
21. $x^4 - 2x^2 + 1 = (x+1)^2(x-1)^2$ and $x^2 - 2x + 1 = (x-1)^2$.
$\therefore$ H.C.F. $= x^2 - 2x + 1$.
22. $ab + 3b + ac + 3c = (a+3)(b+c)$
and $2ab + 6b - 2ac - 6c = 2(a+3)(b-c)$. $\therefore$ H.C.F. $= a + 3$.
23. $c^2 + 3cd + 2d^2 = (c+d)(c+2d)$, $c^2 + 5cd + 6d^2 = (c+2d)(c+3d)$,
and $c^2 + cd - 2d^2 = (c-d)(c+2d)$. $\therefore$ H.C.F. $= c + 2d$.

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1. $12 = 2^2 \cdot 3$ and $18 = 2 \cdot 3^2$. $\therefore$ L.C.M. $= 2^2 \cdot 3^2 = 36$.
2. $32 = 2^5$ and $48 = 2^4 \cdot 3$. $\therefore$ L.C.M. $= 2^5 \cdot 3 = 96$.
3. $20 = 2^2 \cdot 5$ and $28 = 2^2 \cdot 7$. $\therefore$ L.C.M. $= 2^2 \cdot 5 \cdot 7 = 140$.
4. $96 = 2^5 \cdot 3$ and $144 = 2^4 \cdot 3^2$. $\therefore$ L.C.M. $= 2^5 \cdot 3^2 = 288$.
5. $64 = 2^6$, $120 = 2^3 \cdot 3 \cdot 5$, and $216 = 2^3 \cdot 3^3$. $\therefore$ L.C.M. $= 2^6 \cdot 3^3 \cdot 5 = 8640$.
6. $128 = 2^7$, $160 = 2^5 \cdot 5$, and $200 = 2^3 \cdot 5^2$. $\therefore$ L.C.M. $= 2^7 \cdot 5^2 = 3200$.
7. $x^2y = x^2 \cdot y$, $xy^2 = x \cdot y^2$, and $xy^3 = x \cdot y^3$. $\therefore$ L.C.M. $= x^2y^3$.
8. $6cd^3 = 2 \cdot 3 \cdot c \cdot d^3$, $4c^5de = 2^2 \cdot c^5 \cdot d \cdot e$, and $10c^2d^2e^3 = 2 \cdot 5 \cdot c^2 \cdot d^2 \cdot e^3$.
$\therefore$ L.C.M. $= 60c^5d^3e^3$.
9. $8abc = 2^3 \cdot a \cdot b \cdot c$, $3b^2c = 3 \cdot b^2 \cdot c$, and $12c^2 = 2^2 \cdot 3 \cdot c^2$.
$\therefore$ L.C.M. $= 24ab^2c^2$.

10. $18m^2 = 2 \cdot 3^2 \cdot m^2$, $15mn^2p = 3 \cdot 5 \cdot m \cdot n^2 \cdot p$, and $20m^3p^2 = 2^2 \cdot 5m^3 \cdot p^2$.
$\therefore$ L.C.M. $= 180m^3n^2p^2$.

11. $20 = 2^2 \cdot 5$, $18xy^4 = 2 \cdot 3^2 \cdot x \cdot y^4$, and $27ax^3y = 3^3 \cdot a \cdot x^3 \cdot y$.
$\therefore$ L.C.M. $= 540ax^3y^4$.

12. $36ab^2 = 2^2 \cdot 3^2 \cdot a \cdot b^2$, $42abc = 2 \cdot 3 \cdot 7 \cdot a \cdot b \cdot c$, and $63b^2c = 3^2 \cdot 7 \cdot b^2 \cdot c$.
$\therefore$ L.C.M. $= 252ab^2c$.

13. $4a = 2^2 \cdot a$, $a^2 - ab = a(a - b)$. $\therefore$ L.C.M. $= 4a^2 - 4ab$.

14. $12ax = 2^2 \cdot 3 \cdot ax$, $3a^3x^2 - 3ax^3 = 3ax^2(a^2 - x)$.
$\therefore$ L.C.M. $= 12a^3x^2 - 12ax^3$.

15. $cx + cy = c(x + y)$, $dy + dx = d(x + y)$. $\therefore$ L.C.M. $= cdx + cdy$.

16. $3x + 3z = 3(x + z)$, $6a^2x + 6a^2z = 2 \cdot 3a^2(x + z)$.
$\therefore$ L.C.M. $= 6a^2x + 6a^2z$.

17. $x^2 - xy = x(x - y)$, $ax + ay = a(x + y)$. $\therefore$ L.C.M. $= ax^3 - axy^2$.

18. $x^2 - 9 = (x - 3)(x + 3)$, $x^2 - 5x + 6 = (x - 2)(x - 3)$.
$\therefore$ L.C.M. $= x^3 - 2x^2 - 9x + 18$.

19. $c^2 - 4 = (c + 2)(c - 2)$, $c^2 - 8c - 20 = (c - 10)(c + 2)$.
$\therefore$ L.C.M. $= c^3 - 10c^2 - 4c + 40$.

20. $4ax = 2^2 \cdot a \cdot x$, $4x^2 - 1 = (2x + 1)(2x - 1)$,
$4x^2 + 4x + 1 = (2x + 1)^2$.
$\therefore$ L.C.M. $= 32ax^4 - 8ax^2 + 16ax^3 - 4ax$.

21. $x^2 + 1 = x^2 + 1$, $x^4 - 1 = (x^2 + 1)(x + 1)(x - 1)$,
$x^4 - 2x^2 + 1 = (x + 1)^2(x - 1)^2$. $\therefore$ L.C.M. $= x^6 - x^4 - x^2 + 1$.

22. $4 - c^2 = (2 + c)(2 - c)$, $c^3 + 8 = (c + 2)(c^2 - 2c + 4)$,
$c^2 + 6c + 8 = (c + 2)(c + 4)$.
$\therefore$ L.C.M. $= 64 - 16c - 8c^2 + 8c^3 - 2c^4 - c^5$.

23. $ac - 2bd + 2ad - bc = (a - b)(c + 2d)$,
$a^2 + ab - 2b^2 = (a + 2b)(a - b)$.
$\therefore$ L.C.M. $= a^2c + abc - 2b^2c + 2a^2d + 2abd - 4b^2d$.

24. $x^2 - y^2 = (x + y)(x - y)$, $(x^3 - y^3) = (x - y)(x^2 + xy + y^2)$,
$x^2 + 2xy + y^2 = (x + y)^2$.
$\therefore$ L.C.M. $= x^5 + 2x^4y + x^3y^2 - x^2y^3 - 2xy^4 - y^5$.

25. $ax^2 + bxy = x(ax + by)$, $2ax + 2by = 2(ax + by)$,
$a^2x^2 - b^2y^2 = (ax + by)(ax - by)$. $\therefore$ L.C.M. $= 2a^2x^3 - 2b^2xy^2$.

26. $2x^3 - 2x = 2x(x + 1)(x - 1)$, $3x^4 + 15x^3 - 18x^2 = 3x^2(x + 6)(x - 1)$,
$x^2 - 36 = (x + 6)(x - 6)$.
$\therefore$ L.C.M. $= 6x^6 - 222x^4 + 216x^2$.

27. $8 - y^3 = (2 - y)(4 + 2y + y^2)$, $y^2 - 4 = (y + 2)(y - 2)$,
$4y^2 + 2y^3 + y^4 = y^2(4 + 2y + y^2)$.
$\therefore$ L.C.M. $= - y^6 - 2y^5 + 8y^3 + 16y^2$.

28. $2x^3 - 6x^2 + 4x^2y = 2x^2(x - 3 + 2y)$,
$x^2 - 4y^2 - 6x + 9 = (x - 3 + 2y)(x - 3 - 2y)$.
$\therefore$ L.C.M. $= 2x^4 - 8x^2y^2 - 12x^3 + 18x^2$.

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1. $\dfrac{a^3b}{a^2b^4} = \dfrac{a}{b^3}$.

2. $\dfrac{12\,x^2y^6}{18\,xy^7} = \dfrac{2\,x}{3\,y}$.

3. $\dfrac{32\,a^4b^4c}{48\,a^3b^6c^2} = \dfrac{2\,a}{3\,b^2c}$.

4. $\dfrac{45\,cd^5e^2}{20\,c^5d^3} = \dfrac{9\,d^2e^2}{4\,c^4}$.

5. $\dfrac{36\,a^2b^2}{54\,a^5bc^3} = \dfrac{2\,b}{3\,a^3c^3}$.

6. $\dfrac{2\,a}{2\,a+2} = \dfrac{a}{a+1}$.

7. $\dfrac{3\,xy+3\,y^2}{3\,y^4} = \dfrac{x+y}{y^3}$.

8. $\dfrac{21\,d^2+14\,c}{14\,c} = \dfrac{3\,d^2+2\,c}{2\,c}$.

9. $\dfrac{a^2-1}{a^2-2\,a+1} = \dfrac{a+1}{a-1}$.

10. $\dfrac{4\,x^2+4\,x+1}{4\,x^2-1} = \dfrac{2\,x+1}{2\,x-1}$.

11. $\dfrac{2\,a^2-2\,b^2}{4\,a^2-8\,ab+4\,b^2} = \dfrac{a+b}{2\,a-2\,b}$.

12. $\dfrac{18\,ax^3+9\,ay}{12\,bx^3+6\,by} = \dfrac{3\,a}{2\,b}$.

13. $\dfrac{x^2-25}{x^2-x-30} = \dfrac{x-5}{x-6}$.

14. $\dfrac{c^2-5\,cd+4\,d^2}{c^2-16\,d^2} = \dfrac{c-d}{c+4\,d}$.

15. $\dfrac{2\,x^2-2\,x-180}{2\,x^2-162} = \dfrac{x-10}{x-9}$.

16. $\dfrac{21+10\,x+x^2}{x^2-9} = \dfrac{x+7}{x-3}$.

17. $\dfrac{4\,c^3-5\,c^2-4\,c+5}{8\,c^4-10\,c^3+12\,c-15} = \dfrac{(4\,c-5)(c^2-1)}{(4\,c-5)(2\,c^3+3)} = \dfrac{c^2-1}{2\,c^3+3}$.

18. $\dfrac{x^2-y^2}{(x-y)^2} = \dfrac{x+y}{x-y}$.

19. $\dfrac{a^2-b^2}{(a-b)^3} = \dfrac{a+b}{a^2-2\,ab+b^2}$.

20. $\dfrac{c^2-d^2}{c^3-d^3} = \dfrac{(c+d)(c-d)}{(c^2+cd+d^2)(c-d)} = \dfrac{c+d}{c^2+cd+d^2}$.

21. $\dfrac{y^3-z^3}{(y-z)^3} = \dfrac{(y-z)(y^2+yz+z^2)}{(y-z)(y^2-2\,yz+z^2)} = \dfrac{y^2+yz+z^2}{y^2-2\,yz+z^2}$.

22. $\dfrac{x^3-8}{x^2-4} = \dfrac{(x^2+2\,x+4)(x-2)}{(x+2)(x-2)} = \dfrac{x^2+2\,x+4}{x+2}$.

23. $\dfrac{x^6-1}{x^2-1} = \dfrac{(x+1)(x-1)(x^2+x+1)(x^2-x+1)}{(x+1)(x-1)} = x^4+x^2+1$.

24. $\dfrac{x^4-y^4}{x^6-y^6} = \dfrac{(x+y)(x-y)(x^2+y^2)}{(x+y)(x-y)(x^2+xy+y^2)(x^2-xy+y^2)} = \dfrac{x^2+y^2}{x^4+x^2y^2+y^4}$.

25. $\dfrac{64\,x^3+1}{1+8\,x+16\,x^2} = \dfrac{(16\,x^2-4\,x+1)(4\,x+1)}{(1+4\,x)(4\,x+1)} = \dfrac{16\,x^2-4\,x+1}{1+4\,x}$.

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1. $\dfrac{-x}{y} = -\dfrac{x}{y} = \dfrac{x}{-y} = -\dfrac{-x}{-y}$.

2. $\dfrac{x}{-y} = -\dfrac{x}{y} = \dfrac{-x}{y} = -\dfrac{-x}{-y}$.

3. $\dfrac{a}{a-b} = \dfrac{-a}{-a+b} = -\dfrac{-a}{a-b} = -\dfrac{a}{-a+b}$.

4. $\dfrac{x-3}{2x+4}=\dfrac{-x+3}{-2x-4}=-\dfrac{-x+3}{2x+4}=-\dfrac{x-3}{-2x-4}.$

5. $-\dfrac{c^2-d}{-2c+d}=-\dfrac{-c^2+d}{2c-d}=+\dfrac{-c^2+d}{-2c+d}=+\dfrac{c^2-d}{2c-d}.$

6. $-\dfrac{x-y}{x+y-3}=-\dfrac{-x+y}{-x-y+3}=+\dfrac{-x+y}{x+y-3}=+\dfrac{x-y}{-x-y+3}.$

7. $\dfrac{x^2-5x+6}{x^2-7x+12}=\dfrac{-x^2+5x-6}{-x^2+7x-12}=-\dfrac{-x^2+5x-6}{x^2-7x+12}=-\dfrac{x^2-5x+6}{-x^2+7x-12}.$

8. $-\dfrac{-x+x^2-2}{x^3-x^2y-1}=-\dfrac{x-x^2+2}{-x^3+x^2y+1}=+\dfrac{x-x^2+2}{x^3-x^2y-1}=+\dfrac{-x+x^2-2}{-x^3+x^2y+1}.$

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1. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$ equal $\frac{3}{6}$, $\frac{2}{6}$, and $\frac{1}{6}$ respectively.
2. $\frac{5}{24}$, $\frac{7}{32}$ equal $\frac{20}{96}$ and $\frac{21}{96}$ respectively.
3. $\dfrac{4}{3d}$, $\dfrac{2c}{5d}$ equal $\dfrac{20}{15d}$ and $\dfrac{6c}{15d}$ respectively.
4. $\dfrac{3a+b}{12}$, $\dfrac{3a}{16}$ equal $\dfrac{12a+4b}{48}$ and $\dfrac{9a}{48}$ respectively.
5. $\dfrac{5x+y}{21}$, $\dfrac{4x+3y}{35}$ equal $\dfrac{25x+5y}{105}$ and $\dfrac{12x+9y}{105}$ respectively.
6. $\dfrac{4}{5a^2b}$, $\dfrac{3}{abc}$ equal $\dfrac{4c}{5a^2bc}$ and $\dfrac{15a}{5a^2bc}$ respectively.
7. $\dfrac{5c}{3mn^3}$, $\dfrac{12d}{11m^2n^2}$ equal $\dfrac{55cm}{33m^2n^3}$ and $\dfrac{36nd}{33m^2n^3}$ respectively.
8. $\dfrac{3a}{2b^2c}$, $\dfrac{2b}{3a^2c}$, $\dfrac{c}{6ab}$ equal $\dfrac{9a^3}{6a^2b^2c}$, $\dfrac{4b^3}{6a^2b^2c}$ and $\dfrac{abc^2}{6a^2b^2c}$ respectively.
9. $\dfrac{3b}{2cd^3}$, $\dfrac{9a}{4cde}$, $\dfrac{7ab}{5cde^2}$ equal $\dfrac{30be^2}{20cd^3d^2}$, $\dfrac{45ad^2e}{20cd^3e^2}$ and $\dfrac{28abd^2}{20cd^3e^2}$ respectively.
10. $\dfrac{x+y}{3xy^2}$, $\dfrac{x-2y}{2x^2y}$ equal $\dfrac{2x^2+2xy}{6x^2y^2}$ and $\dfrac{3xy-6y^2}{6x^2y^2}$ respectively.
11. $\dfrac{2+m^2}{2m}$, $\dfrac{4+m^2}{4n}$ equal $\dfrac{4n+2m^2n}{4mn}$ and $\dfrac{4m+m^3}{4mn}$ respectively.
12. $\dfrac{2}{x+2}$, $\dfrac{4}{3x+6}$ equal $\dfrac{6}{3x+6}$ and $\dfrac{4}{3x+6}$ respectively.
13. $\dfrac{4}{x+1}$, $\dfrac{5}{x-1}$ equal $\dfrac{4x-4}{x^2-1}$ and $\dfrac{5x+5}{x^2-1}$ respectively.
14. $\dfrac{3c}{c^2-d^2}$, $\dfrac{4}{c+d}$ equal $\dfrac{3c}{c^2-d^2}$ and $\dfrac{4c-4d}{c^2-d^2}$ respectively.
15. $\dfrac{3x}{x+3}$, $\dfrac{5}{x^2+5x+6}$ equal $\dfrac{3x^2+6x}{x^2+5x+6}$ and $\dfrac{5}{x^2+5x+6}$ respectively.

16. $\dfrac{2x}{x^2-xy}$, $\dfrac{3y}{x^2-2xy+y^2}$ equal $\dfrac{2x^2-2xy}{x^3-2x^2y+xy^2}$ and $\dfrac{3xy}{x^3-2x^2y+xy^2}$ respectively.

17. $\dfrac{3c+2}{c^2-d^2}$, $\dfrac{4-c}{c^2-7cd+6d^2}$ equal $\dfrac{3c^2+2c-18cd-12d}{c^3-cd^2-6c^2d+6d^3}$ and $\dfrac{4c-c^2+4d-cd}{c^3-cd^2-6c^2d+6d^3}$ respectively.

18. $\dfrac{2x+5}{x^2-1}$, $\dfrac{2x-4}{x^3-3x^2+2x}$ equal $\dfrac{2x^3+x^2-10x}{x^4-2x^3-x^2+2x}$ and $\dfrac{2x^2-2x-4}{x^4-2x^3-x^2+2x}$ respectively.

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1. $\dfrac{2x}{3}+\dfrac{3x}{5}=\dfrac{10x}{15}+\dfrac{9x}{15}=\dfrac{19x}{15}.$

2. $\dfrac{2a}{3}-\dfrac{5a}{6}+\dfrac{a}{4}=\dfrac{8a}{12}-\dfrac{10a}{12}+\dfrac{3a}{12}=\dfrac{a}{12}.$

3. $\dfrac{3c}{7}+\dfrac{c}{21}-\dfrac{1}{3}=\dfrac{9c}{21}+\dfrac{c}{21}-\dfrac{7}{21}=\dfrac{10c-7}{21}.$

4. $\dfrac{9a}{16}-\dfrac{12m}{8}-\dfrac{5m}{24}=\dfrac{27a}{48}-\dfrac{72m}{48}-\dfrac{10m}{48}=\dfrac{27a-82m}{48}.$

5. $\dfrac{x-3}{4}+\dfrac{2x-5}{6}=\dfrac{3x-9}{12}+\dfrac{4x-10}{12}=\dfrac{7x-19}{12}.$

6. $\dfrac{5a+7}{14}-\dfrac{9a+x}{21}=\dfrac{15a+21}{42}-\dfrac{18a+2x}{42}=\dfrac{21-3a-2x}{42}.$

7. $\dfrac{5c}{3}-\dfrac{3c-x}{5}+\dfrac{x-3c}{10}=\dfrac{50c}{30}-\dfrac{18c-6x}{30}+\dfrac{3x-9c}{30}=\dfrac{23c+9x}{30}.$

8. $$\dfrac{4m-3}{6}-\dfrac{7-9m}{9}-\dfrac{3a+5m-4}{27}$$
$$=\dfrac{36m-27}{54}-\dfrac{42-54m}{54}-\dfrac{6a+10m-8}{54}$$
$$=\dfrac{80m-6a-61}{54}.$$

9. $\dfrac{2}{a}+\dfrac{3}{4a^2}=\dfrac{8a}{4a^2}+\dfrac{3}{4a^2}=\dfrac{8a+3}{4a^2}.$

10. $\dfrac{3}{x^2}+\dfrac{4}{x}-\dfrac{8}{3x^3}=\dfrac{9x}{3x^3}+\dfrac{12x^2}{3x^3}-\dfrac{8}{3x^3}=\dfrac{12x^2+9x-8}{3x^3}.$

11. $\dfrac{x}{y}-\dfrac{5}{2y^3}+\dfrac{a}{4}=\dfrac{4xy^2}{4y^3}-\dfrac{10}{4y^3}+\dfrac{ay^3}{4y^3}=\dfrac{4xy^2+ay^3-10}{4y^3}.$

12. $\frac{a}{b}+\frac{b}{c}-\frac{c}{a}=\frac{a^2c}{abc}+\frac{ab^2}{abc}-\frac{bc^2}{abc}=\frac{a^2c+ab^2-bc^2}{abc}.$

13. $\frac{7}{m^2}-\frac{5}{mn}+\frac{6}{n^2}=\frac{7n^2}{m^2n^2}-\frac{5mn}{m^2n^2}+\frac{6m^2}{m^2n^2}=\frac{7n^2-5mn+6m^2}{m^2n^2}.$

14. $\frac{3}{xy}-\frac{5}{2x^2}-\frac{6}{5xy^3}=\frac{30xy^2}{10x^2y^3}-\frac{25y^3}{10x^2y^3}-\frac{12x}{10x^2y^3}=\frac{30xy^2-25y^3-12x}{10x^2y^3}.$

15. $\frac{3x-1}{6x^2}-\frac{7-2x^2}{4x^3}+\frac{5x^3-8}{9x^4}=\frac{18x^3-6x^2}{36x^4}-\frac{63x-18x^3}{36x^4}+\frac{20x^3-32}{36x^4}$

$=\frac{56x^3-6x^2-63x-32}{36x^4}.$

16. $\frac{4c^2-9}{2cd}-\frac{6-c}{5c^2}-\frac{c^2-4}{cd^2}=\frac{20c^3d-45cd}{10c^2d^2}-\frac{12d^2-2cd^2}{10c^2d^2}-\frac{10c^3-40c}{10c^2d^2}$

$=\frac{20c^3d-45cd-12d^2+2cd^2-10c^3+40c}{10c^2d^2}.$

17. $\frac{3+x}{x-3}+\frac{3}{4}=\frac{(3+x)4}{(x-3)4}+\frac{3(x-3)}{4(x-3)}=\frac{7x+3}{4x-12}.$

18. $\frac{3}{x-5}-\frac{2}{x+5}=\frac{3(x+5)}{(x-5)(x+5)}-\frac{2(x-5)}{(x+5)(x-5)}=\frac{x+25}{x^2-25}.$

19. $\frac{5x}{x^2+xy}-\frac{7}{x}=\frac{5x}{x(x+y)}-\frac{7(x+y)}{x(x+y)}=\frac{-2x-7y}{x^2+xy}.$

20. $\frac{3a-b}{a^2-b^2}+\frac{2}{a-b}=\frac{3a-b}{a^2-b^2}+\frac{2(a+b)}{(a-b)(a+b)}=\frac{5a+b}{a^2-b^2}.$

21. $\frac{10}{25-m^2}-\frac{1}{m^2+16m+55}$

$=\frac{10(m+11)}{(5+m)(5-m)(m+11)}-\frac{1(5-m)}{(m+5)(m+11)(5-m)}$

$=\frac{105+11m}{275+25m-11m^2-m^3}.$

22. $\frac{3}{a^2-16}+\frac{5}{a^2-6a+8}=\frac{3(a-2)}{(a+4)(a-4)(a-2)}+\frac{5(a+4)}{(a-2)(a-4)(a+4)}$

$=\frac{8a+14}{a^3-2a^2-16a+32}.$

23. $\frac{2x+1}{x^2-1}+\frac{4}{x^2-3x+2}=\frac{(2x+1)(x-2)}{(x+1)(x-1)(x-2)}+\frac{4(x+1)}{(x-1)(x-2)(x+1)}$

$=\frac{2x^2+x+2}{x^3-2x^2-x+2}.$

24. $\frac{c-5}{c^2-6c}+\frac{2c-3}{c^2-8c+12}=\frac{(c-5)(c-2)}{c(c-6)(c-2)}+\frac{(2c-3)c}{(c-6)(c-2)c}$

$=\frac{3c^2-10c+10}{c^3-8c^2+12c}.$

25. $\dfrac{x^2-3xy+y^2}{9-6x+x^2}-\dfrac{5x-3y}{9-3x}=\dfrac{(x^2-3xy+y^2)3}{(3-x)^2\cdot 3}-\dfrac{(5x-3y)(3-x)}{3(3-x)(3-x)}$
$=\dfrac{8x^2-12xy-15x+9y+3y^2}{3x^2-18x+27}.$

26. $\dfrac{x+2}{x^2+x}+\dfrac{1}{x}+\dfrac{3-x}{x^2+2x+1}=\dfrac{(x+2)(x+1)}{x(x+1)(x+1)}+\dfrac{1(x+1)^2}{x(x+1)^2}+\dfrac{(3-x)x}{(x+1)^2\cdot x}$
$=\dfrac{x^2+8x+3}{x^3+2x^2+x}.$

27. $\dfrac{a+b}{a^2-ab}+\dfrac{a-2b}{a^2-2ab+b^2}-\dfrac{2a-b}{a^2-b^2}$
$=\dfrac{(a+b)(a+b)(a-b)}{(a^2-ab)(a-b)(a+b)}+\dfrac{(a-2b)(a+b)a}{(a^2-2ab+b^2)(a+b)a}-\dfrac{(2a-b)(a-b)a}{(a^2-b^2)(a-b)a}$
$=\dfrac{-4ab^2+3a^2b-b^3}{a^4-a^3b-a^2b^2+ab^3}.$

28. $\dfrac{m-2n}{m^2+mn+n^2}-\dfrac{m^2-3n^2}{m^3-n^3}+\dfrac{3m-n}{m-n}$
$=\dfrac{(m-2n)(m-n)}{(m^2+mn+n^2)(m-n)}-\dfrac{m^2-3n^2}{m^3-n^3}+\dfrac{(3m-n)(m^2+mn+n^2)}{(m-n)(m^2+mn+n^2)}$
$=\dfrac{3m^3+2m^2n+2mn^2-3mn+5n^2-n^3}{m^3-n^3}.$

29. $\dfrac{c^2+cd+d^2}{c^2-cd+d^2}-\dfrac{c-d}{2c+2d}-\dfrac{2c^2+5cd}{c^3+d^3}$
$=\dfrac{(c^2+cd+d^2)(c+d)2}{(c^2-cd+d^2)(c+d)2}-\dfrac{(c-d)(c^2-cd+d^2)}{(2c+2d)(c^2-cd+d^2)}-\dfrac{(2c^2+5cd)2}{(c^3+d^3)2}$
$=\dfrac{c^3+6c^2d+2cd^2+3d^3-4c^2-10cd}{2c^3+2d^3}.$

30. $\dfrac{x+y}{x^3-y^3}-\dfrac{x-2y}{(x-y)^3}=\dfrac{(x+y)(x-y)^2}{(x^3-y^3)(x-y)^2}-\dfrac{(x-2y)(x^2+xy+y^2)}{(x-y)^3(x^2+xy+y^2)}$
$=\dfrac{3y^3}{x^5-2x^4y+x^3y^2-x^2y^3+2xy^4-y^5}.$

31. $R^2+\dfrac{R^2}{4}=\dfrac{5R^2}{4}.$

32. $x-3-\dfrac{x+3}{5}=\dfrac{(x-3)5}{5}-\dfrac{x+3}{5}=\dfrac{4x-18}{5}.$

33. $\dfrac{m^2+n^2}{m-n}+n+m=\dfrac{m^2+n^2}{m-n}+\dfrac{(n+m)(m-n)}{1(m-n)}=\dfrac{2m^2}{m-n}.$

34. $3a+b-\dfrac{a^2-3b^2}{3a-b}=\dfrac{(3a+b)(3a-b)}{1(3a-b)}-\dfrac{a^2-3b^2}{3a-b}=\dfrac{8a^2+2b^2}{3a-b}.$

35. $x^2+y^2-\dfrac{3x^3-y^3}{x+y}-xy=\dfrac{(x^2-xy+y^2)(x+y)}{1(x+y)}-\dfrac{3x^3-y^3}{x+y}=\dfrac{2y^3-2x^3}{x+y}.$

36. $m^2 - \dfrac{m^4 - 2m^2}{m^2 + m + 1} + 1 - m = \dfrac{(m^2 - m + 1)(m^2 + m + 1)}{1(m^2 + m + 1)} - \dfrac{m^4 - 2m^2}{m^2 + m + 1}$

$= \dfrac{3m^2 + 1}{m^2 + m + 1}.$

37. $\dfrac{m^3 + 8n^3}{m^2 + 2mn + 4n^2} + m - 2n$

$= \dfrac{m^3 + 8n^3}{m^2 + 2mn + 4n^2} + \dfrac{(m - 2n)(m^2 + 2mn + 4n^2)}{m^2 + 2mn + 4n^2}$

$= \dfrac{2m^3}{m^2 + 2mn + 4n^2}.$

38. $a^2 - \dfrac{a^4 + 2a^2b^2 - b^4}{a^2 + ab + b^2} - ab + b^2$

$= \dfrac{(a^2 - ab + b^2)(a^2 + ab + b^2)}{a^2 + ab + b^2} - \dfrac{a^4 + 2a^2b^2 - b^4}{a^2 + ab + b^2}$

$= \dfrac{2b^4 - a^2b^2}{a^2 + ab + b^2}.$

39. $x^3 + x^2y - \dfrac{x^4 + 3y^4}{x - y} + xy^2 + y^3$

$= \dfrac{(x^3 + x^2y + xy^2 + y^3)(x - y)}{x - y} - \dfrac{x^4 + 3y^4}{x - y}$

$= \dfrac{4y^4}{y - x}.$

40. $\dfrac{2c^2 + d^2}{c^2 - d^2} - \dfrac{c}{c + d} + 5 = \dfrac{2c^2 + d^2}{c^2 - d^2} - \dfrac{c(c - d)}{(c + d)(c - d)} + \dfrac{5(c^2 - d^2)}{c^2 - d^2}$

$= \dfrac{6c^2 + cd - 4d^2}{c^2 - d^2}.$

41. $6 + \dfrac{6r^2 - s^2}{r^2 - 9rs + 14s^2} - \dfrac{3r - 5s}{r - 7s} + 5$

$= \dfrac{11(r^2 - 9rs + 14s^2)}{r^2 - 9rs + 14s^2} + \dfrac{6r^2 - s^2}{r^2 - 9rs + 14s^2} - \dfrac{(3r - 5s)(r - 2s)}{(r - 7s)(r - 2s)}$

$= \dfrac{14r^2 - 88rs + 143s^2}{r^2 - 9rs + 14s^2}.$

42. $\left(x - 3 - \dfrac{3}{x + 3}\right) - \left(2x - 7 - \dfrac{4}{x + 4}\right)$

$= -x + 4 - \dfrac{3}{x + 3} + \dfrac{4}{x + 4}$

$= \dfrac{(-x + 4)(x + 3)(x + 4)}{(x + 3)(x + 4)} - \dfrac{3(x + 4)}{(x + 3)(x + 4)} + \dfrac{4(x + 3)}{(x + 4)(x + 3)}$

$= \dfrac{48 + 17x - 3x^2 - x^3}{x^2 + 7x + 12}.$

43. $\left(5m+\frac{6m}{2n}\right)-\left(m-\frac{m-3n}{n}\right)=4m+\frac{3m}{n}+\frac{m-3n}{n}$
$=\frac{4mn+4m-3n}{n}.$

44. $\left(7c-\frac{2d}{3de}\right)-\left(5c+\frac{c}{3de}\right)=2c-\frac{2d+c}{3de}=\frac{6cde-2d-c}{3de}.$

45. $\left(\frac{2a}{a-b}-5a\right)-\left(2a-\frac{4a}{a+b}\right)=-7a+\frac{2a}{a-b}+\frac{4a}{a+b}$
$=-\frac{7a(a-b)(a+b)}{(a-b)(a+b)}+\frac{2a(a+b)}{(a-b)(a+b)}+\frac{4a(a-b)}{(a+b)(a-b)}$
$=\frac{6a^2-2ab+7ab^2-7a^3}{a^2-b^2}.$

46. $\left(2a-3b+\frac{3a}{2a+b}\right)-\left(3a+2b-\frac{5a}{2a-b}\right)$
$=-a-5b+\frac{3a}{2a+b}+\frac{5a}{2a-b}$
$=\frac{(-a-5b)(2a-b)(2a+b)}{(2a-b)(2a+b)}+\frac{3a(2a-b)}{(2a+b)(2a-b)}+\frac{5a(2a+b)}{(2a-b)(2a+b)}$
$=\frac{-4a^3+16a^2-20a^2b+ab^2+2ab+5b^3}{4a^2-b^2}.$

47. $\left(-\frac{4r^2}{2r+s}+2r-s\right)-\left(-\frac{4s^2}{2s-3r}+2s+3r\right)$
$=-r-3s-\frac{4r^2}{2r+s}+\frac{4s^2}{2s-3r}$
$=\frac{(-r-3s)(2r+s)(2s-3r)}{(2r+s)(2s-3r)}-\frac{4r^2(2s-3r)}{(2r+s)(2s-3r)}+\frac{4s^2(2r+s)}{(2s-3r)(2r+s)}$
$=\frac{18r^3+9r^2s+3rs^2-2s^3}{2s^2+rs-6r^2}.$

48. $\frac{x-1}{2-x}+\frac{x^2-3}{x^2-4}=\frac{(1-x)(x+2)}{(x-2)(x+2)}+\frac{x^2-3}{(x+2)(x-2)}=\frac{x+1}{4-x^2}.$

49. $\frac{5}{x-3}+\frac{3}{3-x}=\frac{5}{x-3}-\frac{3}{x-3}=\frac{2}{x-3}.$

50. $\frac{6}{x^2-25}-\frac{3}{5-x}=\frac{6}{(x+5)(x-5)}+\frac{3(x+5)}{(x-5)(x+5)}=\frac{3x+21}{x^2-25}.$

51. $\frac{3a}{a^2-4}+\frac{2a-1}{2-a}=\frac{3a}{(a+2)(a-2)}-\frac{(2a-1)(a+2)}{(a-2)(a+2)}=\frac{2-2a^2}{a^2-4}.$

52. $\frac{3c}{9-c^2}-\frac{4c-2}{c-3}=\frac{3c}{(3+c)(3-c)}+\frac{(4c-2)(3+c)}{(3-c)(3+c)}=\frac{4c^2+13c-6}{9-c^2}.$

53. $\frac{x}{2x-1}-\frac{x}{1+2x}-\frac{x}{1-4x^2}$

$$= \frac{x(2x+1)}{(2x-1)(2x+1)} - \frac{x(2x-1)}{(2x+1)(2x-1)} + \frac{x}{(2x+1)(2x-1)}$$

$$= \frac{3x}{4x^2-1}.$$

54. $$\frac{7}{x^2-13x+42} + \frac{4x-1}{7-x} - \frac{2x+3}{6-x}$$

$$= \frac{7}{(x-6)(x-7)} - \frac{(4x-1)(x-6)}{(x-7)(x-6)} + \frac{(2x+3)(x-7)}{(x-6)(x-7)}$$

$$= \frac{-2x^2+14x-20}{x^2-13x+42}.$$

55. $$\frac{3x-1}{x^2+7x-8} - \frac{4x-1}{1-x} + \frac{x+2}{8+x}$$

$$= \frac{3x-1}{(x+8)(x-1)} + \frac{(4x-1)(x+8)}{(x-1)(x+8)} + \frac{(x+2)(x-1)}{(x+8)(x-1)}$$

$$= \frac{5x^2+35x-11}{x^2+7x-8}.$$

56. $$\frac{1}{(a-b)(a-c)} + \frac{1}{(b-c)(b-a)} - \frac{1}{(c-a)(c-b)}$$

$$= \frac{1}{(a-b)(a-c)} - \frac{1}{(b-c)(a-b)} - \frac{1}{(a-c)(b-c)}$$

$$= \frac{b-c}{(a-b)(a-c)(b-c)} - \frac{a-c}{(b-c)(a-b)(a-c)} - \frac{a-b}{(a-c)(b-c)(a-b)}$$

$$= \frac{-2}{(a-c)(b-c)}.$$

57. $$\frac{2}{(x-y)(x-z)} - \frac{1}{(x-y)(z-x)} = \frac{2}{(x-y)(x-z)} + \frac{1}{(x-y)(x-z)}$$

$$= \frac{3}{(x-y)(x-z)}.$$

58. $$\frac{3a}{(a-4)(a-3)} - \frac{2a}{(3-a)(4-a)} = \frac{3a}{(a-4)(a-3)} - \frac{2a}{(a-3)(a-4)}$$

$$= \frac{a}{(a-3)(a-4)}.$$

59. $$\frac{2}{(m-n)(m+n)} + \frac{3}{(n-m)(m-7)} - \frac{4}{(n-m)(7-m)}$$

$$= \frac{2}{(m-n)(m+n)} - \frac{3}{(m-n)(m-7)} - \frac{4}{(m-n)(m-7)}$$

$$= \frac{2(m-7)-3(m+n)-4(m+n)}{(m-n)(m+n)(m-7)}$$

$$= \frac{-5m-7n-14}{(m-n)(m+n)(m-7)}.$$

60. $$\frac{c}{c^2-10c+24}-\frac{1}{6c-c^2-8}+\frac{3}{(6-c)(2-c)}$$
$$=\frac{c}{(c-4)(c-6)}+\frac{1}{(c-4)(c-2)}+\frac{3}{(c-6)(c-2)}$$
$$=\frac{c(c-2)+1(c-6)+3(c-4)}{(c-4)(c-6)(c-2)}$$
$$=\frac{c^2+2c-18}{(c-6)(c-4)(c-2)}.$$

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Performing the indicated division:

1. $\dfrac{15x^2-10x+2}{5x}=3x-2+\dfrac{2}{5x}.$

2. $\dfrac{24a^3-6a^2-14}{6a}=4a^2-a-\dfrac{14}{6a}.$

3. $\dfrac{c}{c+1}=1-\dfrac{1}{c+1}.$

4. $\dfrac{d^3+1}{d-1}=d^2+d+1+\dfrac{2}{d-1}.$

5. $\dfrac{27x^3-y^3}{3x+y}=9x^2-3xy+y^2-\dfrac{2y^3}{3x+y}.$

6. $\dfrac{y^4+y^2+1}{y^2-y-1}=y^2+y+3+\dfrac{4y+4}{y^2-y-1}.$

7. $\dfrac{a^4+a^2b^2+b^4}{a^2+ab-b^2}=a^2-ab+3b^2+\dfrac{4b^4-4ab^3}{a^2+ab-b^2}.$

8. $\dfrac{3y^3-11}{y+3}=3y^2-9y+27-\dfrac{92}{y+3}.$

9. $\dfrac{(a+b)^3}{a^3+b^3}=1+\dfrac{3a^2b+3ab^2}{a^3+b^3}=1+\dfrac{3ab}{a^2-ab+b^2}.$

10. $\dfrac{16a^4+b^4}{2a-1}=8a^3+4a^2+2a+1+\dfrac{1+b^4}{2a-1}.$

11. $\dfrac{5a^3+3a^2-6}{5a^2+3a+2}=a+\dfrac{-2a-6}{5a^2+3a+2}.$

12. $\dfrac{x^6+y^6}{x^2-y^2}=x^4+x^2y^2+y^4+\dfrac{2y^6}{x^2-y^2}.$

13. $\dfrac{x^4}{x+1}=x^3-x^2+x-1+\dfrac{1}{x+1}.$

14. $\dfrac{(a^2+b)^2}{a^2+b^2}=a^2-b^2+2b+\dfrac{b^2-2b^3+b^4}{a^2+b^2}.$

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1. $\dfrac{3x^2y^3}{4a^4}\cdot\dfrac{8a^3}{12xy}=\dfrac{xy^2}{2a}.$

2. $\dfrac{10a^3b^2}{15c^3}\cdot\dfrac{18c}{7a^2}=\dfrac{12ab^2}{7c^2}.$

3. $\dfrac{3x}{8y}\cdot\dfrac{6ax}{9b}\cdot\dfrac{12b^2y}{16x}=\dfrac{3abx}{16}.$

4. $\dfrac{14m^2n^4}{5n}\cdot\dfrac{25n^3}{6m^2}\cdot\dfrac{4x}{7m}=\dfrac{20n^6x}{3m}.$

5. $\left(\dfrac{2x}{3ay}\right)^2\cdot\dfrac{12a^3}{16b^2xy^3}\cdot 8b^4y^2=\dfrac{8ab^2x}{3y^3}.$

6. $\dfrac{a}{4c^2}\cdot\left(\dfrac{c}{2a}\right)^3\cdot 16=\dfrac{c}{2a^2}.$

7. $\left(\dfrac{2a}{c}\right)^2\cdot\left(\dfrac{2c}{a}\right)^3\cdot\left(\dfrac{1}{4}\right)^2=\dfrac{2c}{a}.$

8. $\left(\dfrac{-3a}{3}\right)^2\cdot\dfrac{c^5}{(2a)^5}\cdot\left(\dfrac{2}{3}\right)^2=\dfrac{c^5}{72a^3}.$

9. $\dfrac{6c}{a}\cdot\left(\dfrac{9c^2}{4a}\right)^2\cdot\left(\dfrac{-2a}{3c}\right)^3=-9c^2.$

10. $\dfrac{(4ax)^2}{225c^4}\cdot\dfrac{(5ac^2)^3}{(2x^2)^3}\cdot\left(\dfrac{-x}{c}\right)^5=\dfrac{-10a^5x}{9c^3}.$

11. $\dfrac{2a+4x}{5y^2}\cdot\dfrac{15y}{a+2x}=\dfrac{6}{y}.$

12. $\dfrac{3c}{9-c^2}\cdot\dfrac{c^2+5c+6}{18cd}=\dfrac{3c}{(3+c)(3-c)}\cdot\dfrac{(c+2)(c+3)}{18cd}=\dfrac{c+2}{18d-6cd}.$

13. $\dfrac{3a-6y}{4a+2y}\cdot\dfrac{8(2a+y)^2}{4a^2-24ay+24y^2}=\dfrac{3(a-2y)}{2(2a+y)}\cdot\dfrac{8(2a+y)^2}{4(a^2-6ay+6y^2)}$

$=\dfrac{6a^2-9ay-6y^2}{a^2-6ay+6y^2}.$

14. $\dfrac{c^2+6ce+9e^2}{6(c+d)^2}\cdot\dfrac{3c^2e-12d^2e}{c^2-2cd+3ce-6de}=\dfrac{(c+3e)^2 3e(c+2d)(c-2d)}{6(c+d)^2(c-2d)(c+3e)}$

$=\dfrac{c^2e+2cde+3ce^2+6de^2}{2(c+d)^2}.$

15. $\dfrac{a^2+4ab+4b^2}{9-a^2}\cdot\dfrac{a^2-5a+6}{a^2-4b^2}=\dfrac{(a+2b)^2}{(3+a)(3-a)}\cdot\dfrac{(a-2)(a-3)}{(a-2b)(a+2b)}$

$=\dfrac{(a+2b)(a-2)}{(a+3)(2b-a)}$

$=\dfrac{2a-2ab+4b-a^2}{a^2-2ab+3a-6b}.$

16. $\dfrac{5c^2-20d^2}{c^3+8d^3}\cdot\dfrac{c^2-2cd+4d^2}{25cd^4}$

$=\dfrac{5(c+2d)(c-2d)}{(c+2d)(c^2-2cd+4d^2)}\cdot\dfrac{c^2-2cd+4d^2}{25cd^4}$

$=\dfrac{c-2d}{5cd^4}.$

17. $\dfrac{(x+2)^2}{3x^3+6x^2+12x}\cdot(15-9x^2)\cdot\dfrac{x^3-8}{4-x^2}$

$=\dfrac{(x+2)^2}{3x(x^2+2x+4)}\cdot\dfrac{3(5-3x^2)}{1}\cdot\dfrac{(x-2)(x^2+2x+4)}{(2-x)(2+x)}$

$=\dfrac{3x^3+6x^2-5x-10}{x}.$

18. $\dfrac{8c-24d}{c^2-6cd+9d^2}\cdot\left(4c-\dfrac{d^2}{c}\right)\cdot\dfrac{4c^2+d^2}{64c^4-4d^4}$

$=\dfrac{8(c-3d)}{(c-3d)^2}\cdot\dfrac{(2c+d)(2c-d)}{c}\cdot\dfrac{4c^2+d^2}{4(4c^2+d^2)(4c^2-d^2)}$

$=\dfrac{2}{c^2-3cd}.$

19. $\left(\dfrac{6m-9}{2m-3}+2m\right)\left(2m-9+\dfrac{36}{2m+3}\right)=\dfrac{(2m+3)(2m-3)}{2m-3}\cdot\dfrac{(2m-3)^2}{(2m+3)}$

$=4m^2-12m+9.$

20. $\left(\dfrac{3}{2a}-1\right)^2\cdot\dfrac{8a^2x}{9-4a^2}\cdot\dfrac{3+2a}{2a-3}=\dfrac{(3-2a)^2}{4a^2}\cdot\dfrac{8a^2x}{(3-2a)(3+2a)}\cdot\dfrac{3+2a}{2a-3}$

$=-2x.$

21. $\left(1+\dfrac{2}{x}-\dfrac{3}{x^2}\right)\cdot\dfrac{9x^3}{3(x^2-18x+17)}=\dfrac{(x+3)(x-1)}{x^2}\cdot\dfrac{9x^3}{3(x-1)(x-17)}$

$=\dfrac{3x^2+9x}{x-17}.$

22. $\dfrac{x^3-8}{6xy}\cdot\left(3-\dfrac{4}{2-x}\right)\cdot\dfrac{18x^2y^2}{x^2+2x+4}$

$=\dfrac{(x-2)(x^2+2x+4)}{6xy}\cdot\dfrac{2-3x}{2-x}\cdot\dfrac{18x^2y^2}{x^2+2x+4}$

$=9x^2y-6xy.$

23. $\dfrac{a^3-b^3}{(a-b)^3}\cdot\dfrac{6a+6b}{2a^2+2ab+2b^2}\cdot\dfrac{(a-b)^2}{a^2-b^2}$

$=\dfrac{(a-b)(a^2+ab+b^2)}{(a-b)^3}\cdot\dfrac{6(a+b)}{2(a^2+ab+b^2)}\cdot\dfrac{(a-b)^2}{(a+b)(a-b)}$

$=\dfrac{3}{a-b}.$

24. $\left(4-\dfrac{20-9x}{5-x}-x\right)\left(\dfrac{1}{x}+\dfrac{1}{x^2}-\dfrac{30}{x^3}\right)=\dfrac{x^2}{5-x}\cdot\dfrac{(x+6)(x-5)}{x^3}=-\dfrac{x+6}{x}.$

25. $\left(x+2y-\dfrac{5x+10y}{x+y}\right)\left(\dfrac{y^2+x^2+2yx}{x^2-3xy-4y^2}\right)\cdot\dfrac{1}{x+2y}$

$=\dfrac{(x+2y)(x+y-5)}{x+y}\cdot\dfrac{(x+y)^2}{(x-4y)(x+y)}\cdot\dfrac{1}{(x+2y)}=\dfrac{x+y-5}{x-4y}.$

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1. $\frac{6a}{3b} \div \frac{4a}{12b^2} = \frac{6a}{3b} \cdot \frac{12b^2}{4a} = 6b.$

2. $\frac{9a^2}{6b^2} \div \frac{14ab}{8x} = \frac{9a^2}{6b^2} \cdot \frac{8x}{14ab} = \frac{6ax}{7b^3}.$

3. $\left(\frac{2ax}{3c}\right)^2 \div \frac{6ax^2}{9c^4} = \left(\frac{2ax}{3c}\right)^2 \cdot \frac{9c^4}{6ax^2} = \frac{2ac^2}{3}.$

4. $\frac{a}{b} \div \frac{c}{d} \div \frac{ad^2}{bc^2} = \frac{a}{b} \cdot \frac{d}{c} \cdot \frac{bc^2}{ad^2} = \frac{c}{d}.$

5. $2a \cdot \left(\frac{2a^2}{3x}\right)^2 \div \frac{(4a^2)^2}{18x^2} = 2a \cdot \frac{(2a^2)^2}{(3x)^2} \cdot \frac{18x^2}{(4a^2)^2} = a.$

6. $\frac{5x}{5y} \div \frac{10x^2}{12y^3} \div \frac{4ax}{3by^2} = \frac{5x}{5y} \cdot \frac{12y^3}{10x^2} \cdot \frac{3by^2}{4ax} = \frac{9by^4}{10ax^2}.$

7. $\frac{6}{(2x)^2} \div \frac{12}{(4x^2)^3} \cdot \left(\frac{2}{x}\right)^2 = \frac{6}{(2x)^2} \cdot \frac{(4x^2)^3}{12} \cdot \frac{2^2}{x^2} = 32x^2.$

8. $\frac{3m^2}{5n} \div \frac{21m^3}{10mn^2} \div 14m^4n = \frac{3m^2}{5n} \cdot \frac{10mn^2}{21m^3} \cdot \frac{1}{14m^4n} = \frac{1}{49m^4}.$

9. $\left(\frac{4x^2}{3a}\right)^2 \div \left(\frac{2x}{3a}\right)^3 \cdot \left(\frac{a}{x}\right)^2 = \frac{(4x^2)^2}{(3a)^2} \cdot \frac{(3a)^3}{(2x)^3} \cdot \frac{a^2}{x^2} = \frac{6a^3}{x}.$

10. $\frac{6a-3}{5x} \div \frac{2a-1}{15bx^3} = \frac{3(2a-1)}{5x} \cdot \frac{15bx^3}{2a-1} = 9bx^2.$

11. $\frac{4-x^2}{3x^2+x^3} \div \frac{4-4x+x^2}{x^3-x^2-12x} = \frac{(2-x)(2+x)}{x^2(3+x)} \cdot \frac{x(x-4)(x+3)}{(2-x)^2}$
$= \frac{x^2-2x-8}{2x-x^2}.$

12. $\frac{a^2-7a+12}{a-1} \div \frac{a^2-16}{1-a^2} = \frac{(a-3)(a-4)}{a-1} \cdot \frac{(1-a)(1+a)}{(a+4)(a-4)}$
$= -\frac{a^2-2a-3}{a+4}.$

13. $(-14-5c+c^2) \div \frac{ac^2-49a}{bc^2+9bc+14b} = \frac{(c-7)(c+2)}{1} \cdot \frac{b(c+2)(c+7)}{a(c+7)(c-7)}$
$= \frac{bc^2+4bc+4b}{a}.$

14. $\frac{4a^2-4ab-3b^2}{8a^3x} \div \left(a - \frac{9b^2}{4a}\right) = \frac{(2a-3b)(2a+b)}{8a^3x} \cdot \frac{4a}{(2a-3b)(2a+3b)}$
$= \frac{2a+b}{4a^3x+6a^2bx}.$

15. $\left(\frac{2a}{x}\right)^4 \div \left(\frac{-6a^2c}{5x}\right)^3 \cdot \left(\frac{3c^3}{10ax}\right)^2 = \left(\frac{2a}{x}\right)^4 \cdot \left(\frac{5x}{-6a^2c}\right)^3 \cdot \left(\frac{3c^3}{10ax}\right)^2 = \frac{-5c^3}{6a^4x^3}.$

16. $\left(\dfrac{-a^2}{x}\right) \div \left(\dfrac{-a}{x^2}\right)^5 \cdot \left(\dfrac{a}{x}\right)^3 \cdot \left(\dfrac{-1}{x^2}\right)^3 = \dfrac{-a^2}{x} \cdot \left(\dfrac{x^2}{-a}\right)^5 \cdot \dfrac{a^3}{x^3} \cdot \dfrac{-1}{x^6} = -1.$

17. $\left(\dfrac{m^2}{n^2} - \dfrac{n^2}{m^2}\right) \div \left(\dfrac{m^4 + 2m^2n^2 + n^4}{m^2n - 4mn^2}\right) = \dfrac{m^4 - n^4}{m^2n^2} \cdot \dfrac{mn(m - 4n)}{(m^2 + n^2)^2}$

$$= \frac{(m^2 - n^2)(m - 4n)}{mn(m^2 + n^2)}$$

$$= \frac{m^3 - 4m^2n - mn^2 + 4n^3}{m^3n + mn^3}.$$

18. $\dfrac{9x^2 + 6xy - 8y^2}{2x + y} \div (3x - 2y)^2 \div \left(2 - \dfrac{x - 2y}{2x + y}\right)$

$$= \frac{(3x + 4y)(3x - 2y)}{2x + y} \cdot \frac{1}{(3x - 2y)^2} \cdot \frac{2x + y}{3x + 4y}$$

$$= \frac{1}{3x - 2y}.$$

19. $\left(\dfrac{a}{c} + \dfrac{c}{a}\right) \div \left(\dfrac{a^6 + c^6}{a^3c^3}\right)\left(a^2 - c^2 + \dfrac{c^4}{a^2}\right) \cdot \dfrac{1}{ac^2}$

$$= \frac{a^2 + c^2}{ac} \cdot \frac{a^3c^3}{a^6 + c^6} \cdot \frac{a^4 - a^2c^2 + c^4}{a^2} \cdot \frac{1}{ac^2} = \frac{1}{a}.$$

20. $\dfrac{2a - 5}{2a^2 + 2} \div \dfrac{4a^4 - 25a^2}{4 - 4a^4} \cdot \left(3a^2 - \dfrac{a^3 - 8a^2}{a - 1}\right)$

$$= \frac{2a - 5}{2(a^2 + 1)} \cdot \frac{4(1 - a^4)}{a^2(2a + 5)(2a - 5)} \cdot \frac{a^2(2a + 5)}{a - 1}$$

$$= -2 - 2a.$$

21. $\dfrac{8m^3 - 125n^3}{m^2 + mn} \div \left(\dfrac{2m}{5n} - \dfrac{5n}{2m}\right) \div \left[3mn\left(2m + 5n + \dfrac{25n^2}{2m}\right)\right]$

$$= \frac{8m^3 - 125n^3}{m(m + n)} \cdot \frac{10mn}{(2m + 5n)(2m - 5n)} \cdot \frac{2m}{3mn(4m^2 + 10mn + 25n^2)}$$

$$= \frac{20}{6m^2 + 21mn + 15n^2}.$$

22. $\left(\dfrac{4y}{x} - \dfrac{15y^2}{x^2} + 4\right) \div \left(4 - \dfrac{16y}{x} + \dfrac{15y^2}{x^2}\right)\left(3 - \dfrac{4x + 20y}{2x + 5y}\right)$

$$= \frac{(2x + 5y)(2x - 3y)}{x^2} \cdot \frac{x^2}{(2x - 3y)(2x - 5y)} \cdot \frac{2x - 5y}{2x + 5y}$$

$$= 1.$$

23. $\left(\dfrac{9c^2 - 4d^2}{6c^2}\right) \div \left(d + 4c + \dfrac{15c^2}{4d}\right) \cdot \dfrac{15c^2d + 6cd^2}{9c^2d + 24cd^2 - 20d^3}$

$$= \frac{(3c + 2d)(3c - 2d)}{6c^2} \cdot \frac{4d}{(2d + 5c)(2d + 3c)} \cdot \frac{3cd(5c + 2d)}{d(3c + 10d)(3c - 2d)}$$

$$= \frac{2d}{3c^2 + 10cd}.$$

24. $$\left(6x - 11 - \frac{7}{x}\right) \div \left(2 + \frac{11}{x} + \frac{5}{x^2}\right) \div \left(\frac{1}{3x^3 - 75x}\right)$$
$$= \frac{(3x-7)(2x+1)}{x} \cdot \frac{x^2}{(x+5)(2x+1)} \cdot \frac{3x(x+5)(x-5)}{1}$$
$$= 9x^4 - 66x^3 + 105x^2.$$

25. $$\left(x^2 - y^2 + \frac{4xy(y+x)}{x-y}\right) \div \frac{x^2 + y(y+2x)}{2x^2 - 3xy + y^2}$$
$$= \frac{(x+y)^3}{x-y} \cdot \frac{(x-y)(2x-y)}{(x+y)^2}$$
$$= 2x^2 + xy - y^2.$$

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1. $$\frac{4 - \frac{1}{4}}{2 + \frac{1}{2}} = \frac{\frac{15}{4}}{\frac{5}{2}} = \frac{15}{4} \cdot \frac{2}{5} = \frac{3}{2}.$$

2. $$\frac{\frac{4}{9} + 1}{\frac{7}{2} - 2} = \frac{\frac{13}{9}}{\frac{3}{2}} = \frac{13}{9} \cdot \frac{2}{3} = \frac{26}{27}.$$

3. $$\frac{9 - (\frac{1}{5})^2}{4 - \frac{4}{5}} = \frac{\frac{224}{25}}{\frac{16}{5}} = \frac{224}{25} \cdot \frac{5}{16} = \frac{14}{5}.$$

4. $$\frac{\frac{3}{5} + \frac{4}{7}}{2 - \frac{3}{5} \cdot \frac{4}{7}} = \frac{\frac{41}{35}}{\frac{58}{35}} = \frac{41}{35} \cdot \frac{35}{58} = \frac{41}{58}.$$

5. $$\frac{(\frac{8}{5})^2 - 2}{3 \cdot \frac{8}{5}} = \frac{\frac{14}{25}}{\frac{24}{5}} = \frac{14}{25} \cdot \frac{5}{24} = \frac{7}{60}.$$

6. $$\frac{4 - \frac{3}{2} + \frac{2}{3}}{3 - \frac{2}{3} + \frac{3}{2}} = \frac{\frac{19}{6}}{\frac{23}{6}} = \frac{19}{23}.$$

7. $$\frac{\frac{2^3}{27} + 1}{\frac{9}{2} - 1} = \frac{\frac{35}{27}}{\frac{7}{2}} = \frac{35}{27} \cdot \frac{2}{7} = \frac{10}{27}.$$

8. $$\frac{2\frac{1}{2} - 3\frac{1}{3} + 4\frac{3}{4}}{2\frac{1}{2} \cdot 3\frac{1}{3} - 4\frac{3}{4}} = \frac{\frac{47}{12}}{\frac{43}{12}} = \frac{47}{43}.$$

9. $$\frac{2 - \frac{9}{2}}{(\frac{1}{2})^2 - (\frac{1}{2})^3 - 12(\frac{1}{2})^4} = \frac{-\frac{5}{2}}{-\frac{5}{8}} = 4.$$

10. $$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}.$$

11. $$\frac{\frac{1+x}{x}}{1 - \frac{1}{x^2}} = \frac{1+x}{x} \cdot \frac{x^2}{x^2-1} = \frac{x}{x-1}.$$

12. $$\frac{c + \frac{c}{d}}{1 + \frac{1}{d}} = \frac{\frac{cd+c}{d}}{\frac{d+1}{d}} = \frac{cd+c}{d} \cdot \frac{d}{d+1} = c.$$

13. $$\frac{9 - \frac{b^2}{4a^2}}{1 - \frac{b}{6a}} = \frac{\frac{36a^2 - b^2}{4a^2}}{\frac{6a-b}{6a}} = \frac{36a^2 - b^2}{4a^2} \cdot \frac{6a}{6a-b} = \frac{18a + 3b}{2a}.$$

14. $$\frac{\frac{a^2}{b^2} - \frac{b^2}{a^2}}{a + \frac{b^2}{a}} = \frac{a^4 - b^4}{a^2b^2} \cdot \frac{a}{a^2+b^2} = \frac{a^2 - b^2}{ab^2}.$$

15. $$\frac{a-\frac{9}{a}}{\frac{1}{a^2}-\frac{1}{a^3}-\frac{12}{a^4}}=\frac{a^2-9}{a}\cdot\frac{a^4}{(a+3)(a-4)}=\frac{a^4-3a^3}{a-4}.$$

16. $$\frac{\frac{m^3}{n^3}+1}{m+\frac{n^2}{m}-n}=\frac{m^3+n^3}{n^3}\cdot\frac{m}{m^2-mn+n^2}=\frac{m^2+mn}{n^3}.$$

17. $$\frac{2+\frac{1}{a-2}+a}{a+\frac{1}{a+2}-2}=\frac{a^2-3}{a-2}\cdot\frac{a+2}{a^2-3}=\frac{a+2}{a-2}.$$

18. $$\frac{\frac{a}{b}}{c}-\frac{a}{\frac{b}{c}}=\frac{a}{b}\cdot\frac{1}{c}-\frac{a}{1}\cdot\frac{c}{b}=\frac{a-ac^2}{bc}.$$

19. $$\frac{1+\frac{b}{a}-\frac{20a}{b}}{\frac{b}{a}-\frac{8a}{b}-2}=\frac{(b+5a)(b-4a)}{ab}\cdot\frac{ab}{(b-4a)(b+2a)}=\frac{b+5a}{b+2a}.$$

20. $$\frac{\frac{16}{x}-x}{\frac{24}{x^4}+\frac{10}{x^3}+\frac{1}{x^2}}=\frac{16-x^2}{x}\cdot\frac{x^4}{x^2+10x+24}=\frac{x^3(4-x)}{6+x}.$$

21. $$\frac{x+\frac{x^2+y^2}{y}}{\frac{x}{y}-1}=\frac{x^2+xy+y^2}{y}\cdot\frac{y}{x-y}=\frac{x^2+xy+y^2}{x-y}.$$

22. $$\frac{2+\frac{a}{b}}{\frac{(a-2b)^2}{4ab}+2}=\frac{2b+a}{b}\cdot\frac{4ab}{a^2+4ab+4b^2}=\frac{4a}{a+2b}.$$

23. $$\frac{\left(\frac{2x^2}{x-y}\right)^2}{\frac{8x^3}{(x-y)^3}}=\frac{4x^4}{(x-y)^2}\cdot\frac{(x-y)^3}{8x^3}=\frac{x(x-y)}{2}.$$

24. $$\frac{\frac{a-b}{a}-\frac{a+b}{b}}{\frac{a-b}{b}+\frac{a+b}{a}}=\frac{-(a^2+b^2)}{ab}\cdot\frac{ab}{a^2+b^2}=-1.$$

25. $$1-\frac{\left(1-\frac{c^4}{9}\right)-\left(1-\frac{c^4}{16}\right)}{1-\frac{7c^4}{144}}=1-\frac{\frac{-7c^4}{144}}{\frac{144-7c^4}{144}}$$
$$=1+\frac{7c^4}{144-7c^4}$$
$$=\frac{144}{144-7c^4}.$$

26. $$\frac{\frac{a}{1+a}+\frac{1-a}{a}}{\frac{a}{1+a}-\frac{1-a}{a}}=\frac{\frac{1}{a(a+1)}}{\frac{2a^2-1}{a(a+1)}}=\frac{1}{2a^2-1}.$$

27. $$\frac{\frac{9x}{8}-\frac{9y}{8}-\frac{7y^2}{2x}}{\frac{3x}{4}+\frac{9y}{4}+\frac{5y^2}{3x}}=\frac{\frac{9x^2-9xy-28y^2}{8x}}{\frac{9x^2+27xy+20y^2}{12x}}$$
$$=\frac{(3x-7y)(3x+4y)}{8x}\cdot\frac{12x}{(3x+5y)(3x+4y)}$$
$$=\frac{9x-21y}{6x+10y}.$$

28. $$\frac{\frac{x}{2ax+3bx+6ab+x^2}}{\frac{1}{x+2a}-\frac{1}{x+3b}}=\frac{x}{(x+2a)(x+3b)}\cdot\frac{(x+2a)(x+3b)}{3b-2a}$$
$$=\frac{x}{3b-2a}.$$

29. $$\frac{\left(\frac{3a+4b}{3a}\right)^2-\frac{6b}{a}}{8a-\frac{(3a+2b)^2}{3b}}=\frac{(3a-8b)(3a-2b)}{9a^2}\cdot\frac{3b}{(2b-3a)(3a-2b)}$$
$$=\frac{b(3a-8b)}{3a^2(2b-3a)}.$$

30. $$\frac{1-\frac{x^2-1}{5x^2-6x+1}}{1+\frac{3x+2}{5x-1}}=\frac{\frac{4x^2-6x+2}{5x^2-6x+1}}{\frac{8x+1}{5x-1}}$$
$$=\frac{2(x-1)(2x-1)}{(5x-1)(x-1)}\cdot\frac{5x-1}{8x+1}$$
$$=\frac{4x-2}{8x+1}.$$

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1. $$\frac{x}{2}+\frac{x}{3}=10. \quad (1)$$

$(1)\cdot 6,$ $3x+2x=60.$

Whence $x=12.$

2. $$\tfrac{4}{3}x+\tfrac{2}{5}x=5\tfrac{1}{5}. \quad (1)$$

$(1)\cdot 15,$ $20x+6x=78.$

Whence $x=3.$

3. $$\frac{x+5}{4}-\frac{2x+4}{9}=1. \quad (1)$$

$(1)\cdot 36,$ $9(x+5)-4(2x+4)=36.$

Whence $x=7.$

4. $$\frac{2x+3}{5}-\frac{1}{3}(x-3)=2. \quad (1)$$

$(1)\cdot 15,$ $3(2x+3)-5(x-3)=30.$

Whence $x=6.$

5. $$\frac{4x+2}{11}-\frac{1}{5}(x+5)=0. \quad (1)$$

$(1)\cdot 55,$ $5(4x+2)-11(x+5)=0.$

Whence $x=5.$

6. $$\frac{3}{4}(x+1)-\frac{5x-7}{6}=\frac{7}{3}. \quad (1)$$

$(1)\cdot 12,$ $9(x+1)-2(5x-7)=28.$

Whence $x=-5.$

7. $$\frac{x+6}{10}+\frac{3}{2}(x+4)=-3. \quad (1)$$

$(1)\cdot 10,$ $x+6+15(x+4)=-30.$

Whence $x=-6.$

8. $$\frac{5x-12}{6}-\frac{4}{11}(2x-7)=\frac{1}{3}. \quad (1)$$

$(1)\cdot 66,$ $11(5x-12)-24(2x-7)=22.$

Whence $x=-2.$

9. $$2x-1-\frac{12x-7}{6}-\frac{1}{6}=0. \quad (1)$$

$(1)\cdot 6,$ $6(2x-1)-(12x-7)-1=0.$

Whence $0=0$, or any number will satisfy the equation.

10. $$\frac{10x-7}{6}+\frac{5}{2}\left(\frac{2}{5}-x\right)=\frac{15x-11}{3}. \quad (1)$$

$(1)\cdot 6,$ $(10x-7)+15(\tfrac{2}{5}-x)=2(15x-11).$

Whence $x=\tfrac{3}{5}.$

11. $$\frac{5x}{6}-\frac{1}{2}-\frac{3}{8}\left(x-\frac{5}{3}\right)+\frac{7}{32}=0. \quad (1)$$

$(1)\cdot 96,$ $80x-48-12(3x-5)+21=0.$

Whence $x=-\tfrac{3}{4}.$

12. $$\frac{5}{x}+\frac{4}{3}=\frac{9}{x}. \quad (1)$$

$(1)\cdot 3x,$ $15+4x=27.$

Whence $x=3.$

13. $$\frac{9x}{4}-\frac{3}{4}-\frac{3x-7}{3}+4=-\frac{17}{24}. \quad (1)$$

$(1)\cdot 24,$ $54x-18-8(3x-7)+96=-17.$

Whence $x=-\tfrac{151}{30}.$

14. $\dfrac{c}{3x} - \dfrac{c}{5x} = \dfrac{1}{15}.$

Whence $x = 2c.$

15. $\dfrac{1}{2x} - \dfrac{13}{24} = \dfrac{8}{3x}.$

Whence $x = -4.$

16. $\dfrac{2a - 3x}{6a} + \dfrac{5a - 2x}{5a} + \dfrac{41}{30} = 0.$

Whence $x = 3a.$

17. $\dfrac{x}{a} - \dfrac{1}{3}(a - 3x) + \dfrac{19a}{3} = -6.$

Whence $x = -6a.$

18. $\dfrac{cx}{3} - \dfrac{n}{5}(3x - 5cn) = cn\left(\dfrac{2c}{3} - \dfrac{n}{5}\right).$ (1)

(1) · 15, $5cx - 9nx + 15cn^2 = 10c^2n - 3cn^2.$

Whence $x(5c - 9n) = 2cn(5c - 9n),$

or $x = 2cn.$

19. $2x - b - \dfrac{b}{a}(3x - 4b) + 2a = \dfrac{2(2a^2 - b^2)}{a}.$

Whence $2ax - 3bx = 2a^2 + ab - 6b^2,$

or $x = a + 2b.$

20. $(x + 5)(x - 6) = x(x - \frac{5}{2}).$

Whence $x^2 - x - 30 = x^2 - \dfrac{5x}{2},$

or $x = 20.$

21. $\left(x + \dfrac{3}{4}\right)\left(3 + \dfrac{x}{2}\right) = \dfrac{x}{2}(x - 5) + 8\frac{1}{8}.$ (1)

(1) · 8, $4\left(x + \dfrac{3}{4}\right)2\left(3 + \dfrac{x}{2}\right) = 4x(x - 5) + 65.$

Whence $x = 1.$

22. $(x - \frac{1}{2})(x + \frac{3}{7}) = (x - 1)(x + 2) + 1\frac{11}{14}.$ (1)

(1) · 14, $2(x - \frac{1}{2})7(x + \frac{3}{7}) = 14(x^2 + x - 2) + 25.$

Whence $x = 0.$

23. $(x - \frac{3}{2})(x + \frac{2}{3}) - (x - \frac{1}{3})^2 - 1\frac{1}{9} = 0.$ (1)

(1) · 18, $6(x - \frac{3}{2})3(x + \frac{2}{3}) - 18(x - \frac{1}{3})^2 - 20 = 0.$

Whence $x = -\frac{40}{3}.$

24. $(x + \frac{2}{5})^2 - (x - \frac{1}{2})(x + \frac{1}{5}) + \frac{31}{50} = 0.$ (1)

(1) · 50, $2 \cdot 25(x + \frac{2}{5})^2 - 10(x - \frac{1}{2})(x + \frac{1}{5})5 + 31 = 0.$

Whence $x = -\frac{4}{5}.$

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1. If $n =$ the number,

then $\frac{n}{4} + \frac{n}{12} = 16.$

Whence $n = 48.$

2. If $n =$ the number,

then $\frac{n}{3} - \frac{n}{17} = 70.$

Whence $n = 255.$

3. If $n =$ the greater number,

then $38 - n =$ the less,

and $\frac{n}{10} = \frac{38 - n}{9}.$

Whence $n = 20,$

and $38 - n = 18.$

4. If $l =$ the length in centimeters,

then $\frac{4l}{5} =$ the width,

and $2l + \frac{8l}{5} = 216.$

Whence $l = 60,$

and $\frac{4l}{5} = 48.$

Therefore the area of the rectangle is 2880 square centimeters.

5. Let $n =$ the number.

Then $\frac{4+n}{7} = \frac{n}{5}.$

Whence $n = 10.$

6. Let n, $n + 1$, and $n + 2$ represent the numbers.

Then $\frac{3n}{4} = \frac{n+1+n+2}{3}.$

Whence $n = 12.$

7. Let n, $n + 2$, and $n + 4$ represent the numbers.

Then $\frac{n}{11} = \frac{n+2+n+4}{24}.$

Whence $n = 33,$

$n + 2 = 35,$

and $n + 4 = 37.$

8. If $n =$ the number,

then $\frac{10+n}{17+n} = \frac{5}{7}.$

Whence $n = \frac{15}{2}.$

9. Let $g =$ the greater part.

Then $42 - g =$ the less,

and $\frac{g-(42-g)}{6} = \frac{1}{3}.$

Whence $g = 22,$

and $42 - g = 20.$

10. Let $n =$ the number.

Then $\frac{3n-4}{4} = \frac{5n-4}{7}.$

Whence $n = 12.$

11. Let $g =$ the greater part.

Then $112 - g =$ the less,

and $\frac{112-g}{g} = \frac{2}{9}.$

Whence $g = 91\frac{7}{11},$

and $112 - g = 20\frac{4}{11}.$

12. Let $n =$ one number.

Then $24 - n =$ the other,

and

$$\frac{n-(24-n)}{n+24-n} = n - (24 - n) - 3\tfrac{5}{6}.$$

Whence $n = 14,$

and $24 - n = 10.$

13. Let $n =$ the number.

Then $\frac{27+7n}{2n} = \frac{90+5n}{3n}.$

Whence $n = 9.$

14. Let $x =$ B's age in years.

Then $\frac{5x}{2} =$ A's age in years,

and $\frac{5x}{2} + 10 = 2(x + 10).$

Whence $x = 20,$

and $\frac{5x}{2} = 50.$

15. Let $x =$ B's age in years.

Then $\frac{2x}{3} =$ A's age in years,

and $\frac{2x}{3} - 14 = \frac{1}{2}(x - 14).$

Whence $x = 42,$

and $\frac{2x}{3} = 28.$

16. Let $x =$ B's age in years.

Then $x + 16 =$ A's age in years,

and $x - 8 = \frac{3}{5}(x + 16 - 8).$

Whence $x = 32,$

and $x + 16 = 48.$

17. Let $x =$ the number of moons of Uranus.

Then $x + 4 =$ the number of moons of Jupiter,

$2x + 2 =$ the number of moons of Saturn,

$x - 2 =$ the number of moons of Mars,

and $\frac{x - 2}{2} =$ the number of moons of Neptune.

Then $x + x + 4 + 2x + 2 + x - 2 + \frac{x - 2}{2} = 25.$

Whence $x = 4,$

$x + 4 = 8, \quad 2x + 2 = 10, \quad x - 2 = 2,$

and $\frac{x - 2}{2} = 1.$

18. Let $x =$ the other base of the trapezoid in meters.

Then $\frac{8 \cdot 30}{2} = \frac{(8 + x)\,10}{2}.$

Whence $x = 16.$

19. Let $d =$ the distance to the target in feet.

Then $\frac{d}{1100} + \frac{d}{1925} = 3.$

Whence $d = 2100,$

and $\frac{2100}{1925} = \frac{12}{11}.$

Therefore $\frac{12}{11}$ seconds is the time the bullet was in the air.

20. Let $v =$ the velocity of the bullet in feet per second.

Then $\frac{3 \cdot 2640}{v} + \frac{3 \cdot 2640}{1100} = 9\frac{2}{5}.$

Whence $v = 3600.$

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1. $$\frac{32}{x} = 5.$$

Then $32 = 5x,$

or $x = \frac{32}{5}.$

2. $$\frac{25}{3x} = 5.$$

Then $5 = 3x,$

or $x = \frac{5}{3}.$

3. $$5x + \frac{x-2}{4} = 4x + 7.$$

Then $20x + x - 2 = 16x + 28,$

or $x = 6.$

4. $$3x - \frac{x-1}{4} - \frac{8x}{5} = 6.$$

Then $60x - 5x + 5 - 32x = 120,$

or $x = 5.$

5. $$\frac{4}{x} - \frac{12+x}{3x} = \frac{4}{3}.$$

Then $x = 0,$

or the equation is "impossible," for division by zero is excluded.

6. $$\frac{5}{6} - \frac{3x-5}{4x} + \frac{x-2}{3x} = 0.$$

Then

$10x - 9x + 15 + 4x - 8 = 0,$

or $x = -\frac{7}{5}.$

7. $$\frac{x+5}{5x} - \frac{3(x+1)}{x} = 3\frac{1}{5}.$$

Then

$x + 5 - 15x - 15 = 16x,$

or $x = -\frac{1}{3}.$

8. $$5x - 4x\left(3 - \frac{2}{x}\right) + \frac{1}{4} = 3.$$

Then

$20x - 48x + 32 + 1 = 12,$

or $x = \frac{3}{4}.$

9. $$x - \frac{3x}{5}\left(\frac{10}{x} - 4\right) + 3\frac{3}{5} = 18,$$

or $5x - 30 + 12x + 18 = 90.$

Then $x = 6.$

10. $$\frac{8x-7}{4x} + 3\frac{1}{2} = \frac{\frac{3}{2} - x}{x}.$$

Then $8x - 7 + 14x = 6 - 4x,$

or $x = \frac{1}{2}.$

11. $$\frac{2x-3}{3+2x} = 4,$$

or $2x - 3 = 12 + 8x.$

Then $x = -\frac{5}{2}.$

12. $$\frac{x-2}{x-3} = \frac{15}{16}.$$

Then $16(x-2) = 15(x-3),$

or $x = -13.$

13. $$\frac{5}{7x+5} - \frac{1}{8} = 0,$$

or $40 - 7x - 5 = 0.$

Then $x = 5.$

14. $$\frac{3x^2 - 7x - 4}{4x^2 - 10x - 8} = \frac{3}{4}.$$

Then $6x^2 - 14x - 8 = 6x^2 - 15x - 12,$

or $x = -4.$

15. $$\frac{1}{x-2} = \frac{3}{x-3},$$

or $x - 3 = 3(x-2).$

Then $x = \frac{3}{2}.$

16. $$\frac{1}{x-3}=\frac{x-6}{x+3}+\frac{x}{x+3}.$$

Then $$x+3=x^2-9x+18+x^2-3x.$$

Whence $$x=\tfrac{3}{2} \text{ or } 5.$$

17. $$\frac{x-3}{x+4}=\frac{x-9}{x+5},$$

or $$(x-3)(x+5)=(x+4)(x-9).$$

Then $$x=-3.$$

18. $$\frac{x-2}{x+2}+\frac{4}{x+2}+2=0,$$

or $$x-2+4+2x+4=0.$$

Then $$x=-2.$$

Since $x=-2$ the equation is "impossible," for division by zero is excluded.

19. $$\frac{3x}{4}-\frac{2}{x-2}=\frac{3x-2}{4},$$

or $$3x(x-2)-8=(3x-2)(x-2).$$

Then $$x=6.$$

20. $$\frac{4}{x-3}+\frac{3x+4}{6}=\frac{x}{2},$$

or $$24+(3x+4)(x-3)=3x(x-3).$$

Then $$x=-3.$$

21. $$\frac{2x+3}{x-5}+4=\frac{x+8}{x-5},$$

or $$2x+3+4(x-5)=x+8.$$

Then $$x=5.$$

Since $x=5$, the equation is "impossible," for division by zero is excluded.

22. $$\frac{x}{4}-\frac{5}{4x-12}=\frac{2x+\frac{5}{4}}{3},$$

or $$3x(x-3)-15=4(x-3)(2x+\tfrac{5}{4}).$$

Then $$5x^2-10x=0,$$

and $$x=2 \text{ or } 0.$$

23. $$\frac{x-4}{x+5}+\frac{7}{5}=\frac{3}{x+5},$$

or $$5(x-4)+7(x+5)=15.$$

Then $$x=0.$$

24. $$\frac{x+4}{15x-5}-\frac{x}{5}=\frac{2x+\frac{20}{3}}{10}.$$

Then $36x^2+42x-44=0,$

or $(3x-2)(6x+11)=0.$

Therefore $x=\frac{2}{3}$ or $-\frac{11}{6}.$

25. $$\frac{4}{3x+6}+\frac{5}{7x+14}+43=0,$$

or $28+15+43\cdot 21\cdot(x+2)=0.$

Then $x=-\frac{43}{21}.$

26. $$\frac{7}{4x-12}+\frac{47}{220}=\frac{-3}{5x-15},$$

or $7(55)+47(x-3)=-132.$

Then $x=-8.$

27. $$\frac{4x}{x+3}-\frac{6}{2x+6}=\frac{10x+11}{3x+9},$$

or $24x-18=(10x+11)2.$

Then $x=10.$

28. $$\frac{3}{x-2}=\frac{5}{x^2-25}+\frac{3x}{x^2-25},$$

or $3(x^2-25)=5(x-2)+3x(x-2).$

Then $x=65.$

29. $$\frac{1}{x-3}+\frac{2}{x+3}=\frac{-3}{x^2-9},$$

or $x+3+2(x-3)=-3.$

Then $x=0.$

30. $$\frac{x+2}{x-2}=\frac{10-x^2}{4-x^2}-\frac{10}{x^2-4},$$

or $$\frac{x+2}{x-2}=\frac{x^2-20}{x^2-4},$$

or $(x+2)(x+2)=x^2-20.$

Then $x=-6.$

31. $$\frac{x-4}{x-5}+\frac{x-15}{x+4}=\frac{2x^2-10x-1}{x^2-x-20},$$

or $(x-4)(x+4)+(x-15)(x-5)$

$=2x^2-10x-1.$

Then $x=6.$

32. $$\frac{x+2}{x+3}+\frac{x+3}{x+2}=\frac{4x+9}{x^2+5x+6},$$

or $(x+2)^2+(x+3)^2=4x+9,$

or $(x+2)(x+1)=0.$

Then $x=-1,$

for -2 does not satisfy, since division by zero is excluded.

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1. $$.3x + 4 = .25. \qquad (1)$$
$[(1) \cdot 100] \div 30,$ $\quad x = -\frac{25}{2}.$

2. $$.15x - .4x = 235x - 2352.5. \qquad (1)$$
$[(1) \cdot 100,$ collected$] \div -23{,}525,$ $\quad x = 10.$

3. $$1.3x + 8.24 = -5.26 - 3.2x. \qquad (1)$$
$[(1) \cdot 100,$ collected$] \div 450,$ $\quad x = -3.$

4. $$3x - 1.245x + .6x = 1.5 + .355x. \qquad (1)$$
$[(1) \cdot 1000,$ collected$] \div 2000,$ $\quad x = .75.$

5. $$3.5x + .0564 - .1x = 4.9128 - .02x. \qquad (1)$$
$[(1) \cdot 10{,}000,$ collected$] \div 34{,}200,$ $\quad x = 1.42.$

6. $$.12(2x + .05) - .15(1.5x - 2) = 0.246. \qquad (1)$$
$$.24x + .006 - .225x + .30 = .246. \qquad (2)$$
$[(2) \cdot 1000,$ collected$] \div 15,$ $\quad x = -4.$

7. $$\frac{.01x + .003}{6} + \frac{.02x + .0008}{7} = .0017. \qquad (1)$$
$(1) \cdot 10{,}000,$ $$\frac{100x + 30}{6} + \frac{200x + 8}{7} = 17. \qquad (2)$$
$[(2) \cdot 42] \div 1900,$ $\quad x = .24.$

8. $$\frac{.3(x + 5)}{8} - \frac{4(.25x - .35)}{7} = \frac{14.325}{56}. \qquad (1)$$
$(1) \cdot 1000,$ $$\frac{300(x + 5)}{8} - \frac{4(250x - 350)}{7} = \frac{14325}{56}. \qquad (2)$$
$[(2) \div 56] \div -5900,$ $\quad x = \frac{5}{4}.$

9. $$\frac{0.5(6 - .2x)}{.80} - \frac{.3(.4x - 3)}{.16} = 5. \qquad (1)$$
100 (numerator and denominator of each fraction),
$$\frac{5(60 - 2x)}{80} - \frac{3(4x - 30)}{16} = 5. \qquad (2)$$
$(2) \cdot 80$ and solved gives $\quad x = 5.$

10. $$\frac{.32x}{.05} + \frac{.045x}{.125} = 13.52. \qquad (1)$$
(First numerator and denominator) $\cdot$ 100, and (second numerator and denominator) $\cdot$ 1000,
$$\frac{32x}{5} + \frac{45x}{125} = 13.52. \qquad (2)$$
$[(2) \cdot (125)] \div 845,$ $\quad x = 2.$

11. $$\frac{33}{x+5}+\frac{3.75}{.5(x-8.5)}=0. \qquad (1)$$

(Numerator and denominator of second fraction) · 100,

$$\frac{33}{x+5}+\frac{375}{5(10x-85)}=0. \qquad (2)$$

$[(2)\cdot(x+5)(10x-85)\cdot 5]\div 2025, \qquad x=6.$

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1. $5cx-8c^2=4c^2-cx.$
$6cx=12c^2.$
Therefore $x=2c.$

2. $2(x+1)-4k=2.$
$2x=4k.$
Therefore $x=2k.$

3. $3(2x-a)=2(x-2a).$
$6x-3a=2x-4a.$
Therefore $x=-\frac{a}{4}.$

4. $ax+bx=a^2+ab.$
$x(a+b)=a(a+b).$
Therefore $x=a.$

5. $cx+b^2=bx+bc.$
$x(c-b)=b(c-b).$
Therefore $x=b.$

6. $mx+n^2=m^2-nx.$
$x(m+n)=(m^2-n^2).$
Therefore $x=m-n.$

7. $6ac+cx+4a^2=2ax+3c^2+2ca.$
$x(c-2a)=3c(c-2a)+2a(c-2a).$
Therefore $x=3c+2a.$

8. $5ax-5a^2+6b^2=7ab+3bx.$
$x(5a-3b)=(5a-3b)(a+2b).$
Therefore $x=a+2b.$

9. $\frac{x}{2a}=b.$
Therefore $x=2ab.$

10. $\frac{3ab}{x}=a.$
Therefore $x=3b.$

11. $\frac{a}{x}+\frac{3a}{2x}=\frac{5}{4}.$
$4a+6a=5x.$
Therefore $x=2a.$

12. $\frac{4a}{3x}+\frac{4a}{x}=\frac{3}{2}+\frac{5a}{6x}.$
$8a+24a=9x+5a.$
Therefore $x=3a.$

13. $\frac{x}{a}+\frac{x}{b}=a+b.$
$x(a+b)=ab(a+b).$
Therefore $x=ab.$

14. $\frac{c^2}{x}-c=\frac{d^2}{x}+d.$
$-x(c+d)=d^2-c^2.$
Therefore $x=c-d.$

15. $\frac{ax}{2b}-4b^2=\frac{2bx}{a}-a^2.$
$x(a^2-4b^2)=-2ab(a^2-4b^2).$
Therefore $x=-2ab.$

16. $$\frac{x}{c}+\frac{c-2x}{3}-3c=-4.$$

$$x(3-2c)=-4c(3-2c).$$

Therefore $$x=-4c.$$

17. $$\frac{x}{a}+\frac{x}{c}+ac=bc+ab+\frac{x}{b}.$$

$$x(bc+ab-ac)=abc(bc+ab-ac).$$

Therefore $$x=abc.$$

18. $$\frac{3b+4x}{5b}+\frac{3b+2x}{4b}=\frac{1}{20}.$$

$$26x=b-27b.$$

Therefore $$x=-b.$$

19. $$\frac{ax}{2}-\frac{3b}{5}\left(x-\frac{2ab}{3}\right)=ab\left(\frac{a}{2}-\frac{b}{5}\right).$$

$$5ax-6bx=5a^2b-6ab^2.$$

Therefore $$x=ab.$$

20. $$\frac{2x-3b}{a}+\frac{2}{b}\left(\frac{3x}{2}-a\right)+5+\frac{9b}{a}=\frac{4a}{b}.$$

$$x(3a+2b)=6a^2-5ab-6b^2.$$

Therefore $$x=2a-3b.$$

21. $$\frac{x-m^2}{x-n^2}=\frac{n}{m}.$$

$$x(m-n)=m^3-n^3.$$

Therefore $$x=m^2+mn+n^2.$$

22. $$\frac{x}{c}-\frac{d}{c}=\frac{x+2c}{d}-3.$$

$$x(d-c)=d^2-3cd+2c^2.$$

Therefore $$x=d-2c.$$

23. $$\frac{b}{b(b-x)}+\frac{3}{a(b-x)}+\frac{3a+9}{2ab}=0.$$

$$2ab+6b+3ab+9b-3ax-9x=0.$$

Therefore $$x=\frac{5b}{3}.$$

24. $$\frac{c}{a(x+c)}+\frac{a}{c(x-a)}=\frac{c^2-ac+2a^2}{2ac(x-a)}.$$

$$2c^2x-2ac^2+2a^2x+2a^2c=c^2x-acx+2a^2x+c^3-ac^2+2a^2c.$$

$$x(c+a)=c^2+ac.$$

Therefore $$x=c.$$

25. $$\frac{1}{ab} + 1 - \frac{ab}{x} + \frac{1}{abx} = 0.$$

$$x + abx - a^2b^2 + 1 = 0.$$

Therefore $$x = ab - 1.$$

26. $$\frac{a^2}{bx} + \frac{b^2}{ax} = \frac{1}{a} + \frac{1}{b}.$$

$$a^3 + b^3 = x(a + b).$$

Therefore $$x = a^2 - ab + b^2.$$

27. $$\frac{c^2}{dx} - \frac{d^2}{cx} - \frac{3c - 3d}{x} = \frac{c - d}{cd}.$$

$$x(c - d) = c^3 - 3c^2d + 3cd^2 + d^3.$$

Therefore $$x = c^2 - 2cd + d^2.$$

28. $$\frac{a + x}{b + x} - \frac{a - x}{b - x} = \frac{2}{x^2 - b^2}.$$

$$(a + x)(b - x) - (a - x)(b + x) = -2.$$

Whence $$x = \frac{1}{a - b}.$$

29. $$\frac{\dfrac{x}{a - b} - a}{\dfrac{x}{a - b} + a} + 1 = \frac{2}{a^2 + ab + 1}.$$

$$\frac{x - a^2 + ab}{x + a^2 - ab} + 1 = \frac{2}{a^2 + ab + 1}.$$

$$2a^2 + 2abx + 2x = 2x + 2a^2 - 2ab,$$

or $$ax + bx = a - b.$$

Therefore $$x = \frac{a - b}{a + b}.$$

30. $$\frac{\dfrac{a}{2} + \dfrac{x}{3}}{\dfrac{a}{3} - \dfrac{x}{2}} + \frac{7a}{4a - 6x} = -7.$$

$$\frac{3a + 2x}{2a - 3x} + \frac{7a}{2(2a - 3x)} = -7,$$

or $$(3a + 2x)2 + 7a = -14(2a - 3x).$$

Therefore $$x = \frac{41a}{38}.$$

31. $$\frac{a^2 + ac}{x + 3c} + \frac{2cx(a + c)}{x^2 + 5cx + 6c^2} = \frac{a^2 + 2ac + c^2}{x + 2c}.$$

$$(a^2 + ac)(x + 2c) + 2acx + 2c^2x = (a^2 + 2ac + c^2)(x + 3c).$$

Therefore $$x = a + 3c.$$

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1. Solve for R, $K = 2\pi RH$.

$$R = \frac{K}{2\pi H}.$$

2. Solve for a, $A = \frac{ab}{2}$.

$$a = \frac{2A}{b}.$$

3. Solve for R, $C = 2\pi R$.

$$R = \frac{C}{2\pi}.$$

4. Solve for r and t, $d = rt$.

$$r = \frac{d}{t}, \quad t = \frac{d}{r}.$$

5. Solve for a and A, $\frac{a}{A} = \frac{D}{360}$.

$$a = \frac{AD}{360}, \quad A = \frac{360\,a}{D}.$$

6. Solve for C, $\frac{D}{360} = \frac{l}{C}$.

$$C = \frac{360\,l}{D}.$$

7. Solve for r, $C = \frac{E}{R + r}$.

$$r = \frac{E - CR}{C}.$$

8. Solve for r and n, $C = \frac{E}{R + nr}$.

$$r = \frac{E - CR}{Cn}, \quad n = \frac{E - CR}{Cr}.$$

9. Solve for r and n, $C = \frac{n \cdot e}{R + nr}$.

$$r = \frac{n \cdot e - CR}{Cn},$$

$$n = \frac{CR}{e - Cr}.$$

10. Solve for F, $C = \frac{5}{9}(F - 32)$.

$$F = \frac{9C}{5} + 32.$$

11. Solve for W_2, $\frac{W_1}{W_2} = \frac{L_1}{L_2}$.

$$W_2 = \frac{W_1 L_2}{L_1}.$$

12. Solve for r and t, $A = P(1 + rt)$.

$$r = \frac{A - P}{Pt}, \quad t = \frac{A - P}{Pr}.$$

13. Solve for P_2, $\frac{V_1}{V_2} = \frac{P_2}{P_1}$.

$$P_2 = \frac{V_1 P_1}{V_2}.$$

14. Solve for n and l, $s \quad \frac{n(a + l)}{2}$.

$$n = \frac{2s}{a + l}, \quad l = \frac{2s - an}{n}.$$

15. Solve for a, l, and r, $s = \frac{rl - a}{r - 1}$.

$$a = rl + s - rs,$$

$$l = \frac{a + rs - s}{r},$$

$$r = \frac{s - a}{s - l}.$$

16. Solve for θ, $\frac{D}{180} = \frac{\theta}{\pi}$.

$$\theta = \frac{\pi D}{180}.$$

17. Solve for t_1,

$$V_1 = V_0(1 + .00365\,t_1).$$

$$t_1 = \frac{V_1 - V_0}{.00365\,V_0}.$$

18. Solve for b_2, $A = \frac{(b_1 + b_2)a}{2}$.

$$b_2 = \frac{2A - ab_1}{a}.$$

19. Solve for x, $\frac{a}{b} = \frac{x}{c - x}$.

$$x = \frac{ac}{a + b}.$$

20. Solve for F, D_1, and D_0, $\frac{1}{F}=\frac{1}{D_1}+\frac{1}{D_0}$.

$$F=\frac{D_1D_0}{D_1+D_0},\quad D_1=\frac{FD_0}{D_0-F},\quad D_0=\frac{FD_1}{D_1-F}.$$

21. $C(t_m-t_1)H_c+W_c(t_m-t_1)=W_h(t_2-t_m)$.

Find the value of t_m in the preceding equation, when $C=80$, $t_1=20$, $t_2=99$, $H_c=.09$, $W_c=1000$, and $W_h=800$.

$t_m=54.9+$.

22. $C(t_m-t_w)H_c+W(t_m-t_w)=I(t-t_m)H_I$.

Solve the preceding equation for t, when $C=80$, $t_m=54$, $t_w=18$, $H_c=.09$, $W=100$, $I=440$, and $H_I=.11$.

$t=133.7+$.

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1. Let $b=$ the weight of B in pounds.
Then $4\cdot 96=6\cdot b$.
Whence $b=64$.

2. Let $a=$ the distance in feet of A from the fulcrum.
Then $100\cdot a=6\frac{2}{3}\cdot 120$.
Whence $a=8$.

3. Let $a=$ the distance in feet of A from the fulcrum.
Then $9-a=$ the distance in feet of B from the fulcrum,
and $125\cdot a=100\cdot(9-a)$.
Whence $a=4$,
and $9-a=5$.

4. Let $a=$ the weight in pounds of A.
Then $210-a=$ the weight in pounds of B,
and $3\frac{3}{4}\cdot a=5(210-a)$.
Whence $a=120$,
and $b=90$.

5. Let $x=$ C's distance in feet from the fulcrum.
Then $108\cdot x=90\cdot 4+60\cdot 3$,
or $x=5$.

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1. Let $x=$ the greater part.
Then $300-x=$ the less part,
and $\frac{x}{300-x}=5$.

Whence $x = 250,$
and $300 - x = 50.$

2. Let $x =$ the greater part.
Then $60 - x =$ the less part,
and $$\frac{2x}{3} = \frac{3}{4}(60 - x).$$
Whence $x = 31\frac{13}{17},$
and $60 - x = 28\frac{4}{17}.$

3. Let $x =$ the greater part.
Then $45 - x =$ the less part,
and $\frac{4}{9}x + \frac{2}{3}(45 - x) = 24.$
Whence $x = 27,$
and $45 - x = 18.$

4. Let $x =$ the less part.
Then $\frac{3}{5} - x =$ the greater part,
and $\frac{3}{4}x = \frac{1}{3}(\frac{3}{5} - x).$
Whence $x = \frac{12}{65},$
and $\frac{3}{5} - x = \frac{27}{65}.$

6. Let $g =$ the greater part.
Then $126 - g =$ the less part,
and $$\frac{g}{126 - g} = 6 + \frac{7}{126 - g}.$$
Whence $g = 109,$
and $126 - g = 17.$

7. Let $l =$ the less number.
Then $1906 - l =$ the greater number,
and $$\frac{1906 - l}{l} = 41 + \frac{16}{l}.$$
Whence $l = 45,$
and $1906 - l = 1861.$

8. Let $g =$ the greater part.
Then $\frac{8}{5} - g =$ the less part,
and $g(\frac{8}{5} - g) = g^2 - \frac{2}{5}.$
Whence $g = 1$ or $-\frac{1}{5},$
and $\frac{8}{5} - g = \frac{3}{5}$ or $\frac{9}{5}$ respectively.

9. Let $x =$ one part.
Then $71 - x =$ the other part,

and $40 - \frac{2x}{3} = 71 - x - 16.$

Whence $x = 45,$
and $71 - x = 26.$

10. Let $x =$ the boy's age in years.
Then $x = \frac{2}{5}(x + 12)$
Therefore $x = 8.$

11. Let $m =$ the man's age in years.
Then $\frac{2m}{3} = \frac{6}{5}(m - 30).$
Therefore $m = 67\frac{1}{2}.$

12. Let $m =$ the man's age in years.
Then $\frac{m-8}{6} = \frac{m+12}{8}.$
Therefore $m = 68.$

13. Let $x =$ the number of dollars invested at 6%.
Then $3100 - x =$ the number of dollars invested at 5%,
and $.05(3100 - x) - .06x = 18.60.$
Whence $x = 1240,$
and $3100 - x = 1860.$

14. Let $n =$ the number of dollars invested at 5%.
Then $5360 - n =$ the number of dollars invested at 6%,
and $.05n - .06(5360 - n) = 63.40.$
Whence $n = 3500,$
and $5360 - n = 1860.$

15. Let $n =$ the number of dollars invested at 4%.
Then $3880 - n =$ the number of dollars invested at 6%,
and $.04n + .06(3880 - n) = 171.20.$
Whence $n = 3080,$
and $3880 - n = 800.$

16. Let $n =$ the number of quarters.
Then $80 - n =$ the number of nickels,
and $25n + 5(80 - n) = 1600.$
Whence $n = 60,$
and $80 - n = 20.$

17. Let $n =$ the number of quarters.
Then $28 - n =$ the number of dimes,

and $$25n + 10(28 - n) = 505.$$

Whence $$n = 15,$$

and $$28 - n = 13.$$

18. Let n, $n + 2$, and $n + 4$ represent the required numbers.

Then $$\frac{n^2}{4} = \frac{1}{4}(n + 2)(n + 4) - 11.$$

Whence $$n = 6,$$

$$n + 2 = 8,$$

and $$n + 4 = 10.$$

19. Let n, $n + 1$, and $n + 3$ represent the required numbers.

Then $$\frac{16n^2}{9} = \frac{16}{9}(n + 1)(n + 3) - 2864.$$

Whence $$n = 402,$$

$$n + 1 = 403,$$

and $$n + 3 = 405.$$

20. Let $m =$ the number of meters in width.

Then $4m =$ the number of meters in length,

and $$4m^2 = (4m - 4)(m + \tfrac{8}{2}) - 11.$$

Whence $$m = 8\tfrac{1}{2},$$

and $$4m = 34.$$

21. Let $m =$ the number of meters in width.

Then $\frac{5m}{2} =$ the number of meters in length,

and $$\frac{5m^2}{2} = \left(\frac{5m}{2} + 5\right)\left(m - 4\right) + 50.$$

Whence $$m = 6,$$

and $$\frac{5m}{2} = 15.$$

22. Let $s =$ the number of meters in the side of the square.

Then $$20 \cdot s^2 = 80 \cdot 4s.$$

Whence $$s = 16.$$

23. Let $w =$ the width in yards.

Then $2w =$ the length in yards,

and $$50 \cdot 6w = 15 \cdot 2w^2.$$

Whence $$w = 10,$$

and $$2w = 20.$$

24. Let $w =$ the width in inches.

Then $\dfrac{5w}{2} =$ the length in inches,

and $$(w+4)\left(\frac{5w}{2}+4\right)-\frac{5w^2}{2}=128.$$

Whence $$w = 8,$$

and $$\frac{5w}{2} = 20.$$

25. Let $s =$ the side of the square in yards.

Then $s + 2\frac{7}{9}$ and $s - 2\frac{1}{2}$ represent the length and width of the rectangle respectively.

Hence $$s^2 = (s + 2\tfrac{7}{9})(s - \tfrac{5}{2}).$$

Therefore $$s = 25,$$

$$s + 2\tfrac{7}{9} = 27\tfrac{7}{9},$$

and $$s - 2\tfrac{1}{2} = 22\tfrac{1}{2}.$$

26. Let $n =$ the number of apples bought.

Then $$\frac{n}{5}\cdot\frac{4}{3}+\frac{4n}{5}\cdot\frac{3}{4}=n\cdot\frac{3}{2}-76.$$

Whence $$n = 120.$$

28. Let $n =$ the number of days required for both.

Then $$\frac{2}{5}+\frac{4}{15}=\frac{1}{n}.$$

Whence $$n = \tfrac{3}{2}.$$

29. Let $n =$ the number of days required for all three.

Then $$\frac{1}{2}+\frac{5}{14}+\frac{2}{7}=\frac{1}{n}.$$

Whence $$n = \tfrac{7}{8}.$$

30. Let $x =$ the number of days required by B alone.

Then $$\frac{5}{24}-\frac{1}{8}=\frac{1}{x}.$$

Whence $$x = 12.$$

31. Let $x =$ the number of days required by C alone.

Then $$\frac{6}{7}-\frac{5}{14}-\frac{2}{7}=\frac{1}{x}.$$

Whence $$x = 4\tfrac{2}{3}.$$

32. Let $n =$ the number of days they work together.

Then $\frac{3}{12} + \frac{n}{12} + \frac{n}{15} = 1.$

Whence $n = 5.$

35. Let $r =$ the rate of automobile in miles per hour.

Then $10 \cdot 5\frac{2}{3} = r \cdot 4.$

Whence $r = 14\frac{1}{6}.$

36. Let $r =$ the rate of A in miles per hour.

Then $r + 3 =$ the rate of B in miles per hour,

and $\frac{54}{r} = \frac{72}{r+3}.$

Whence $r = 9,$

and $r + 3 = 12.$

37. Let $t =$ the required time in hours.

Then $4\frac{1}{2} \cdot t + 10\frac{1}{2}(t - 2\frac{1}{2}) = 40.$

Whence $t = 4\frac{5}{12}.$

38. Let $r =$ the rate of return in miles per hour.

Then $\frac{80}{8} + \frac{80}{r} = 18.$

Whence $r = 10.$

39. Let $t =$ A's time of actual traveling in hours.

Then $t - \frac{3}{2} =$ B's time of actual traveling in hours,

and $9t + 12(t - \frac{3}{2}) = 150.$

Whence $t = 8,$

and $t - \frac{3}{2} = 6\frac{1}{2}.$

Therefore A's distance is 72 miles and B's 78 miles.

40. Let $r =$ B's rate in miles per hour.

Then $\frac{7r}{2} =$ A's rate in miles per hour,

and $8r + 8 \cdot \frac{7r}{2} = 126.$

Whence $r = 3\frac{1}{2},$

and $\frac{7r}{2} = 12\frac{1}{4}.$

41. From Hint. $5(4\frac{1}{4} - x) = 3(4\frac{1}{4} + x).$

Whence $x = 1\frac{1}{16},$

or the rate of the current is $1\frac{1}{16}$ miles per hour.

42. Let $d =$ the distance in miles upstream.

Then $$\frac{d}{4-2}+\frac{d}{4+2}=12.$$

Whence $d=18.$

43. Let $t =$ the time, in hours, for the return.

Then $13-t =$ the time, in hours, going,

and $(4\frac{1}{3}+2\frac{1}{4})(13-t)=(4\frac{1}{3}-2\frac{1}{4})t.$

Whence $t=9\frac{7}{8},$

and $13-t=3\frac{1}{8}.$

44. $5x^2-7x-6=0$

becomes $(5x+3)(x-2)=0.$

Whence $x=2, \text{ or } -\frac{3}{5},$

and $x+1=3, \text{ or } \frac{2}{5}.$

The second set of values is rejected.

45. Let $x =$ the number of days required by B alone.

Then $x+2 =$ the number of days required by A alone,

and $$\frac{1}{x}+\frac{1}{x+2}=\frac{5}{12},$$

or $(x-4)(5x+6)=0.$

Whence $x=4, \text{ or } -\frac{6}{5},$

and $x+2=6, \text{ or } \frac{4}{5}.$

The second set of values is rejected.

46. Let $r =$ the rate downstream in miles per hour.

Then $$\frac{10}{2\frac{1}{2}}+\frac{10}{r}=6.$$

Whence $r=5.$

Then $\frac{5-2\frac{1}{2}}{2}=1\frac{1}{4},$ the rate of the current,

and $2\frac{1}{2}+1\frac{1}{4}=3\frac{3}{4},$ the rate in still water.

47. Let $x =$ the number of sheep bought.

Then $x-2 =$ the number of sheep sold,

and $$\frac{96}{x}=\frac{100}{x-2}-2,$$

or $(x-12)(x+8)=0.$

Whence $x=12, \text{ or } -8,$

and $x-2=10, \text{ or } -10.$

The second set of values is rejected.

48. Let $n =$ the number of yards bought.

Then $n - 4 =$ the number of yards sold,

and $$\frac{64}{n} = \frac{72}{n-4} - 2,$$

or $$(x - 16)(x + 8) = 0.$$

Whence $n = 16,$

and $\frac{64}{16} = 4$, the cost in dollars per yard.

49. Let $r =$ the rate going in miles per hour.

Then $r + 5 =$ the rate returning in miles per hour,

and $$\frac{100}{r} - \frac{100}{r+5} = 1,$$

or $$(r + 25)(r - 20) = 0.$$

Whence $r = 20$, or -25,

and $r + 5 = 25$, or -20.

The second set of values is rejected.

50. Let $r =$ the rate going in miles per hour.

Then $r + 4 =$ the rate returning in miles per hour,

and $$\frac{120}{r} - \frac{120}{r+4} = 5,$$

or $$(r + 12)(r - 8) = 0.$$

Whence $r = 8$, or -12,

and $r + 4 = 12$, or -8.

The second set of values is rejected.

51. Let $r =$ the rate of slower automobile in miles per hour.

Then $r + 2 =$ the rate of faster automobile in miles per hour,

and $$\frac{72}{r} - \frac{72}{r+2} = \frac{1}{2},$$

or $$(r + 18)(r - 16) = 0.$$

Whence $r = 16$, or -18,

and $r + 2 = 18$, or -16.

The second set of values is rejected.

52. Let $r =$ A's rate in miles per hour.

Then $r + 4 =$ B's rate in miles per hour,

and $$\frac{72}{r} = \frac{72}{r+4} + 3,$$

or $$(r + 12)(r - 8) = 0.$$

Whence $r = 8$, or -12.

and $r + 4 = 12$, or -8.

The second set of values is rejected.

53. Let $r =$ the rate of B in miles per hour.

Then $r + 1 =$ the rate of A in miles per hour,

and
$$2 + \frac{\frac{172 - 28}{2}}{r} = \frac{100}{r + 1},$$

or
$$2 + \frac{72}{r} = \frac{100}{r + 1},$$

or
$$(r - 4)(r - 9) = 0.$$

Whence $r = 9$ or 4,

and $r + 1 = 10$ or 5 respectively.

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1. $5 : 10 = \frac{5}{10} = \frac{1}{2}$.

2. $10 : 5 = \frac{10}{5} = 2$.

3. $16\,a^2 : 8\,a = \dfrac{16\,a^2}{8\,a} = 2\,a$.

4. $3\frac{1}{16} : 3\frac{1}{2} = \dfrac{\frac{49}{16}}{\frac{7}{2}} = \frac{7}{8}$.

5. $8\frac{2}{3} : 5\frac{7}{9} = \dfrac{\frac{26}{3}}{\frac{52}{9}} = \frac{3}{2}$.

6. 3 days : 9 hours $= \frac{72}{9} = 8$.

7. 150 pounds : 1 ton $= \frac{150}{2000} = \frac{3}{40}$.

8. $(x^2 - y^2) : (x + y) = \dfrac{(x + y)(x - y)}{x + y} = x - y$.

9. $(a^3 + b^3) : (a + b) = \dfrac{a^3 + b^3}{a + b} = a^2 - ab + b^2$.

10. $\left(1 - \dfrac{1}{a^2}\right) : \left(1 + \dfrac{1}{a}\right) = \dfrac{(a^2 - 1)}{a^2} : \left(\dfrac{a + 1}{a}\right) = \dfrac{\frac{(a + 1)(a - 1)}{a^2}}{\frac{a + 1}{a}} = \dfrac{a - 1}{a}$.

11. $\left(2 + \dfrac{1}{x^2}\right) : \left(4 - \dfrac{1}{x^4}\right) = \dfrac{2\,x^2 + 1}{x^2} : \dfrac{4\,x^4 - 1}{x^4} = \dfrac{\frac{2\,x^2 + 1}{x^2}}{\frac{(2\,x^2 + 1)(2\,x^2 - 1)}{x^4}}$

$$= \frac{x^2}{2\,x^2 - 1}.$$

12. $\dfrac{1}{x - 3} : \dfrac{1}{x^2 - 5\,x + 6} = \dfrac{\frac{1}{x - 3}}{\frac{1}{(x - 2)(x - 3)}} = x - 2$.

13. $(x^2 - xy + y^2) : (x^3 + y^3) = \dfrac{x^2 - xy + y^2}{(x + y)(x^2 - xy + y^2)} = \dfrac{1}{x + y}$.

14. $\left(x^2-\frac{y^3}{x}\right):\left(1-\frac{y}{x}\right)=\frac{(x^3-y^3)}{x}:\left(\frac{x-y}{x}\right)=\frac{(x-y)(x^2+xy+y^2)}{x-y}$

$=x^2+xy+y^2.$

15. $\left(\frac{16}{x}-x\right):\left(\frac{24}{x^4}+\frac{10}{x^3}+\frac{1}{x^2}\right)=\left(\frac{16-x^2}{x}\right):\left(\frac{24+10x+x^2}{x^4}\right)$

$$=\frac{\frac{(4+x)(4-x)}{x}}{\frac{(4+x)(6+x)}{x^4}}=\frac{x^3(4-x)}{6+x}.$$

16. Let $2x=$ one part.
Then $3x=$ the other part.
and $6x=$ the third part.
$\therefore 4x+5x+6x=135,$
or $x=9,$
$4x=36,$
$5x=45,$
and $6x=54.$

$\therefore 2x+3x=40,$
or $x=8,$
$2x=16,$
and $3x=24.$

17. Let $5x=$ one part.
Then $3x=$ the other part.
$\therefore 3x+5x=16,$
or $x=2,$
$5x=10,$
and $3x=6.$

18. Let $3x=$ one part.
Then $11x=$ the other part.
$\therefore 3x+11x=84,$
or $x=6,$
$3x=18,$
and $11x=66.$

19. Let $2x=$ the first part.
Then $3x=$ the second part,
and $4x=$ the third part.
$\therefore 2x+3x+4x=36,$
or $x=4,$
$2x=8,$
$3x=12,$
and $4x=16.$

20. Let $4x=$ the first part.
Then $5x=$ the second part,

21. Let $x=$ the number.
Then $\frac{5+x}{8+x}=\frac{21}{28}.$
Transposing,
$7x=28,$
or $x=4.$

22. Let $x=$ the number.
Then $\frac{13-x}{27-x}=\frac{2}{9}.$
Transposing,
$-7x=-63,$
or $x=9.$

23. $\frac{4+2a}{4+3a}=\frac{16+24a+8a^2}{(4+3a)(4+4a)},$
$\frac{4+3a}{4+4a}=\frac{16+24a+9a^2}{(4+4a)(4+3a)}.$
The second is the greater.

24. $\frac{a+4b}{a+5b}=\frac{a^2+11ab+28b^2}{(a+5b)(a+7b)},$
$\frac{a+6b}{a+7b}=\frac{a^2+11ab+30b^2}{(a+7b)(a+5b)}.$
The second is the greater.

25. It is increased.

Page 179 (First set)

1. $\frac{3}{4} = \frac{6}{x}$. $\therefore x = 8$.
2. $\frac{4}{7} = \frac{16}{x}$. $\therefore x = 28$.
3. $\frac{3}{2} = \frac{x}{5}$. $\therefore x = 7\frac{1}{2}$.
4. $3 : x = 7 : 9$. $\therefore x = 3\frac{6}{7}$.
5. $x : 4 = 3 : 6$. $\therefore x = 2$.
6. $\frac{4}{3} = 2 : \frac{1}{x}$. $\therefore x = \frac{2}{3}$.
7. $\frac{2}{3} = \frac{\frac{1}{x}}{4}$. $\therefore x = \frac{3}{8}$.
8. $\frac{2}{\frac{1}{x}} = \frac{3\frac{1}{3}}{4}$. $\therefore x = \frac{5}{12}$.
9. $\frac{1}{x} : 2 = 3 : 4\frac{1}{2}$. $\therefore x = \frac{3}{4}$.
10. $\frac{a}{b} = \frac{c}{x}$. $\therefore x = \frac{bc}{a}$.
11. $\frac{a}{b} = \frac{c}{\frac{1}{x}}$. $\therefore x = \frac{a}{bc}$.
12. $\frac{a}{x^2} = \frac{1}{x} : a^3$. $\therefore x = a^4$.
13. $4 : 3\frac{1}{3} = 3 : x - 3$. $\therefore x = 5\frac{1}{2}$.
14. $5 : x - 3 = 7 : 2x + 6$. $\therefore x = -17$.

Page 179 (Second set)

1. $m^2 = 1 \cdot 4$, or $m = \pm 2$.
2. $m^2 = 4 \cdot 9$, or $m = \pm 6$.
3. $m^2 = 16 \cdot 4$, or $m = \pm 8$.
4. $m^2 = 3 \cdot 12$, or $m = \pm 6$.
5. $m^2 = (a - b)^2 \cdot 4$, or $m = \pm 2(a - b)$.
6. $m^2 = \frac{1}{2} \cdot \frac{1}{8}$, or $m = \pm \frac{1}{4}$.
7. $m^2 = \frac{1}{3} \cdot \frac{1}{27}$, or $m = \pm \frac{1}{9}$.
8. $m^2 = \frac{4}{a^3} \cdot \frac{9}{ax^2}$, or $m = \pm \frac{6}{a^2x}$.
9. $1 : 4 = 4 : t$, or $t = 16$.
   $4 : 9 = 9 : t$, or $t = 20\frac{1}{4}$.
   $16 : 4 = 4 : t$, or $t = 1$.
   $3 : 12 = 12 : t$, or $t = 48$.
   $(a - b)^2 : 4 = 4 : t$,
   or $t = \frac{16}{(a - b)^2}$.
   $\frac{1}{2} : \frac{1}{8} = \frac{1}{8} : t$, or $t = \frac{1}{32}$.
   $\frac{1}{3} : \frac{1}{27} = \frac{1}{27} : t$, or $t = \frac{1}{243}$.
10. $1 : 2 = 3 : f$, or $f = 6$.
11. $4 : 5 = 6 : f$, or $f = \frac{15}{2}$.
12. $7 : 14 = 5 : f$, or $f = 10$.
13. $5 : 12 = a : f$, or $f = \frac{12a}{5}$.
14. $7 : 21x = 6x : f$, or $f = 18x^2$.
15. $a : a^2 = a^3 : f$, or $f = a^4$.
16. $a^3 : a^5 = a^4 : f$, or $f = a^6$.
17. $(a + b) : (a - b) = (a^2 - b^2) : f$,
    or $f = (a - b)^2$.
18. $\frac{a - b}{2} : \frac{1}{a + b} = (a^2 - b^2) : f$,
    or $f = 2$.

Page 180

1. $3 : 6 = 2 : 4$; $3 : 2 = 6 : 4$; $6 : 3 = 4 : 2$.
2. $5 : 10 = 3 : 6$; $10 : 5 = 6 : 3$; $10 : 6 = 5 : 3$.
3. $3 : 2 = x : 6$; $2 : 3 = 6 : x$; $3 : x = 2 : 6$.
4. $a : b = c : d$; $a : c = b : d$; $b : a = d : c$.
5. $a - b : 2 = 3 : a + b$; $2 : a - b = a + b : 3$; $a - b : 3 = 2 : a + b$.

6. $a+b:a+2=a+3:a-b$; $a+2:a+b=a-b:a+3$; $a+b:a+3=a+2:a-b$.

7. $a-b:2=3:a+b$.

8. $a-b:3=6:a-b$.

9. $a-2:4=2:a-3$.

10. $a-4:a+b=a+b:a-3$.

11. $a-3:a-2=a-8:a-3$.

12. $x-2y:a+5b$
$=a+5b:x-2y$.

13. $x:2=2:y$.

14. $x:3=1:y$.

15. $a:1=1:b$.

16. $ab:d=e:c$.

17. $mn:xy=z:p$.

18. $a:a+b=1:b$.

19. $5:3=4:x$.

20. $4:9=7:x$.

21. $c:b=a:x$.

22. $p:q=r:x$.

23. $ac:b=d:x$.

24. $a:1=1:x$.

25. $c:a=b:x$.

26. $y:y+1=1:x$.

27. $\frac{2}{4}=\frac{3}{6}$.

28. $\frac{4}{x}=\frac{3}{y}$.

29. $\frac{4}{6}=\frac{5}{x}$.

30. $\frac{3}{a}=\frac{1}{b}$.

31. $\frac{8}{4}=\frac{6}{3}$.

32. $\frac{2}{3}=\frac{b}{a}$.

33. $\frac{P_1}{P}=\frac{W}{W_1}$.

34. $\frac{V_2}{V_1}=\frac{P_1}{P_2}$.

35. $\frac{\frac{1}{x}}{3}=\frac{a}{2}$.

36. $\frac{\frac{1}{y}}{\frac{1}{x}}=\frac{z}{1}$.

37. $\frac{N_2}{N_1}=\frac{\sqrt{T_1}}{\sqrt{T_2}}$.

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1. $\frac{5}{3}=\frac{10}{6}$.

2. $\frac{16}{12}=\frac{32}{24}$.

3. $\frac{a+x}{x}=\frac{3}{2}$.

4. $\frac{7}{3}=\frac{f+x}{x}$.

5. $\frac{A_1+A_2}{A_2}=\frac{S_1^2+S_2^2}{S_2^2}$.

6. $\frac{-1}{3}=\frac{-2}{6}$; $\frac{-8}{12}=\frac{-16}{24}$; $\frac{a-x}{x}=\frac{-1}{2}$; $\frac{1}{3}=\frac{f-x}{x}$.

7. $\frac{5}{-1}=\frac{10}{-2}$; $\frac{16}{-8}=\frac{32}{-16}$; $\frac{a+x}{a-x}=\frac{3}{-1}$; $\frac{7}{1}=\frac{f+x}{f-x}$.

8. If $\frac{a}{b}=\frac{c}{d}$,

then $\frac{b}{a}=\frac{d}{c}$,

and $\frac{b}{a}+1=\frac{d}{c}+1$.

Whence $\frac{b+a}{a}=\frac{d+c}{c}$.

9. If $\frac{a}{b}=\frac{c}{d}$,

then $\frac{b}{a}=\frac{d}{c}$,

and $1-\frac{b}{a}=1-\frac{d}{c}$.

Whence $\frac{a-b}{a}=\frac{c-d}{c}$.

10. If $\frac{a}{b}=\frac{c}{d}$, then $\frac{2a}{b}=\frac{2c}{d}$. Whence $\frac{2a+b}{b}=\frac{2c+d}{d}$.

11. If $$\frac{a}{b}=\frac{c}{d},$$

then $$\frac{a}{3b}=\frac{c}{3d}.$$

Whence $$\frac{a+3b}{3b}=\frac{c+3d}{3d},$$

or $$\frac{a+3b}{b}=\frac{c+3d}{d}.$$

12. $$\frac{3x+4}{3x-4}=\frac{3+2}{3-2}.$$

Then $$\frac{6x}{8}=\frac{6}{4},$$

or $$x=2.$$

13. $$\frac{6x+3}{6x-3}=\frac{2a+b}{2a-b}.$$

Then $$\frac{12x}{6}=\frac{4a}{2b},$$

or $$x=\frac{a}{b}.$$

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1. From $\frac{1}{2}=\frac{3}{6}=\frac{5}{10}$, $\frac{9}{18}=\frac{1}{2}$, etc.
2. From $\frac{1}{a}=\frac{2}{2a}=\frac{3b}{3ab}$, $\frac{3+3b}{3a+3ab}=\frac{1}{a}$, etc.
3. From $\frac{3}{4}=\frac{6}{8}=\frac{12}{16}$, $\frac{21}{28}=\frac{3}{4}$, etc.
4. From $\frac{1}{x-y}=\frac{a}{ax-ay}=\frac{b}{bx-by}$, $\frac{1+a+b}{(x-y)(1+a+b)}=\frac{1}{x-y}$, etc.
5. $\frac{3}{9}=\frac{4}{12}=\frac{5}{15}=\frac{1}{3}$. Also $\frac{3}{9}=\frac{4}{12}=\frac{5}{15}=\frac{12}{36}=\frac{1}{3}$. The perimeters are in the same ratio as the corresponding sides.
6. $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$. Multiply both terms of $\frac{a}{b}$ by 2, of $\frac{c}{d}$ by 3, and of $\frac{e}{f}$ by 4. Then use (10) on p. 183 in text, which gives
$$\frac{2a+3c+4e}{2b+3d+4f}=\frac{a}{b}=\frac{c}{d}=\frac{e}{f}.$$

Page 184

1. If $\frac{a}{b}=\frac{4}{x}$ and $\frac{a}{b}=\frac{4}{\frac{1}{x}}$, then $\frac{4}{x}=\frac{4}{\frac{1}{x}}$, or $x=\pm 1$.
2. If $\frac{a}{b}=\frac{9}{y}$ and $\frac{a}{b}=\frac{1}{\frac{1}{y}}$, then $\frac{9}{y}=\frac{1}{\frac{1}{y}}$, or $y=\pm 3$.
3. If $\frac{p}{r}=\frac{7}{6}$ and $\frac{q}{r}=\frac{3}{5}$, then, by dividing, $\frac{p}{q}=\frac{35}{18}$.
4. If $\frac{p}{q}=\frac{4}{a}$ and $\frac{q}{r}=\frac{a}{7}$, then, by multiplying, $\frac{p}{r}=\frac{4}{7}$.

5. Let the parts of 12 be x and $12-x$ respectively.

Then $\dfrac{8}{10}=\dfrac{x}{12-x}$,

or $x=5\frac{1}{3}$, and $12-x=6\frac{2}{3}$.

6. Let $p=$ one part.

Then $21-p=$ the other part,

and $\dfrac{p}{21-p}=\dfrac{18}{24}$,

or $p=9$, and $21-p=12$.

7. Let $h=$ the height of the pole in feet.

Then $\frac{70}{12}:h=\frac{35}{12}:36$,

or $h=72$.

8. Let x and x_1 be the required sides respectively.

Then $\dfrac{10}{12}=\dfrac{15}{x}$. Also $\dfrac{10}{12}=\dfrac{20}{x_1}$.

Clearing, $10x=180$. Clearing, $10x_1=240$.

$x=18$. $x_1=24$.

$\frac{10}{12}=\frac{15}{18}=\frac{20}{24}=\frac{45}{54}=\frac{5}{6}$.

Therefore they have the same ratio.

9. Let x, x_1, and x_2 be the required sides respectively.

Then $\dfrac{9}{x}=\dfrac{9+10+17}{108}=\dfrac{36}{108}=\dfrac{1}{3}$. $\dfrac{10}{x_1}=\dfrac{1}{3}$. $\dfrac{17}{x_2}=\dfrac{1}{3}$.

Clearing, $108=4x$. $30=x_1$. $51=x_2$.

$x=27$.

10. (*a*) $\dfrac{AD}{DB}=\dfrac{AE}{EC}$.

Substituting, $\dfrac{6}{4}=\dfrac{10}{EC}$.

Clearing, $6\,EC=40$.

$EC=6\frac{2}{3}$.

(*b*) $\dfrac{AD}{AB}=\dfrac{DE}{BC}$.

Substituting, $\dfrac{8}{8+4}=\dfrac{6}{BC}$.

Clearing, $8\,BC=72$. $BC=9$.

(*c*) $\dfrac{AD}{AB}=\dfrac{AE}{AC}$.

Substituting, $\dfrac{8}{8+4}=\dfrac{AE}{10}$.

Clearing, $80=12\,AE$.

$AE=6\frac{2}{3}$.

(*d*) $\dfrac{AD}{AB}=\dfrac{AE}{AC}$.

Substituting, $\dfrac{10}{16}=\dfrac{AE}{16}$.

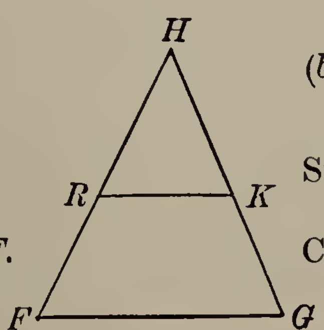
Clearing, $AE=10$.

11. (*a*) $\dfrac{FG}{RK}=\dfrac{HF}{RH}$.

Subst., $\dfrac{15}{10}=\dfrac{HF}{8}$.

Clearing, $120=10\,HF$.

$HF=12$.



(*b*) $\dfrac{FG-RK}{FG}=\dfrac{HF-HR}{HF}$.

Subst., $\dfrac{4}{20}=\dfrac{RF}{10}$.

Clearing, $40=20\,RF$.

$RF=2$.

(c) $\frac{FG}{RK} = \frac{HR + RF}{HR}.$

Substituting, $\frac{20}{15} = \frac{HR + 6}{HR}.$

Clearing, $20\,HR = 15\,HR + 90.$

Transposing, $5\,HR = 90.$

$HR = 18.$

(d) $\frac{FG}{RK} = \frac{HK + KG}{HK}.$

Substituting, $\frac{18}{15} = \frac{KH + 4}{HK}.$

Clearing, $18\,HK = 15\,HK + 60.$

Transposing, $3\,HK = 60.$

$HK = 20.$

12. Let $x =$ the third side of the triangle.

Then $\frac{12}{24} = \frac{12}{x}.$

Clearing, $x = 24.$

Therefore the third side of the triangle is 24 centimeters.

13. Let x, x_1, and x_2 be the required lengths in centimeters.

Then $\frac{5}{10} = \frac{x}{16}.$ $\quad \frac{6}{12} = \frac{x_1}{10}.$ $\quad \frac{8}{16} = \frac{x^2}{12}.$

$x = 8.$ $\quad x_1 = 5.$ $\quad x_2 = 6.$

14. (a) $\frac{AD}{BD} = \frac{BD}{DC}.$

Substituting, $\frac{9}{6} = \frac{6}{DC}.$

Clearing, $9\,DC = 36.$

$DC = 4.$

(b) $\frac{AD}{BD} = \frac{BD}{DC}.$

Substituting, $\frac{4}{BD} = \frac{BD}{20 - 4}.$

Clearing, $64 = \overline{BD}^2.$

$BD = 8.$

15. (a) $\frac{AD}{CD} = \frac{CD}{BD}.$

Substituting, $\frac{2}{CD} = \frac{CD}{18}.$

Clearing, $36 = \overline{CD}^2.$

$CD = 6.$

(b) $\frac{AD}{CD} = \frac{CD}{BD}.$

Substituting, $\frac{AD}{8} = \frac{8}{34 - AD}.$

Clearing, $34\,AD - \overline{AD}^2 = 64.$

$\overline{AD}^2 - 34\,AD + 64 = 0.$

$(AD - 32)(AD - 2) = 0.$

$AD = 32,\ 2.$

16. $\frac{DC}{BE} = \frac{AB + BC}{AB}.$

Substituting, $\frac{480}{420} = \frac{8}{7} = \frac{AB + 160}{AB}.$

Clearing, $8\,AB = 7\,AB + 1120.$

$AB = 1120.$

Therefore the computed value of AB was 1120 feet.

17. Let x, x_1, and x_2 be the sides of the second triangle respectively.

Then $\frac{45}{135} = \frac{11}{x}$. $\frac{45}{135} = \frac{19}{x_1}$. $\frac{45}{135} = \frac{45 - 11 - 19}{x_2}$.

$x = 33$. $x_1 = 57$. $x_2 = 45$.

18. Let $2x =$ the rate of the first man in miles per hour.

Then $3x =$ the rate of the second man in miles per hour.

Therefore $10x + 15x = 100,$

or $x = 4,$

$2x = 8,$

and $3x = 12.$

19. Let $7x =$ the amount A contributed.

Then $8x =$ the amount B contributed.

Also $\frac{7x + 8x}{3}$, or $5x =$ the amount each ate.

Therefore $7x - 5x$, or $2x =$ the amount A should be paid for.

$8x - 5x$, or $3x =$ the amount B should be paid for.

Therefore $2x + 3x = 30,$

or $x = 6,$

$2x = 12$ dollars,

and $3x = 18$ dollars.

20. From February 21 to March 28 is 35 days.

Now $\frac{\text{true number of minutes in a day}}{\text{false number of minutes in a day}} = \frac{\text{true number of days}}{\text{false number of days}}$.

Therefore $\frac{1440}{1450} = \frac{x}{35}$.

Whence $x = 34.758$ days.

34.758 days = 34 days, 18 hours, 12+ minutes.

Therefore the correct time was 10.12 A.M. March 28.

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1. It begins at 80° and ends at 83°.
2. The highest temperature recorded is 88°; the lowest, 54°.
3. The highest temperature was recorded at 7 P.M. Wednesday. The lowest temperature was recorded at 9.30 A.M. Thursday.
4. The instrument recorded 80° five times, 72° three times, 78° four times, and 62° twice.
5. The 80° records were made at 2 P.M. Wednesday, 10 P.M. Wednesday, 1 P.M. Thursday, 10 P.M. Thursday, 10.30 A.M. Friday.

The 72° records were made at 1 A.M. Thursday, 9 A.M. Thursday, 11 A.M. Thursday.

The 78° records were made at 10.30 P.M. Wednesday, 12.30 P.M. Thursday, 12 P.M. Thursday, 1 A.M. Friday, 9.30 A.M. Friday.

The 62° records were made at 9 A.M. Thursday, 9.30 A.M. Thursday.

6. A graph such as the one here explained might be used as a check against the waste of coal in overheating a building when not in actual use. It might be used to test the care exercised in keeping a room or a car at a uniform temperature.

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1. The line through 8 seconds cuts the curve at a distance of 1024 feet, the line through 3 seconds cuts it at 144 feet, the one through 7 seconds cuts it at 784 feet, the one through $2\frac{1}{2}$ seconds cuts it at 100 feet, and the one through 6.2 seconds cuts it at 615 feet.

2. The line through 400 feet cuts the curve at the line for 5 seconds.
The line through 196 feet cuts the curve at the line for $3\frac{1}{2}$ seconds.
The line through 100 feet cuts the curve at the line for $2\frac{1}{2}$ seconds.
The line through 25 feet cuts the curve at the line for $1\frac{1}{4}$ seconds.
The line through 120 feet cuts the curve at the line for $2\frac{3}{4}$ seconds.
The line through 750 feet cuts the curve at the line for $6\frac{4}{5}$ seconds.

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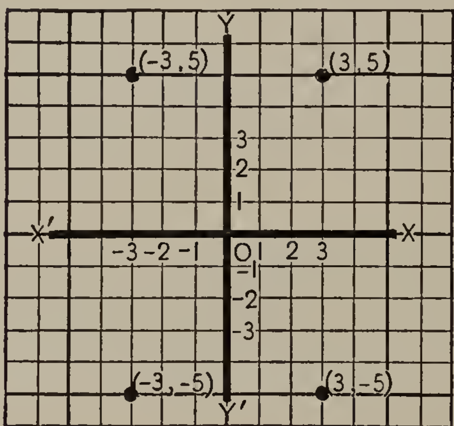
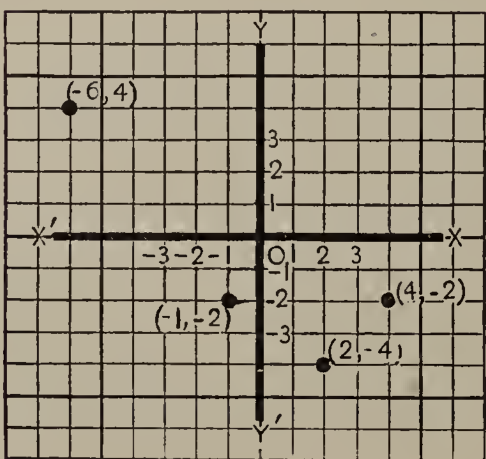
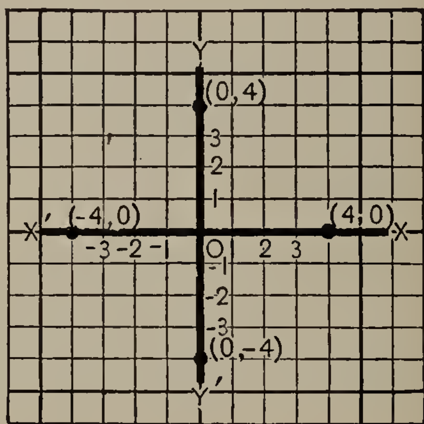
1.



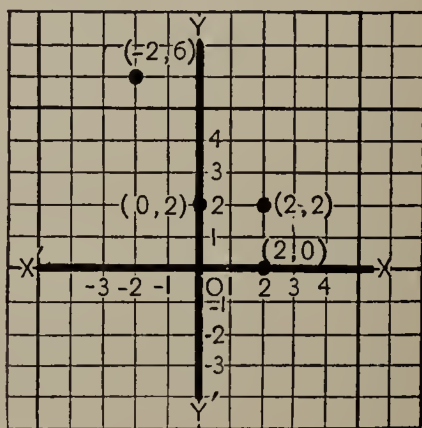
2.



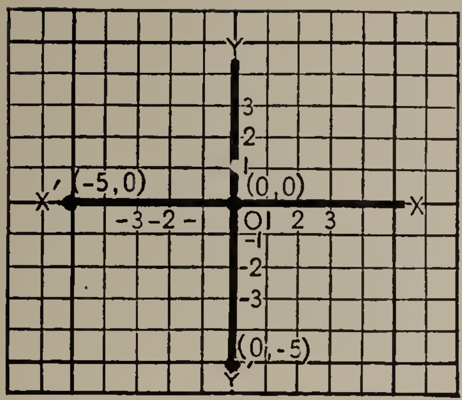
3.



4.



5.



6. If one coördinate of a point is zero, the point is located on one of the axes.

If both are zero, it is located at the origin.

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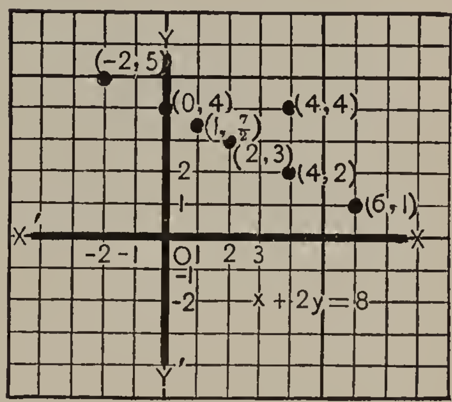
1 $\qquad x + 2y = 8.$

x	-2	0	1	2	4	6
y	5	4	$\frac{7}{2}$	3	2	1

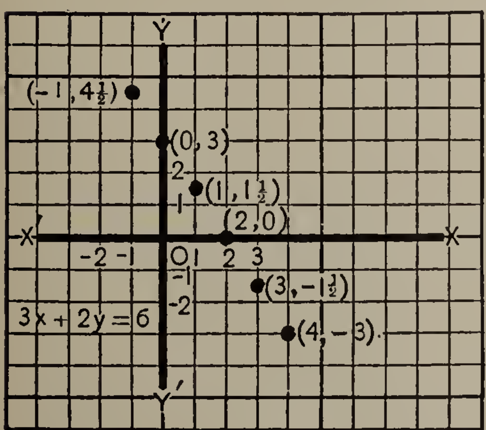


The points are in a straight line which cuts the x-axis at (8, 0) and the y-axis at (0, 4).

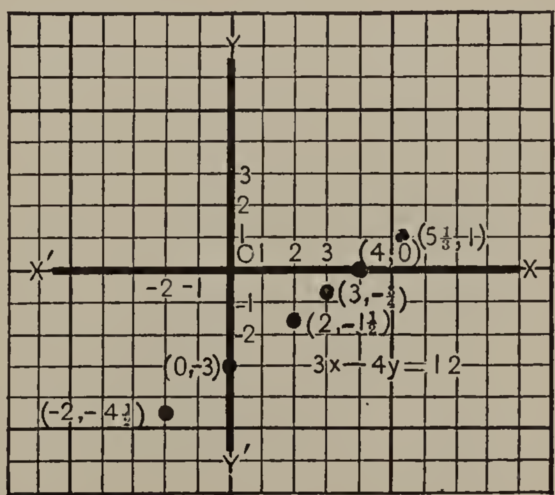
The graph of $x + 2y = 8$ is a straight line which passes through these points.

No, (4, 4) does not satisfy the equation. The point (4, 4) is not on the graph of the equation. If the x- and y-distances of a point satisfy the equation $x + 2y = 8$, the point is located on the graph of the equation. Moreover only such points are there located.

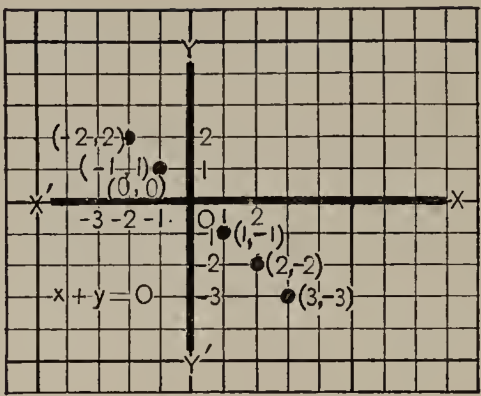
2.



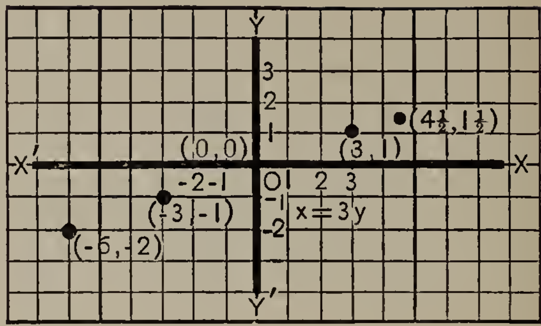
3.



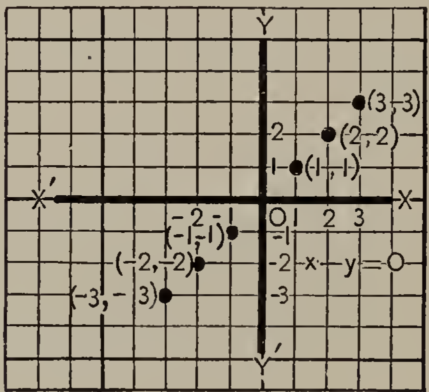
4.



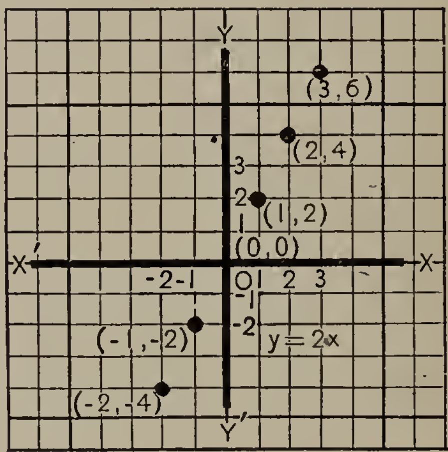
6.



5.



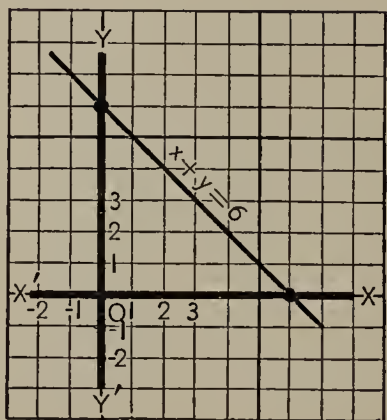
7.



Page 197

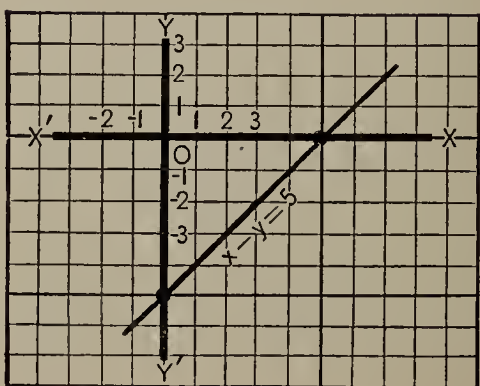
1. $x + y = 6.$

x	0	6
y	6	0



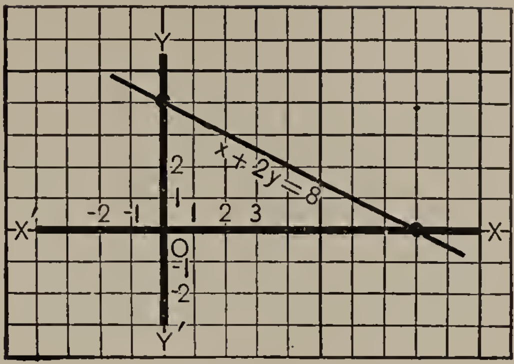
2. $x - y = 5.$

x	0	5
y	-5	0



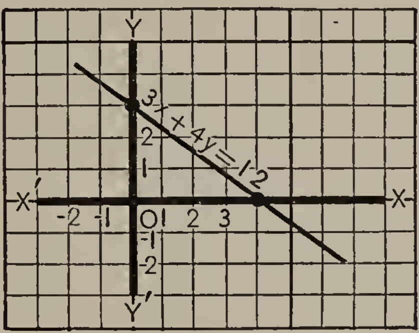
3. $x + 2y = 8.$

x	0	8
y	4	0



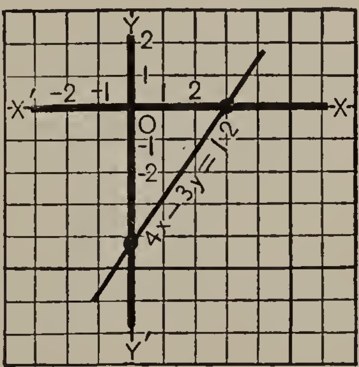
4. $3x + 4y = 12.$

x	0	4
y	3	0



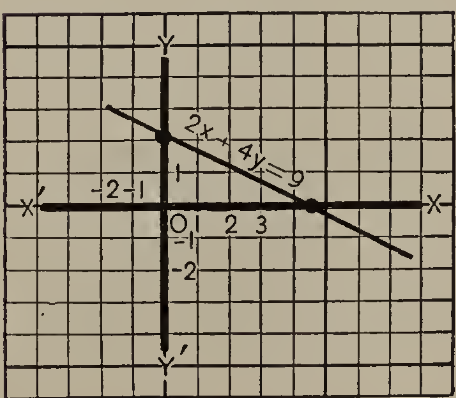
5. $4x - 3y = 12.$

x	0	3
y	-4	0



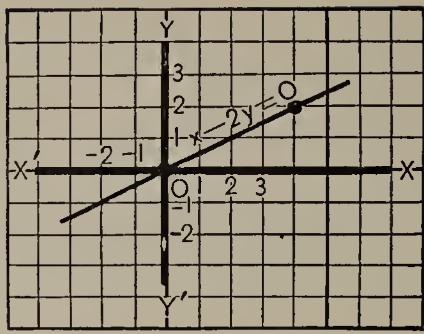
6. $2x + 4y = 9.$

x	0	$4\frac{1}{2}$
y	$2\frac{1}{4}$	0



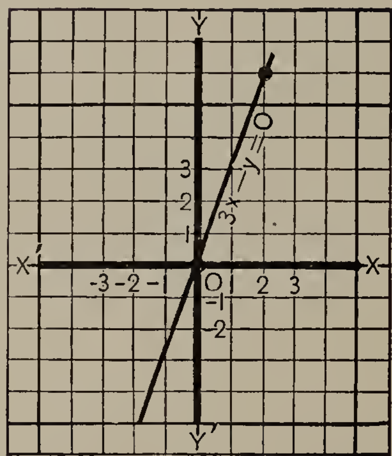
7. $x - 2y = 0.$

x	0	4
y	0	2

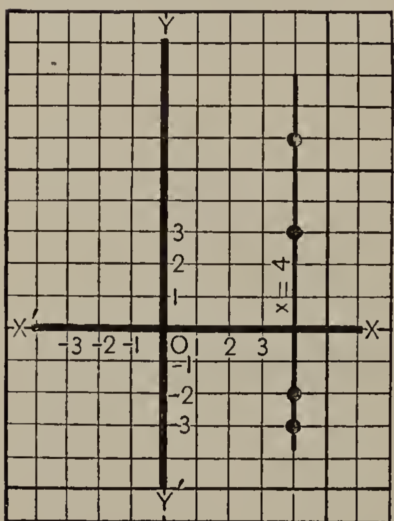


8. $3x - y = 0.$

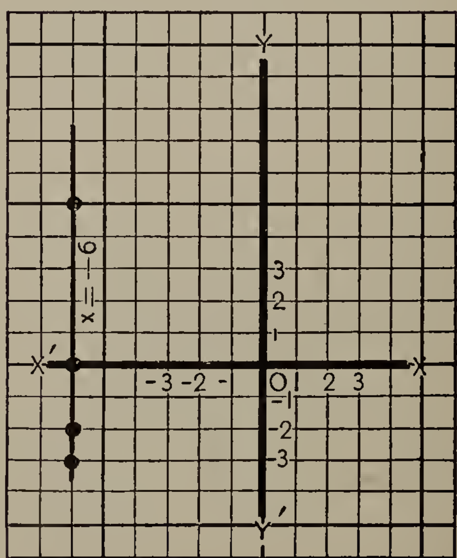
x	0	6
y	0	2



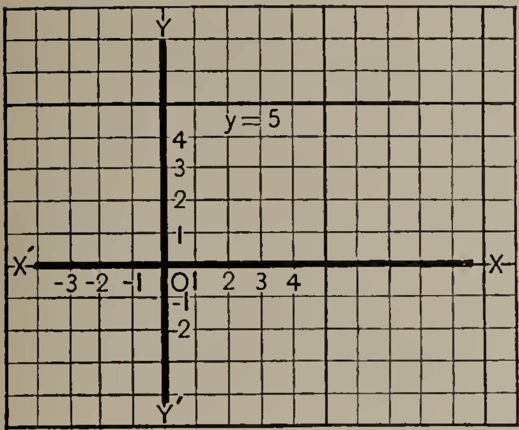
9. $x = 4$ for all values of y.



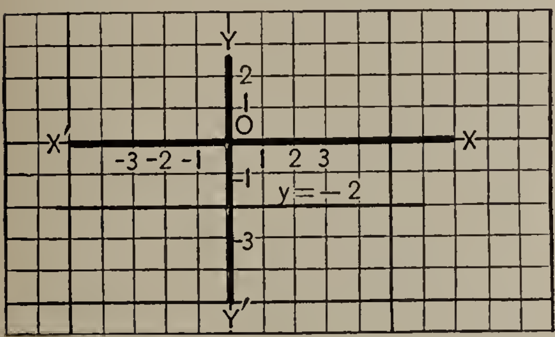
10. $x = -6$ for all values of y.



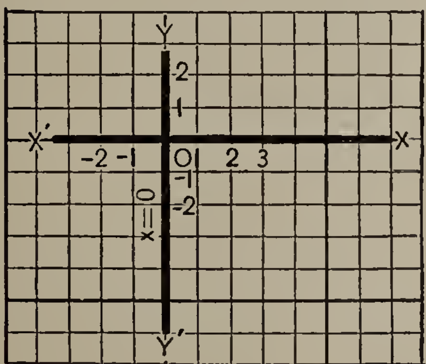
11. $y = 5$ for all values of x.



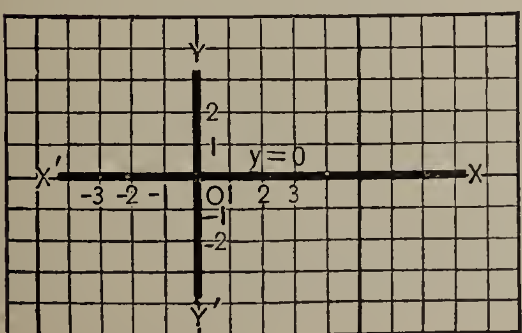
12. $y = -2$ for all values of x.



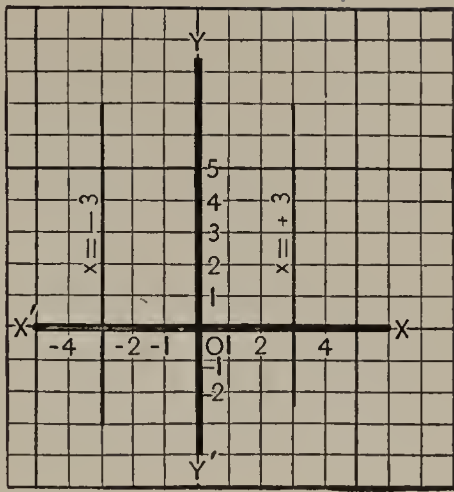
13. $x = 0$ for all values of y.



14. $y = 0$ for all values of x.



15.

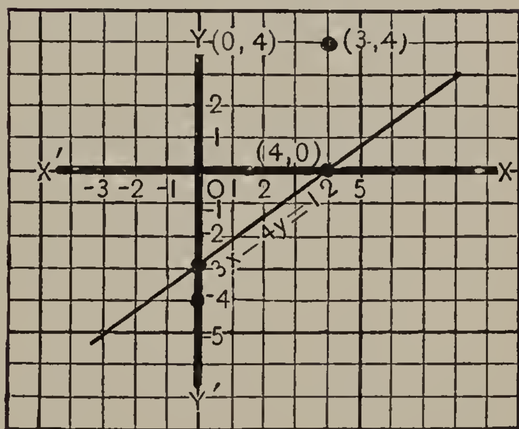


16. Yes.

17. Yes.

18. $3x - 4y = 12.$

x	0	4
y	-3	0



Point (3, 4) is not on the graph of $3x - 4y = 12$.

Point (0, 4) is not on the graph of $3x - 4y = 12$.

Point (4, 0) is on the graph of $3x - 4y = 12$.

19. Point (2, 6) will be on a graph if it satisfies the equation of that graph. It satisfies the equation of Exercise 8, and therefore lies on the graph of $3x - y = 0$.

20. The graphs of equations in Exercises 7, 8, 13, and 14 pass through the origin, for their constant terms are zero.

21. An equation whose graph passes through the origin has zero for its constant term. The constant term of an equation is never zero if the graph does not pass through the origin.

22. Yes. If the constant term of the equation is zero, and x is set equal to zero, then y equals zero also.

23. The graph of a linear equation in x and y will be parallel to the x-axis if the coefficient of y is zero.

Likewise, the graph will be parallel to the y-axis if the coefficient of x is zero.

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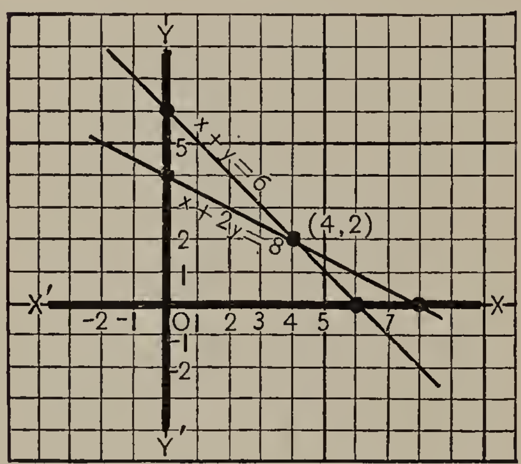
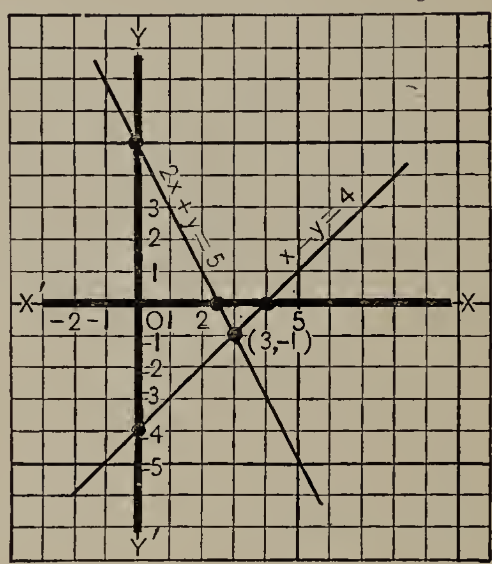
1.



2.



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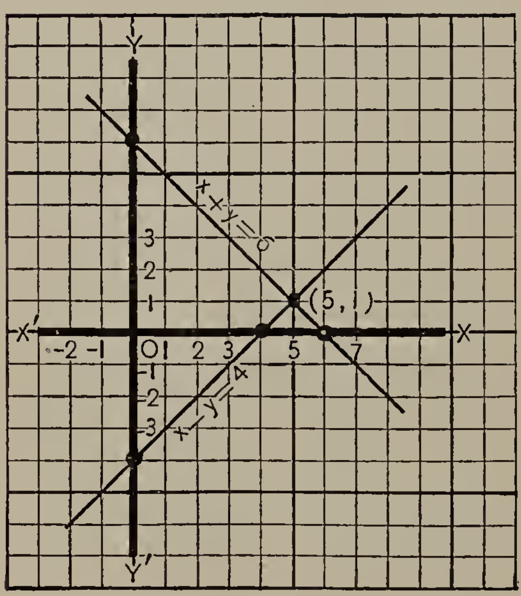

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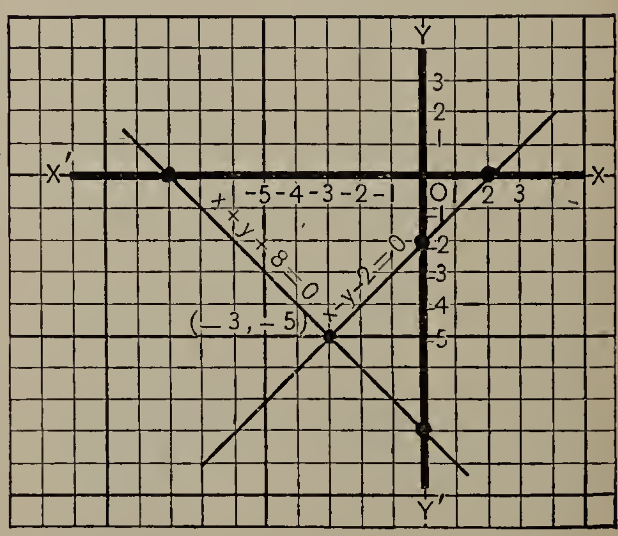

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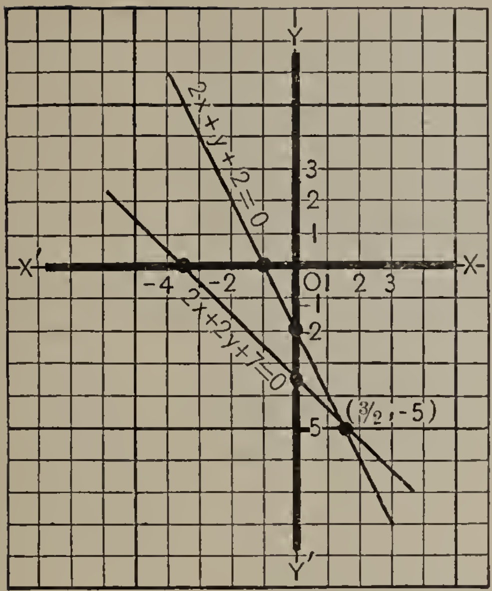
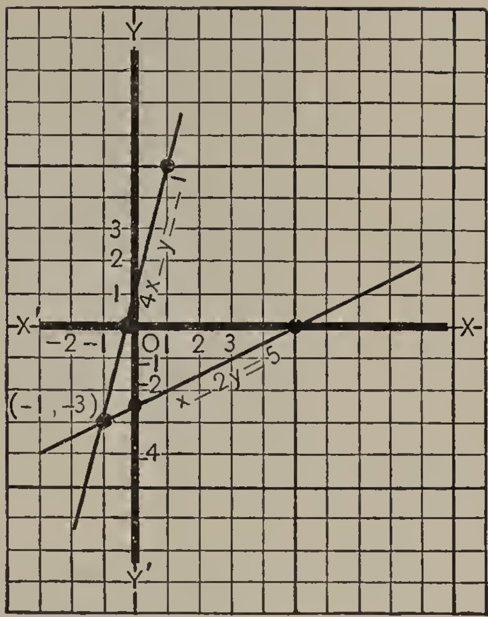
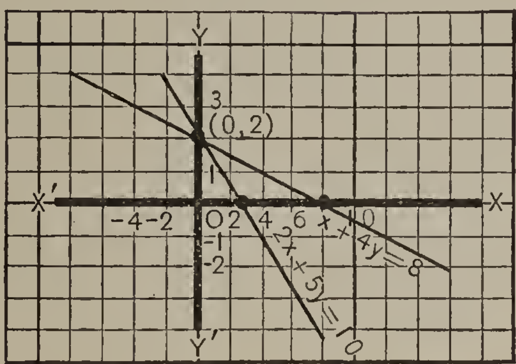

2x+y+2=0
2x+2y+7=0
(3/2, -5)

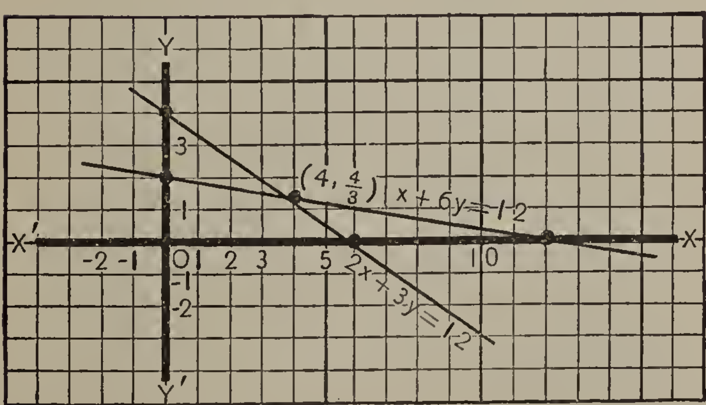
6.


4x−y=−1
x−2y=5
(−1, −3)

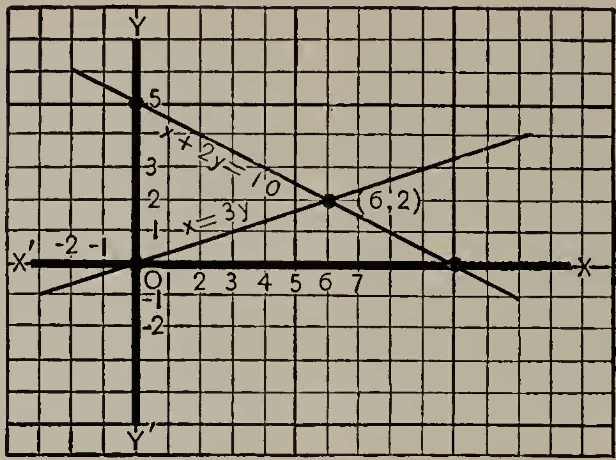
7.


(0,2)
x+4y=8
2x+5y=10

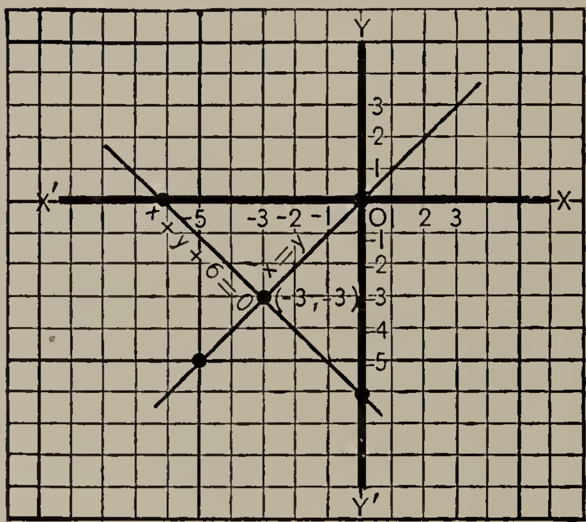
8.


(4, 4/3)
x+6y=12
2x+3y=12

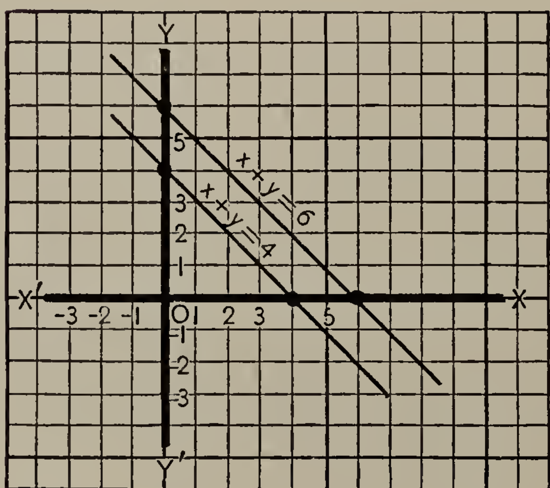
9.



10.

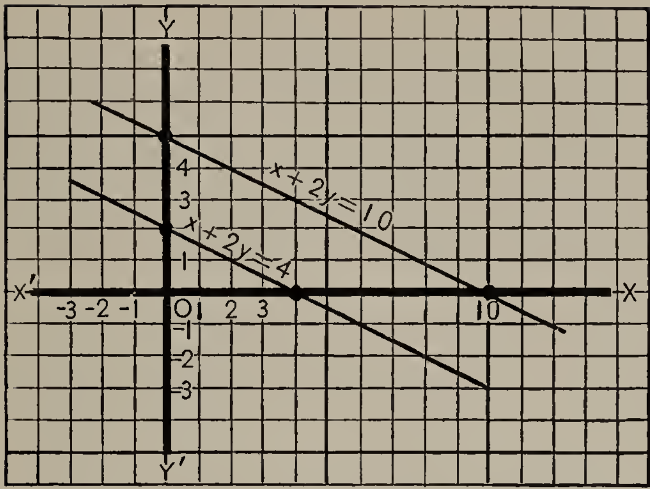


11.



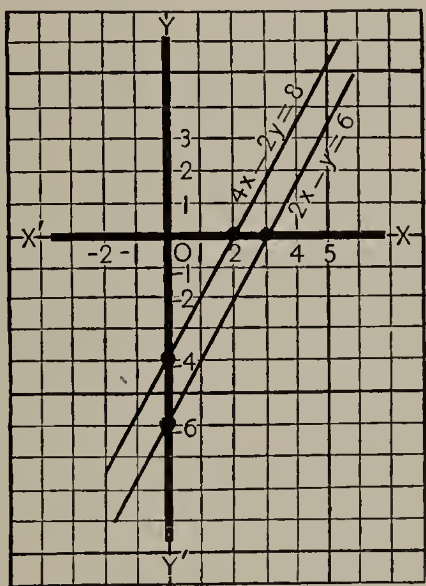
The system is inconsistent, for there is no graphical solution.

12.



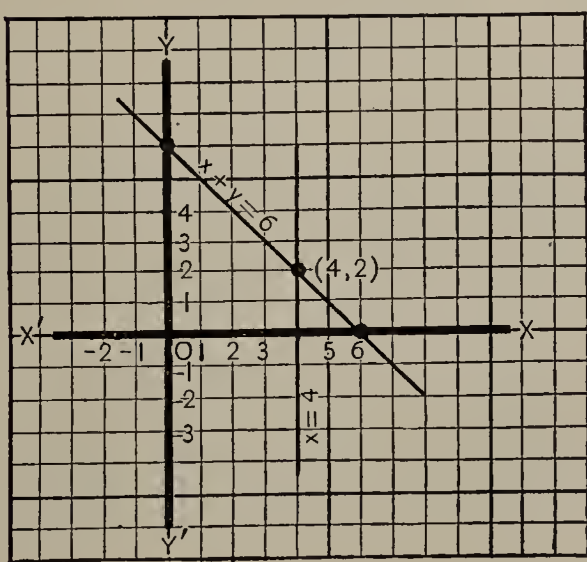
The system is inconsistent, for there is no graphical solution.

13.

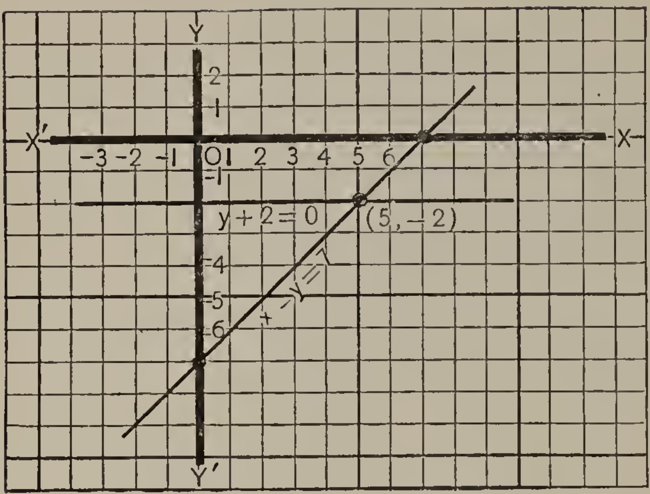


The system is inconsistent, for there is no graphical solution.

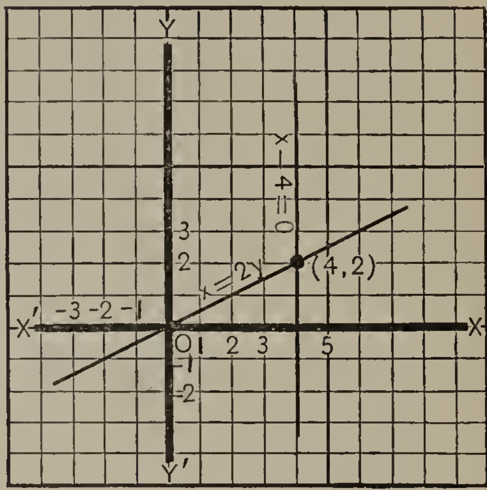
14.



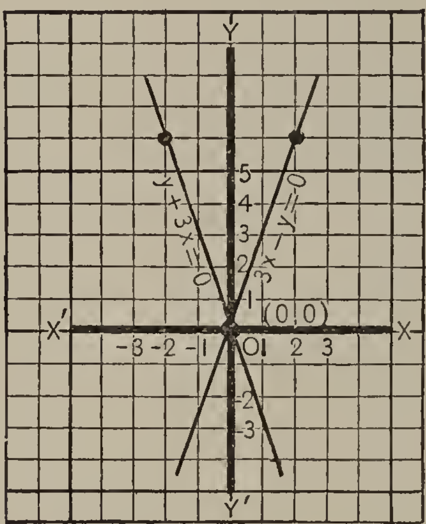
15.



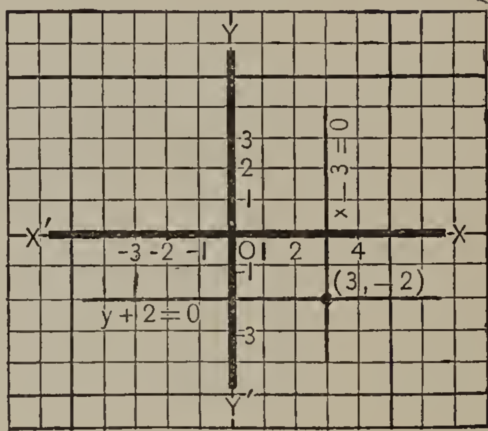
16.



17.



18.



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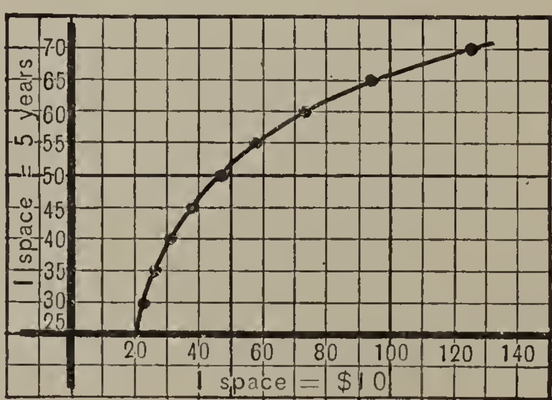
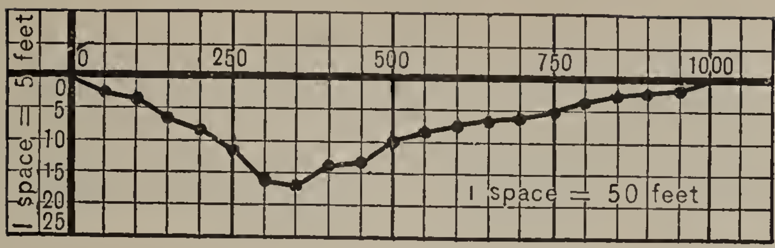
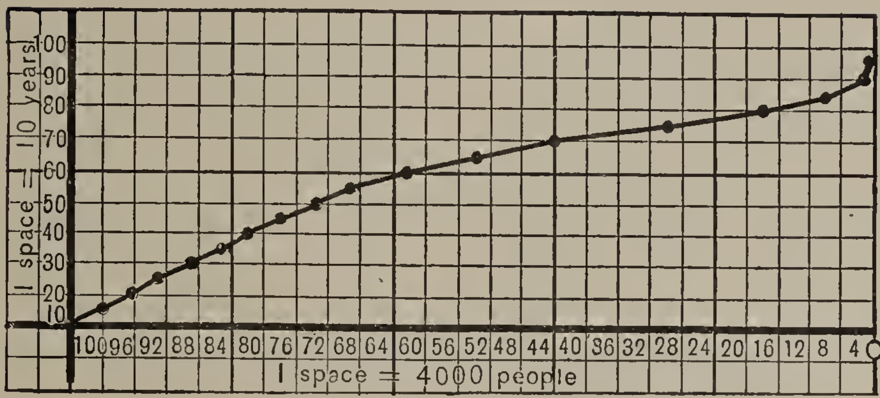
1.



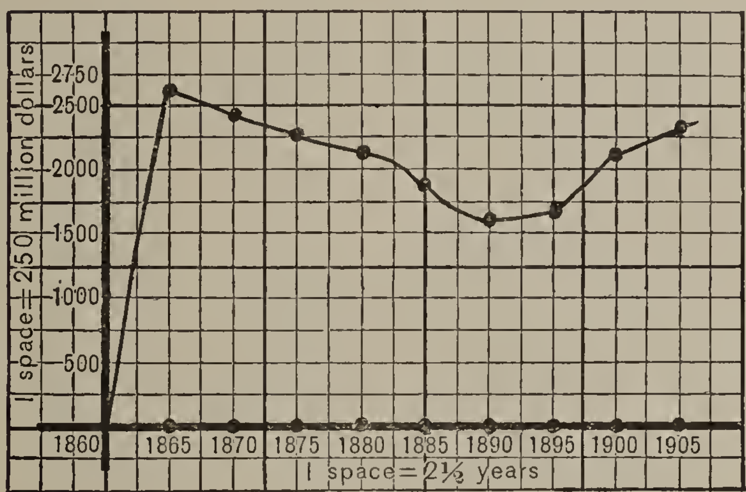
2.


1 space = 5 feet
0 250 500 750 1000
0 5 10 15 20 25
1 space = 50 feet

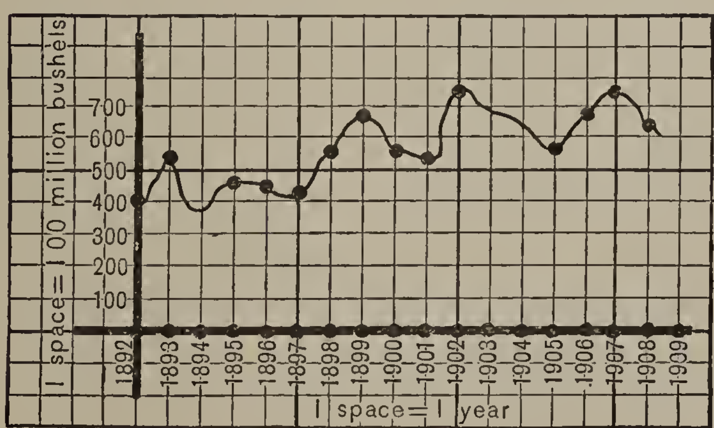
3.


1 space = 10 years
100 90 80 70 60 50 40 30 20 10
100 96 92 88 84 80 76 72 68 64 60 56 52 48 44 40 36 32 28 24 20 16 12 8 4 0
1 space = 4000 people

4.


1 space = 250 million dollars
2750 2500 2000 1500 1000 500
1860 1865 1870 1875 1880 1885 1890 1895 1900 1905
1 space = 2½ years

5.


1 space = 100 million bushels
700 600 500 400 300 200 100
1892 1893 1894 1895 1896 1897 1898 1899 1900 1901 1902 1903 1904 1905 1906 1907 1908 1909
1 space = 1 year

Page 206

1. $x + 2y = 7,$ (1)

$5x - 2y = 11.$ (2)

$[(1)+(2)] \div 6,$ $x = 3.$

Then from (1), $y = 2.$

2. $2x + y = 4,$ (1)

$3x - y = 21.$ (2)

$[(1)+(2)] \div 5,$ $x = 5.$

Then from (1), $y = -6.$

3. $7m - n = 2,$ (1)

$n - 2m = -3.$ (2)

$[(1)+(2)] \div 5,$ $m = -\frac{1}{5}.$

Then $n = -\frac{17}{5}.$

4. $10h - k = -3,$ (1)

$12h + 12k = 102.$ (2)

$[(1) \cdot 12 + (2)] \div 132,$ $h = \frac{1}{2}.$

Then $k = 8.$

5. $7r - 8s = -30,$ (1)

$r + 11s = 20.$ (2)

$[(1) - (2) \cdot 7] \div -85,$ $s = 2.$

Then $r = -2.$

6. $8x + y = 7,$ (1)

$11x + 2y = 28.$ (2)

$[(1) \cdot 2 - (2)] \div 5,$ $x = -\frac{14}{5}.$

Then $y = \frac{147}{5}.$

7. $5l + 2p = 0,$ (1)

$3l + p = 3.$ (2)

$[(1) - (2) \cdot 2] \div -1,$ $l = 6.$

Then $p = -15.$

8. $10v + 2u = 22,$ (1)

$u + 5v = 11.$ (2)

$[(1) - (2) \cdot 2]$ gives $0 = 0.$

The system has an infinite number of sets of roots.

9. $12x + 5y = 14,$ (1)

$3x - 10y = 8.$ (2)

$[(1) \cdot 2 + (2)] \div 27,$ $x = \frac{4}{3}.$

Then $y = -\frac{2}{5}.$

10. $2x - y = -1,$ (1)

$15x - 9y = 20.$ (2)

$[(1) \cdot 9 - (2)] \div 3,$ $x = -\frac{29}{3}.$

Then $y = -\frac{55}{3}.$

11. $3s - t = 12,$ (1)

$2t - 6s = 10.$ (2)

$[(1) \cdot 2 + (2)]$ gives $0 = 14.$

Therefore the system is not consistent.

12. $5x - 3w = 2,$ (1)

$15x + 12w = -5.$ (2)

$[(1) \cdot 3 - (2)] \div -21,$

$w = -\frac{11}{21}.$

Then $x = \frac{3}{35}.$

13. $x_1 - 6x_2 = 7,$ (1)

$12x_2 - x_1 = 0.$ (2)

$[(1) + (2)] \div 6,$ $x_2 = \frac{7}{6}.$

Then $x_1 = 14.$

14. $27h + 32k = 6,$ (1)

$16k - 9h = 8.$ (2)

$[(1) + (2) \cdot 3] \div 80,$ $k = \frac{3}{8}.$

Then $h = -\frac{2}{9}.$

15. $2r + 25r_1 = 15,$ (1)

$3r = 10r_1 - 44.$ (2)

$[(1) \cdot 3 - (2) \cdot 2] \div 95,$

$r_1 = \frac{7}{5}.$

Then $r = -10.$

16. $12n - 2m = 18,$ (1)

$3m = 18n + 10.$ (2)

$[(1) \cdot 3 + (2) \cdot 2]$ gives

$0 = 74.$

Therefore the system is not consistent.

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1. $x - 2y = 8,$ (1)

$3x + 2y = 7.$ (2)

Substituting $2y + 8$ for x from (1) into (2),

$3(2y + 8) + 2y = 7.$

Therefore $y = -\frac{17}{8}.$

Then from (1),

$x = \frac{15}{4}.$

2. $x - 2y = -12,$ (1)

$4x - y = 1.$ (2)

Substituting $2y - 12$ for x from (1) into (2),

$8y - 48 - y = +1.$

Therefore $y = 7,$

and $x = 2.$

3. $14m - 2n = 1,$ (1)

$n - 6m = 0.$ (2)

Substituting $6m$ for n from (2) into (1),

$m = \frac{1}{2}.$

Then $n = 3.$

4. $6h + 10k = 19,$ (1)

$2k = 3h.$ (2)

Substituting $\frac{3h}{2}$ for k from (2) into (1),

$h = \frac{19}{21}.$

Then $k = \frac{19}{14}.$

5. $3s + 12 = 3 + t,$ (1)

$t = s + 1.$ (2)

Substituting $s + 1$ for t in (1),

$s = -4,$

and $t = -3.$

6. $18 + 2p = q,$ (1)

$p + q = -9.$ (2)

Substituting $2p + 18$ for q from (1) into (2),

$p = -9.$

Then $q = 0.$

7. $3r + 15s = 7,$ (1)

$12 + 5s = -r.$ (2)

Substituting $-12 - 5s$ for r from (2) into (1),

$36 = 7.$

Therefore the system is inconsistent.

8. $20y - 3z = 1,$ (1)

$z - 6y = 0.$ (2)

Substituting $6y$ for z from (2) into (1),

$y = \frac{1}{2}.$

Therefore $z = 3.$

9. $.75p + 1.5q = 3,$ (1)

$q = p - 16.$ (2)

Substituting $p - 16$ for q into $(1) \cdot 100,$

$75p + 150p - 2400 = 300.$

Therefore $p = 12,$

and $q = -4.$

10. $\frac{3x - 20}{2} = \frac{2x + 5y}{3},$ (1)

$10 = x - y.$ (2)

Substituting $10 + y$ for x from (2) into $(1) \cdot 6,$

$y = -2.$

Then $x = 8.$

11. $\frac{1}{7 + 2m_1} = \frac{-7}{m_2 - 1},$ (1)

$m_1 = m_2 + 3.$ (2)

Substituting $m_2 + 3$ for m_1 from (2) into (1) and solving,

$m_2 = -6.$

Then $m_1 = -3.$

12. $$\frac{5R_1+2R_2}{2}=2(R_2+2), \quad (1)$$

$$R_1-\tfrac{2}{3}R_2=0. \quad (2)$$

Substituting $\frac{2R_2}{3}$ for R_1 from (2) into (1) and solving,

$$R_2=6.$$

Then $R_1=4.$

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1. $$\frac{2x}{3}+4y=\frac{26}{3}, \quad (1)$$

$$3x-\frac{7y}{2}=-4. \quad (2)$$

(1) · 3, $2x+12y=26.$ (3)
(2) · 2, $6x-7y=-8.$ (4)
[(3) · 3 − (4)] ÷ 43, $y=2.$
Whence $x=1.$

2. $$m-\frac{3n}{5}=\frac{18}{5}, \quad (1)$$

$$\frac{8n}{3}+7m=-16. \quad (2)$$

(1) · 5, $5m-3n=18.$ (3)
(2) · 3, $21m+8n=-48.$ (4)
[(3) · 8 + (4) · 3] ÷ 103,
$m=0.$
Whence $n=-6.$

3. $$\frac{5R_1}{6}+\frac{R_2}{4}=7, \quad (1)$$

$$\frac{2R_1}{3}-\frac{R_2}{8}=3. \quad (2)$$

[(1) · 12 + (2) · 24] ÷ 26,
$R_1=6.$
Whence $R_2=8.$

4. $$3k-\frac{12h}{5}=18, \quad (1)$$

$$\frac{11k}{10}+\frac{17h}{2}=53\tfrac{1}{2}. \quad (2)$$

(1) · 5, $15k-12h=90.$ (3)
(2) · 10, $11k+85h=535.$ (4)
[(3) · 11 + (4) · 15] ÷ − 1407,
$h=5.$
Whence $k=10.$

5. $$\frac{2x}{9}-\frac{y}{2}=-1, \quad (1)$$

$$x=\frac{9y}{4}-4\tfrac{1}{2}. \quad (2)$$

(1) · 18, $4x-9y=-18,$ (3)
(2) · 4, $4x-9y=-18,$ (4)
(3) − (4), $0=0.$

Therefore the system has an infinite number of sets of roots.

6. $$.4x+.9y=5.7, \quad (1)$$

$$2x-y=1. \quad (2)$$

[(1) · 10 − (2) · 2] ÷ 11,
$y=5.$
Whence $x=3.$

7. $$12m=\frac{11y}{9}+17, \quad (1)$$

$$\frac{2m}{3}-\frac{49}{12}=\frac{5y}{12}. \quad (2)$$

(1) · 9, $108m-11y=153.$ (3)
(2) · 12, $8m-5y=49.$ (4)
[(3) · 5 − (4) · 11] ÷ 452,
$m=\tfrac{1}{2}.$
Whence $y=-9.$

8. $$9s=\frac{t}{3}+\frac{23}{4}, \quad (1)$$

$$\frac{7t}{3}-\frac{9}{4}=\frac{-3s}{4}. \quad (2)$$

$(1)\cdot 12,\quad 108\,s - 4\,t = 69.\quad (3)$

$(2)\cdot 12,\quad 28\,t + 9\,s = 27.\quad (4)$

$[(3)\cdot 7 + (4)\cdot 1] \div 765,$

$s = \frac{2}{3}.$

Whence $t = \frac{3}{4}.$

9. $.04\,m + .75\,n = 10,\quad (1)$

$.8\,m - 1.25\,n = 5.\quad (2)$

$(1)\cdot 100,\quad 4\,m + 75\,n = 1000.\quad (3)$

$(2)\cdot 100,\quad 80\,m - 125\,n = 500.\quad (4)$

$[(3)\cdot 20 - (4)\cdot 1] \div 1625,$

$n = 12.$

Whence $m = 25.$

10. $28\,x - 16\,y = 56,\quad (1)$

$\frac{2\,x + 7}{2} - y = 4.\quad (2)$

$[(1) - (2)\cdot 16] \div 12,\quad x = 4.$

Whence $y = \frac{7}{2}.$

11. $\frac{r + 3}{7} - \frac{l}{5} = 0,\quad (1)$

$5\,r + 2 = 7\,l.\quad (2)$

$[(1)\cdot 35 - (2)\cdot 1]$ gives

$13 = 0.$

Therefore the system is inconsistent.

12. $\frac{x + 3}{5} + \frac{y + 4}{10} = \frac{3}{2},\quad (1)$

$\frac{7\,x + 1}{3} - \frac{11\,y - 4}{7} = 4.\quad (2)$

$(1)\cdot 10,\quad 2\,x + y = 5.\quad (3)$

$(2)\cdot 21,\quad 49\,x - 33\,y = 65.\quad (4)$

$[(3)\cdot 33 + (4)\cdot 1] \div 115,$

$x = 2.$

Whence $y = 1.$

13. $\frac{20\,m + 9}{7} = \frac{n}{7} + 1,\quad (1)$

$m + \frac{29}{5} = n.\quad (2)$

$[(1)\cdot 7 - (2)\cdot 20] \div - 19,$

$n = 6.$

Whence $m = \frac{1}{5}.$

14. $\frac{1}{x} + \frac{1}{y} = \frac{5}{6},\quad (1)$

$\frac{1}{x} - \frac{1}{y} = \frac{1}{6}.\quad (2)$

$[(1) + (2)]\cdot x,\quad x = 2.$

Whence $y = 3.$

15. $\frac{1}{m_1} + \frac{1}{m_2} = \frac{2}{15},\quad (1)$

$\frac{1}{m_1} - \frac{1}{m_2} = -\frac{1}{3}.\quad (2)$

$(1) + (2)\cdot 5\,m_1,\quad m_1 = -10.$

Whence $m_2 = \frac{30}{7}.$

16. $\frac{6}{x} + \frac{7}{y} = \frac{3}{2},\quad (1)$

$\frac{7}{x} - \frac{6}{y} + 5\frac{1}{3} = 0.\quad (2)$

$(1)\cdot 6 + (2)\cdot 7,\quad x = -3.$

Whence $y = 2.$

17. $\frac{1}{x} + \frac{1}{y} = 10,\quad (1)$

$\frac{3}{x} + \frac{3}{y} = 14.\quad (2)$

$(1)\cdot 3 - (2),\quad 0 = 16.$

Therefore the system is inconsistent.

18. $\frac{5}{x} + 12 = 17,\quad (1)$

$\frac{2}{x} - 15\,y = 0.\quad (2)$

$[(1)\cdot 2 - (2)\cdot 5] \div 75,\quad y = \frac{2}{15}.$

Whence $x = 1.$

19. $\frac{4\,r + q}{6\,r + q} = \frac{2}{5},\quad (1)$

$\frac{2\,r}{3} - q = 5.\quad (2)$

$(1)\cdot 5\,(6\,r + q),\quad 8\,r + 3\,q = 0.\quad (3)$

$(2)\cdot 3,\quad 2\,r - 3\,q = 15.\quad (4)$

$[(3) + (4)] \div 10,\quad r = \frac{3}{2}.$

Whence $q = -4.$

20. $$\frac{1}{t_2}+\frac{2}{t_1}=\frac{40}{t_1t_2}, \quad (1)$$

$$2t_2-3t_1=0. \quad (2)$$

$[(1)\cdot 2\cdot t_1t_2-(2)]\div 5,$

$$t_1=10.$$

Whence $t_2=15.$

21. $$\frac{5m+3n-1}{3}=n+\frac{1}{3}, \quad (1)$$

$$\frac{25m}{10m+2n}=1. \quad (2)$$

$(1)\cdot 3\div 5,$ $\quad m=\frac{2}{5}.$

Whence $n=3.$

22. $$\frac{k+5}{k+1}=\frac{l+2}{l-2}, \quad (1)$$

$$\frac{11k+4}{l+k+1}=4. \quad (2)$$

$\dfrac{[(1)\cdot(k+1)(l-2)+(2)\cdot(l+k+1)]}{3},$

$$k=4.$$

Whence $l=7.$

23. $$\frac{2x_1-3x_2}{x_2-4x_1}=3, \quad (1)$$

$$\frac{8}{x_2+x_1}+\frac{8}{x_1-x_2}=0. \quad (2)$$

$(1)\cdot(x_2-4x_1),$

$$14x_1-6x_2=0. \quad (3)$$

$(2)\cdot(x_2+x_1)(x_1-x_2),$

$$16x_1=0. \quad (4)$$

$(4)\div 16,$ $\quad x_1=0.$

Then $x_2=0.$

But this set makes the first member of (1) indeterminate, also the first member of (2) is not a number.

Therefore the system has no set of roots.

24. $$3x-2y=7, \quad (1)$$

$$\frac{2}{x-5}=-\frac{5}{y-4}. \quad (2)$$

$[(1)\cdot 1+(2)\cdot(x-y)(y-4)]\div 8,$

$$x=5.$$

Whence $x=4.$

But this set does not satisfy (2).

Hence the system has no set of roots.

25. $$\frac{2}{6+s}-\frac{4}{4-t}=0, \quad (1)$$

$$\frac{t}{4}+\frac{s}{2}+2=0. \quad (2)$$

$(1)\cdot(6+s)(4-t),$

$$-2t-4s=16. \quad (3)$$

$(2)\cdot 4,$ $\quad t+2s=-8. \quad (4)$

$[(3)\cdot 1+(4)\cdot 2]$ gives $0=0.$

Therefore the system has an infinite number of sets of roots.

26. $$\frac{2}{n_1+2+n_2}=\frac{1}{5n_2+3n_1-5}, \quad (1)$$

$$n_1+2n_2=4. \quad (2)$$

(1) cleared, $\quad 5n_1+9n_2=12. \quad (3)$

$(2)\cdot 5,$ $\quad 5n_1+10n_2=20. \quad (4)$

$(4)-(3),$ $\quad n_2=8.$

Whence $n_1=-12.$

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1. Let $g=$ the greater number,

and $l=$ the less.

Then $g-l=25, \quad (1)$

and $g+l=46. \quad (2)$

$[(1)+(2)]\div 2,$

$$g=\tfrac{71}{2}.$$

Whence $l=\frac{21}{2}.$

2. Let $g=$ the greater number,

and $l=$ the less.

Then $\dfrac{g}{l}=6,$

or $g=6l, \quad (1)$

and $g+l=49. \quad (2)$

$[(1)-(2)]\div -7,$ $l=7.$

Whence $g=42.$

3. Let $g =$ the greater number,
and $l =$ the less.

Then $g - l = 36,$ (1)

and $\frac{g}{l} = 3,$

or $g - 3l = 0.$ (2)

$[(1) - (2)] \div 2,$ $l = 18.$

Whence $g = 54.$

4. Let $\frac{n}{d} =$ the fraction.

Then $\frac{n}{d} = \frac{3}{5},$

or $5n - 3d = 0,$ (1)

and $\frac{n + 3}{d + 1} = \frac{3}{4},$

or $4n - 3d = -9.$ (2)

$(1) - (2),$ $n = 9.$

Whence $d = 15,$

and $\frac{n}{d} = \frac{9}{15}.$

5. Let $g =$ the greater number,
and $l =$ the less.

Then $\frac{g}{l} = 3 + \frac{6}{l},$

or $g - 3l = 6,$ (1)

and $\frac{l}{g} = .3,$

or $10l - 3g = 0.$ (2)

$(1) \cdot 3 + (2),$ $l = 18.$

Whence $g = 60.$

6. Let $n =$ the number of nickels,
and $q =$ the number of quarters.

Then $n + q = 77,$ (1)

and $5n + 25q = 985.$ (2)

$[(1) \cdot 5 - (2) \cdot 1] \div -20,$

$q = 30.$

Whence $n = 47.$

7. Let $\frac{n}{d} =$ the fraction.

Then $\frac{n - \frac{4}{3}}{d + \frac{1}{9}} = \frac{3}{5},$

or $5n - 3d = 7,$ (1)

and $n + d = 19.$ (2)

$[(1) \cdot 1 + (2) \cdot 3] \div 8,$

$n = 8.$

Whence $d = 11,$

and $\frac{n}{d} = \frac{8}{11}.$

8. Let $\frac{n}{d} =$ the fraction.

Then $d - n = 24,$ (1)

and $\frac{n + \frac{3}{8}}{d - \frac{1}{4}} = \frac{3}{14},$

or $14n - 3d = -6.$ (2)

$[(1) \cdot 3 + (2) \cdot 1] \div 11,$

$n = 6.$

Whence $d = 30,$

and $\frac{n}{d} = \frac{6}{30}.$

9. Let w_1 and w_2 represent the weights in pounds.

Then $12w_1 = 8w_2,$ (1)

and $10(w_1 + 2) = 8w_2.$ (2)

$[(1) - (2)] \div 2,$ $w_1 = 10,$

and $w_2 = 15.$

10. Let w_1 and w_2 represent the weights in pounds.

Then $12w_1 = 18w_2,$ (1)

and $12(w_1 - 12) = 15w_2.$ (2)

$[(1) - (2)] \div -3,$

$w_2 = 48.$

Whence $w_1 = 72.$

11. Let w_1 and w_2 represent the weights in pounds.

Then $15w_1 = 10w_2,$ (1)

and $14(w_1 - 6) = 10(w_2 - 12).$ (2)

$(1) - (2),$ $w_1 = 36.$

Whence $w_2 = 54.$

12. Let x = A's age in years,
and y = B's age in years.

Then $x - 2y = 0,$ (1)

and $y - 7 = \dfrac{x-7}{3}.$ (2)

$[(1) + (2)],$ $y = 14.$

Whence $x = 28.$

13. Let x = A's age now in years,
and y = B's age now in years.

Then $x + 5 = 2(y + 5),$ (1)

and $x - 5 = 3(y - 5).$ (2)

$(1) - (2),$ $y = 15.$

Whence $x = 35.$

14. Let l = the length,
and w = the width.

Then $2l + 2w = 232,$ (1)

$l - 2w = 8.$ (2)

$[(1)+(2)] \div 3,$ $l = 80.$

Whence $w = 36.$

15. Let p_1 = the number of dollars at 6%,
and p_2 = the number of dollars at 5%.

Then $p_1 + p_2 = 1000,$ (1)

and $.06 p_1 + .05 p_2 = 54.$ (2)

$[(1) \cdot 5 - (2) \cdot 100] \div - 1,$

$p_1 = 400.$

Whence $p_2 = 600.$

16. Let x = the number of dollars at $4\frac{1}{2}\%$,
and y = the number of dollars at $3\frac{1}{2}\%$.

Then $x + y = 2000,$ (1)

and $.035y - .045x = 10.$ (2)

$[(1) \cdot 35 - (2) \cdot 1000] \div 80,$

$x = 750.$

Whence $y = 1250.$

17. Let n_1 = the number of dollars at 4%,
and n_2 = the number of dollars at 6%.

Then $n_1 + n_2 = 5000,$ (1)

and

$5\left(\dfrac{4n_1}{100}\right) - 3\left(\dfrac{6n_2}{100}\right) = 126.$ (2)

$[(2) \cdot 100 + (1) \cdot 18] \div 38,$

$n_1 = 2700,$

Whence $n_2 = 2300.$

18. Let r = the value of a ruble in cents,
and m = the value of a mark in cents.

Then

$20m - 10r = 10,$ (1)

and $12m - 4r = 100.$ (2)

$[(1) \cdot 2 + (2) \cdot 5] \div 20,$

$m = 24.$

Whence $r = 47.$

19. Let f = the value of a franc in cents,
and F = the value of a florin in cents.

Then $5f - 2F = 19,$ (1)

and $3f + F = 95.$ (2)

$[(1) + (2) \cdot 2] \div 11,$

$f = 19.$

Whence $F = 38.$

21. Let t = the digit in tens' place,
and u = the digit in units' place.

Then $t + u = 7,$ (1)

and $10t + u + 27 = 10u + t,$

or $t - u = -3.$ (2)

$[(1)+(2)] \div 2,$ $t = 2,$

and $u = 5.$

Therefore the number is 25.

22. Let t = the digit in tens' place,
and u = the digit in units' place.

Then $t - 2u = 0$, (1)
and $10t + u - 36 = 10u + t$,
or $t - u = 4$. (2)
$[(1) - (2)] \div -1$,
$$u = 4,$$
and $t = 8$.
Therefore the number is 84.

23. Let t = the digit in tens' place,
and u = the digit in units' place.

Then $\dfrac{10t + u}{t + u} = 4$,
or $6t - 3u = 0$; (1)
and $10t + u + 36 = 10u + t$,
or $3t - 3u = -12$. (2)
$[(1) - (2)] \div 3$, $t = 4$,
and $u = 8$.
Therefore the number is 48.

24. Let t = the digit in tens' place,
and u = the digit in units' place.

Then
$$\frac{10t + u + 3}{t + u} = 9,$$
or $t - 8u = -3$; (1)
and $\dfrac{10t + u}{3u} = 17$,
or $t - 5u = 0$. (2)
$[(1) - (2)] \div -3$,
$$u = 1,$$
and $t = 5$.
Therefore the number is 51.

25. Let t = the digit in tens' place,
and u = the digit in units' place.

Then $\dfrac{10t + u}{t + u} = 7$,
or $t - 2u = 0$; (1)
and $\dfrac{10u + t}{t + u + 3} = 3$,
or $-2t + 7u = 9$. (2)
$[(1) \cdot 2 + (2)] \div 3$,
$$u = 3,$$
and $t = 6$.
Therefore the number is 63.

26. Let n_1 = the first number,
and n_2 = the second number.

Then $\dfrac{1}{n_1} + \dfrac{1}{n_2} = \dfrac{5}{24}$, (1)
and $\dfrac{1}{n_1} - \dfrac{1}{n_2} = \dfrac{1}{24}$. (2)
$[(1) + (2)]$ gives $n_1 = 8$.
Whence $n_2 = 12$.

27. Let n_1 = the first number,
and n_2 = the second number.

Then $\dfrac{1}{n_1} - \dfrac{1}{n_2} = \dfrac{7}{6}$, (1)
and $\dfrac{n_2}{n_1} = \dfrac{15}{8}$. (2)

Substituting $\dfrac{15n_1}{8}$ for n_2 from (2) into (1),
$$\frac{1}{n_1} - \frac{8}{15n_1} = \frac{7}{6}. \quad (3)$$
$[(3) \cdot 30n_1] \div 35$,
$$n_1 = \tfrac{2}{5}.$$
Then $n_2 = \tfrac{3}{4}$.

28. Let w_1 = the number of grams in first pan,
and w_2 = the number of grams in second pan.

Then $w_1 - 15 = \dfrac{w_2 + 15}{2}$,
or $2w_1 - w_2 = 45$; (1)
and $w_2 - 85 = w_1 + 85$,
or $w_2 - w_1 = 170$. (2)
$(1) + (2)$, $w_1 = 215$.
Then $w_2 = 385$.

29. Let $a =$ the number of dollars A had,
and $b =$ the number of dollars B had.

Then $2(a-20) = b + 20,$

or $2a - b = 60;$ (1)

and $a - 20 + 75 = 5(b + 20 - 75),$

or $a - 5b = -330.$ (2)

$[(1) - (2) \cdot 2] \div 4,$ $b = 80.$

Then $a = 70.$

30. Let $c_1 =$ the number of feet in the circumference of the fore wheel,
and $c_2 =$ the number of feet in the circumference of the rear wheel.

Then $c_2 - c_1 = 2,$ (1)

and $$\frac{155}{c_1} = \frac{186}{c_2},$$

or $5c_2 - 6c_1 = 0.$ (2)

$[(1) \cdot 5 - (6)],$ $c_1 = 10.$

Then $c_2 = 12.$

31. Let $l =$ the number of feet in the length of the rectangle,
and $w =$ the number of feet in the width of the rectangle.

Then $(l+1)(w+1) = lw + 18,$

or $w + l = 17;$ (1)

and $(l-1)(w-1) = lw - 16,$

or $w + l = 17.$ (2)

$(1) - (2),$ $0 = 0.$

Therefore an infinite number of sets of dimensions satisfy the problem. The sum of each set is 17 feet.

32. Let $a =$ the number of days A requires,
and $b =$ the number of days B requires.

Then $$\frac{1}{a} + \frac{1}{b} = \frac{5}{12}, \quad (1)$$

and $$a - \frac{2b}{3} = 0. \quad (2)$$

Substituting $\frac{2b}{3}$ for a from (2) into (1),

$$\frac{3}{2b} + \frac{1}{b} = \frac{5}{12}.$$

Whence $b = 6.$

Then $a = 4.$

33. Let $a =$ the number of days A requires,
and $b =$ the number of days B requires.

Then $$\frac{1}{a} + \frac{1}{b} = \frac{5}{36}, \quad (1)$$

and $$\frac{5}{a} + \frac{5}{b} + \frac{11}{3a} = 1. \quad (2)$$

$[(1) \cdot 5 - (2)]$ gives

$$-\frac{11}{3a} = -\frac{11}{36},$$

or $a = 12.$

Whence $b = 18.$

34. Let l = the length in feet,
and w = the width in feet.

Then

$$(l + 4)(w - 2) = lw + 8,$$

or $$2w - l = 8; \quad (1)$$

and $$(l - 1)(w + 3) = lw + 33,$$

or $$-w + 3l = 36. \quad (2)$$

$[(1) + (2) \cdot 2] \div 5,$

$$l = 16.$$

Whence $$w = 12.$$

$$\text{Area} = \frac{16 \cdot 12}{9} = 21\tfrac{1}{3} \text{ square yards.}$$

35. Let l = the length in feet,
and w = the width in feet.

Then $$lw = (l + 10)(w - 6),$$

or $$5w - 3l = 30; \quad (1)$$

and $$lw = (l + 4)(w - 3),$$

or $$4w - 3l = 12. \quad (2)$$

$(1) - (2),$ $$w = 18.$$

Whence $$l = 20.$$

36. Let x = the first number of the first pair,
and y = the second number of the first pair.

Then $x + 2$ = the first number of the second pair,
$y - 15$ = the second number of the second pair,
$x + 3$ = the first number of the third pair,
and $y - 18$ = the second number of the third pair.

Therefore $$xy = (x + 2)(y - 15),$$

or $$2y - 15x = 30; \quad (1)$$

and $$xy = (x + 3)(y - 18),$$

or $$3y - 18x = 54. \quad (2)$$

$[(1) \cdot 3 - (2) \cdot 2] \div -9,$ $$x = 2.$$

Then $$y = 30.$$

The pairs are 2, 30; 4, 15; and 5, 12 respectively.

37. Let x = the number of men,
and y = the number of dollars each paid.

Then $$(x + 3)(y - 200) = xy,$$

or $$3y - 200x = 600; \quad (1)$$

and $$(x - 4)(y + 500) = xy,$$

or $$-4y + 500x = 2000. \quad (2)$$

$[(1) \cdot 4 + (2) \cdot 3] \div 700,$ $$x = 12.$$

Then $$y = 1000,$$

and $$xy = 12{,}000.$$

38. Let x = the rate of the man in still water in miles per hour,
and y = the rate of the river in miles per hour.

Then $$\frac{10}{x + y} = 2,$$

or $$x + y = 5; \quad (1)$$

and $$\frac{10}{x - y} = 2\tfrac{1}{2},$$

or $$x - y = 4. \tag{2}$$
$[(1) + (2)]\,2,$ $$x = \tfrac{9}{2}.$$
Then $$y = \tfrac{1}{2}.$$

39. Let $x =$ the rate of the boat in still water in miles per hour,
and $y =$ the rate of the current in miles per hour.

Then $$\frac{36}{x+y} = 3,$$
or $$x + y = 12; \tag{1}$$
and $$\frac{24}{x-y} = 3,$$
or $$x - y = 8. \tag{2}$$
$[(1) + (2)] \div 2,$ $$x = 10.$$
Then $$y = 2.$$

40. Let $x =$ the rate of the current in miles per hour,
and $y =$ the number of miles from A to B.

Then $$\frac{y}{x + 8\frac{1}{2}} = 14,$$
or $$14x - y = -119; \tag{1}$$
and $$\frac{\frac{y}{2}}{8\frac{1}{2} - x} = 10,$$
or $$20x + y = 170. \tag{2}$$
$[(1) + (2)] \div 34,$ $$x = \tfrac{3}{2}.$$
Then $$y = 140.$$

41. Let $x =$ the rate of the stream in miles per hour,
and $y =$ the number of miles from A to C.

Then $$\frac{y}{12 + x} = 7,$$
or $$7x - y = -84; \tag{1}$$
and $$\frac{y - 36}{12 - x} = 5,$$
or $$5x + y = 96. \tag{2}$$
$[(1) + (2)] \div 12,$ $$x = 1.$$
Then $$y = 91.$$

42. Let $d =$ the distance from A to B in miles,
and $r =$ the usual rate in miles per hour.

Then $$\frac{d}{\frac{5r}{4}} = \frac{d}{r} - 1,$$

or $$d - 5r = 0; \qquad (1)$$

and $$\frac{d}{24} = \frac{d}{r} - \frac{5}{6},$$

or $$rd - 24d + 20r = 0. \qquad (2)$$

Substituting $5r$ for d from (1) into (2),
$$5r^2 - 100r = 0.$$

Whence $$r = 0 \text{ or } 20.$$

Then, from (1), $$d = 0 \text{ or } 100.$$

The set $\begin{cases} r = 0 \\ d = 0 \end{cases}$ is rejected.

43. Let $d =$ the distance in miles from A to B,

and $r =$ the usual rate in miles per hour.

Then $$\frac{d}{\frac{6r}{5}} = \frac{d}{r} - \frac{2}{5},$$

or $$5d - 12r = 0; \qquad (1)$$

and $$\frac{15}{r} + \frac{d-15}{\frac{6r}{5}} = \frac{d}{r} - \frac{3}{20},$$

or $$5d - 9r = 75. \qquad (2)$$

$[(1) - (2)] \div -3,$ $$r = 25.$$

Whence $$d = 60.$$

44. Let $x =$ the number of feet in the length of the passenger train,

and $y =$ the number of feet in the length of the freight train.

Then $$2x - y = 0, \qquad (1)$$

and $$\frac{x+y}{66+44} = 15,$$

or $$x + y = 1650. \qquad (2)$$

$[(1) + (2)] \div 3,$ $$x = 550.$$

Whence $$y = 1100.$$

45. Let $p =$ the number of feet in the length of the passenger train,

and $f =$ the number of feet in the length of the freight train.

Then $$f - p = 350, \qquad (1)$$

and $$\frac{p+f}{5280(45-15)} = \frac{75}{60 \cdot 60},$$

or $$p + f = 1650. \qquad (2)$$

$[(1) + (2)] \div 2,$ $$f = 1000.$$

Whence $$p = 650.$$

46. Let $p =$ the rate of the passenger train in feet per minute,

and $f =$ the rate of the freight train in feet per minute.

Then $$\frac{1540+660}{p+f}=\frac{1}{3},$$

or $$p+f=6600; \qquad (1)$$

and $$\frac{1540+660}{p-f}=\frac{5}{3},$$

or $$p-f=1320. \qquad (2)$$

$[(1)+(2)]\div 2,$ $p=3960$, feet per minute or 45 miles per hour,

and $f=2640$, feet per minute or 30 miles per hour.

47. Let $R_1=$ the rate of faster in miles per hour,

and $R_2=$ the rate of slower in miles per hour.

Then $$\tfrac{22}{3600}(R_1+R_2)=\tfrac{1}{4},$$

or $$R_1+R_2=\tfrac{900}{22}; \qquad (1)$$

and $$\frac{220\,R_1-220\,R_2}{3600}=\frac{1}{4},$$

or $$R_1-R_2=\tfrac{90}{22}. \qquad (2)$$

$[(1)+(2)]\div 2,$ $$R_1=22\tfrac{1}{2}.$$

Whence $$R_2=18\tfrac{9}{22}.$$

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1. $$3x+7y=17a, \qquad (1)$$
$$10x-4y=2a. \qquad (2)$$

$[(1)\cdot 10-(2)\cdot 3]\div 82,$
$$y=2a.$$
Whence $$x=a.$$

2. $$3x-y=10b, \qquad (1)$$
$$4x+9y=3b. \qquad (2)$$

$[(1)\cdot 9+(2)]\div 31,$ $x=3b.$

Whence $$y=-b.$$

3. $$5m-4n=10a-4, \qquad (1)$$
$$m-2na=0. \qquad (2)$$

$[(1)-(2)\cdot 5]\div(10a-4),$
$$n=1.$$
Whence $$m=2a.$$

4. $$11h+5k=33c, \qquad (1)$$
$$\frac{h}{c}-\frac{k}{2c}=3. \qquad (2)$$

$[(1)+(2)\cdot 10c]\div 21,$
$$h=3c.$$
Whence $$k=0.$$

5. $$12R_1-11R_2=a+12b, \qquad (1)$$
$$R_1+R_2=2a+b. \qquad (2)$$

$[(1)+(2)\cdot 11]\div 23,$
$$R_1=a+b.$$
Whence $$R_2=a.$$

6. $$8p+9q=4a+9a_1, \qquad (1)$$
$$\frac{p}{2}-3q=\frac{a-12a_1}{4}. \qquad (2)$$

$[(1)-(2)\cdot 16]\div 57,$
$$q=a_1.$$
Whence $$p=\frac{a}{2}.$$

7. $$\frac{7cx}{3}-\frac{5y}{2}=-3c, \qquad (1)$$
$$x+\frac{11y}{4}=11c+3. \qquad (2)$$

$[(1)\cdot 12-(2)\cdot 28c]\div(-30-77c),$
$$y=4c.$$
Whence $$x=3.$$

8. $$7.5x + 3y = 6a, \qquad (1)$$
$$.25x + .5y = 0. \qquad (2)$$
$[(1) \cdot 10 - (2) \cdot 300] \div -120,$
$$y = \frac{-a}{2}.$$
Whence $x = a.$

9. $$\frac{h}{2a} - \frac{2k}{a} = -5, \qquad (1)$$
$$\frac{3h}{a} - \frac{7k}{4a} = \frac{3}{4}. \qquad (2)$$
$[(1) \cdot 24a - (2) \cdot 4a] \div -41,$
$$k = 3a.$$
Whence $h = 2a.$

10. $$dr + 3s = 1 - d, \qquad (1)$$
$$7dr + 36s = 7 - 12d. \qquad (2)$$
$[(1) \cdot 7 - (2)] \div -15,$
$$s = -\frac{d}{3}.$$
Whence $r = \dfrac{1}{d}.$

11. $$h - k = 0, \qquad (1)$$
$$4 - \frac{h+k}{5c} = \frac{h-k}{2c}. \qquad (2)$$
Substituting h for k from (1) into (2),
$$h = 10c.$$
Whence $k = 10c.$

12. $$(a+b)n = 1 - cm, \qquad (1)$$
$$(a+b)m - 1 = -cn. \qquad (2)$$
$[(1) \cdot (a+b) - (2) \cdot c] \div [(a+b)^2 - c^2],$
$$n = \frac{1}{a+b+c}.$$
Whence $m = \dfrac{1}{a+b+c}.$

13. $$\frac{m}{b_1 - b_2} + \frac{n}{b_1 + b_2} = 2, \qquad (1)$$
$$m + n = 2b_1. \qquad (2)$$
Substituting $2b_1 - n$ for m from (2) into (1) and solving for n,
$$n = b_1 + b_2,$$
and $m = b_1 - b_2.$

14. $$\frac{1}{x} + \frac{1}{y} = 2b, \qquad (1)$$
$$\frac{2}{x} - \frac{3}{y} = 5c - b. \qquad (2)$$
$[(1) \cdot 3 + (2)]$ solved for x gives
$$x = \frac{1}{b+c}.$$
Whence $y = \dfrac{1}{b-c}.$

15. $$\frac{1}{x+a_1} + \frac{1}{y} = \frac{a_1 + a_2}{2a_1a_2}, \qquad (1)$$
$$\frac{a_1}{x+a_1} - \frac{a_2}{y} = 0. \qquad (2)$$
$[(1) \cdot a_1 - (2)]$ solved gives
$$y = 2a_2.$$
Whence $x = a_1.$

16. $$\frac{m+2a}{n-a} = 1,$$
or $$m - n = -3a; \qquad (1)$$
$$\frac{\frac{6}{5}n - a}{3m - 2a} - \frac{2n}{5m} = 0,$$
or $$-5am + 4an = 0. \qquad (2)$$
$[(1) \cdot 5a + (2)] \div -a,$
$$n = 15a.$$
Whence $m = 12a.$

17. $$kx - ry = 0, \qquad (1)$$
$$x + y - h = 0. \qquad (2)$$
$[(1) + (2) \cdot r] \div (k + r),$
$$x = \frac{rh}{r+k}.$$
Whence $y = \dfrac{hk}{r+k}.$

18. $$x + dy = 3, \qquad (1)$$
$$d(x-3) - y = 0. \qquad (2)$$
$[(1) + (2) \cdot d] \div (d^2 + 1),$
$$x = 3.$$
Whence $y = 0.$

19.
$$ax - by = c, \quad (1)$$
$$x + y = b. \quad (2)$$
$[(1) + (2) \cdot b] \div (a + b)$,
$$x = \frac{b^2 + c}{a + b}.$$
Whence
$$y = \frac{ab - c}{a + b}.$$

20.
$$ax + by = c, \quad (1)$$
$$dx + ey = f. \quad (2)$$
$[(1) \cdot d - (2) \cdot a] \div (bd - ae)$,
$$y = \frac{cd - af}{bd - ae}.$$
Whence
$$x = \frac{bf - ec}{bd - ae}.$$

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1. ac dollars.

2. $\frac{b}{a}$ dollars; $\frac{bc}{a}$ dollars.

3. (a) $2a + 2b$, ab;
(b) $6a + 6b$, $9ab$;
(c) 3;
(d) 9.

4. $\frac{8 \cdot 10}{2} = 40.$

5. $\frac{8 \cdot b}{2} = 4b.$

6. $\frac{a \cdot b}{2}.$

7. $\frac{(a + b)(a - b)}{2} = \frac{a^2 - b^2}{2}.$

8. $\frac{(x - 2y)(x + 2y)}{2} = \frac{x^2}{2} - 2y^2.$

9. $\frac{2k}{b}.$

10. Let x be the required number of inches the base must be increased.

Then
$$\frac{(a - 2)(10 + x)}{2} = \frac{a \cdot 10}{2}.$$
Whence
$$x = \frac{20}{a - 2}.$$

11. Let x be the required number of feet the base must be decreased.

Then
$$\frac{(a + h)(b - x)}{2} = \frac{ab}{2}.$$
Whence
$$x = \frac{bh}{a + h}.$$

12. Let $x =$ the greater number,
and $y =$ the less number.
Then
$$x + y = s, \quad (1)$$
and
$$x - y = d. \quad (2)$$
$[(1) + (2)] \div 2$, $x = \frac{s + d}{2}.$
Whence
$$y = \frac{s - d}{2}.$$

13. Let $f =$ the first number,
and $s =$ the second number.
Then
$$f - as = 0, \quad (1)$$
and
$$f - s = b. \quad (2)$$
$[(1) - (2)] \div (1 - a)$,
$$s = \frac{b}{a - 1}.$$
Whence
$$f = \frac{ab}{a - 1}.$$

14. Let $x =$ the first number,
and $y =$ the second number.
Then
$$x + y = b, \quad (1)$$
and
$$\frac{x}{y} = a. \quad (2)$$
$[(1) - (2) \cdot y] \div (a + 1)$,
$$y = \frac{b}{a + 1}.$$
Whence
$$x = \frac{ab}{a + 1}.$$

15. Let $\frac{n}{d} =$ the fraction.

Then
$$\frac{n + a}{d} = 2,$$

or $\quad n - 2d = -a;$ (1)

and $\quad \dfrac{n}{d+b} = 1,$

or $\quad n - d = b.$ (2)

$[(1) - (2)] \div -1,$

$$d = b + a.$$

Whence $\quad n = a + 2b.$

The fraction is $\dfrac{a+2b}{a+b}$.

16. Let n = the numerator,

and $\quad d$ = the denominator.

Then $\quad \dfrac{n+1}{d} = x,$

or $\quad n - dx = -1;$ (1)

and $\quad \dfrac{n}{d+2} = y,$

or $\quad n - dy + 2y = 0.$ (2)

$[(1) - (2)] \div (y - x),$

$$d = \frac{2y-1}{y-x}.$$

Whence $\quad n = \dfrac{2xy - y}{y - x}.$

17. Let n = the numerator,

and $\quad d$ = the denominator.

Then $\quad \dfrac{n}{d} = b,$

or $\quad n - db = 0;$ (1)

and $\quad \dfrac{n+2}{d} = c,$

or $\quad n - cd = -2.$ (2)

$[(1) - (2)] \div (c - b),$

$$d = \frac{2}{c-b}.$$

Whence $\quad n = \dfrac{2b}{c-b}.$

18. Let x = the number of feet the a-pound boy is from the fulcrum,

and $\quad y$ = the number of feet the b-pound boy is from the fulcrum.

Then $\quad ax - by = 0,$ (1)

and $\quad x + y = l.$ (2)

$[(1) + (2) \cdot b] \div (a + b),$

$$x = \frac{bl}{a+b}.$$

Whence $\quad y = \dfrac{al}{a+b}.$

19. Let x = the number of books at 80 cents each,

and $\quad y$ = the number of books at 110 cents each.

Then

$80x + 110y = 100h,$ (1)

and $\quad 110x + 80y = 100k.$ (2)

$[(1) \cdot 11 - (2) \cdot 8] \div 570,$

$$y = \frac{110h - 80k}{57}.$$

Whence $\quad x = \dfrac{110k - 80h}{57}.$

20. Let x = the cost of the first book in cents,

and $\quad y$ = the cost of the second book in cents.

Then $\quad x - y = d,$ (1)

and $\quad x + y = 100c.$ (2)

$[(1) + (2)] \div 2, \quad x = \dfrac{d + 100c}{2}.$

Whence $\quad y = \dfrac{100c - d}{2}.$

21. Let x = the number of dollars A has,

and $\quad y$ = the number of dollars B has.

Then $\quad x + y = k,$ (1)

and $\quad x - h = y + h.$ (2)

$[(1)+(2)] \div 2, \quad x = \dfrac{2h+k}{2}.$

Whence $\quad y = \dfrac{k-2h}{2}.$

22. Let $x =$ the number of dollars A has,

and $\quad y =$ the number of dollars B has.

Then $\quad x - h = y + h, \quad (1)$

and $\quad x + k = 2y - 2k. \quad (2)$

$[(1)-(2)], \quad y = 3k + 2h.$

Whence $\quad x = 3k + 4h.$

23. Let $x =$ the number of dollars A has,

and $\quad y =$ the number of dollars B has.

Then $x - 10 + h = y + 10, \quad (1)$

and $\quad x + k = 3(y - k). \quad (2)$

$[(1)-(2)] \div 2, \quad x = 30 + 2k - \dfrac{3h}{2}.$

Whence $\quad y = 10 + 2k - \dfrac{h}{2}.$

24. Let $x =$ the number of dollars A has,

and $\quad y =$ the number of dollars B has.

Then $\quad x + y = 40, \quad (1)$

and $\quad x - h + k = y + h - k. \quad (2)$

$[(1)-(2)] \div 2, \quad y = 20 - h + k.$

Whence $\quad x = 20 + h - k.$

25. Let $x =$ the number of dollars A had at first,

and $\quad y =$ the number of dollars B had at first.

Then $\quad x - r = \dfrac{y+r}{2},$

or $\quad 2x - y = 3r; \quad (1)$

and $\frac{3}{4}(x - r + 8) = y + r - 8,$

or $\quad -3x + 4y = -7r + 56. \quad (2)$

$[(1)\cdot 3 + (2)\cdot 2] \div 5,$

$$y = \frac{112 - 5r}{5}.$$

Whence $\quad x = \dfrac{56 + 5r}{5}.$

26. Let $x =$ the number of dollars at $a\%$,

and $\quad y =$ the number of dollars at $b\%$.

Then $\quad x + y = 1000, \quad (1)$

and $\quad \dfrac{ax}{100} + \dfrac{by}{100} = c. \quad (2)$

$[(1)\cdot a - (2)\cdot 100] \div (a - b),$

$$y = \frac{1000a - 100c}{a - b}.$$

Then $\quad x = \dfrac{1000b - 100c}{b - a}.$

27. Let $m =$ the number of dollars at 5%,

and $\quad n =$ the number of dollars at 4%.

Then $\quad m + n = x, \quad (1)$

and $\quad \dfrac{5m}{100} + \dfrac{4n}{100} = y. \quad (2)$

$[(1)\cdot 5 - (2)\cdot 100],$

$$n = 5x - 100y.$$

Whence $\quad m = 100y - 4x.$

28. Let $x =$ the number of days A requires,

and $\quad y =$ the number of days B requires.

Then $\quad 3x - y = 0, \quad (1)$

and $\quad \dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{c}. \quad (2)$

Substituting $3x$ for y from (1) into (2),

$$x = \frac{4c}{3}.$$

Whence $\quad y = 4c.$

29. Let $x =$ the number of days A requires,

and $y =$ the number of days B requires.

Then $hx - y = 0,$ (1)

and $\frac{1}{x} + \frac{1}{y} = \frac{1}{4}.$ (2)

Substituting hx for y from (1) into (2),

$$x = \frac{4h+4}{h}.$$

Whence $y = 4h + 4.$

30. Let $x =$ the number of days A requires,

and $y =$ the number of days B requires.

Then $\frac{1}{x} + \frac{1}{y} = \frac{1}{h},$ (1)

and $\frac{6}{x} = \frac{2}{3}.$ (2)

From (2), $x = 9.$

Whence $y = \frac{9h}{9-h}.$

31. Let $x =$ the number of days A requires,

and $y =$ the number of days B requires.

Then $\frac{1}{x} + \frac{1}{y} = \frac{1}{5},$ (1)

and $\frac{k}{x} = \frac{2}{5}.$ (2)

From (2), $x = \frac{5k}{2}.$

Whence $y = \frac{5k}{k-2}.$

32. Let $x =$ the number of days A requires,

and $y =$ the number of days B requires.

Then $2x - y = 0,$ (1)

and $\frac{1}{x} + \frac{1}{y} = \frac{1}{n}.$ (2)

Substituting $2x$ for y from (1) into (2),

$$x = \frac{3n}{2}.$$

Whence $y = 3n.$

33. Let $x =$ the number of days A requires,

and $y =$ the number of days B requires.

Then $\frac{1}{x} + \frac{1}{y} = \frac{1}{p},$ (1)

and $qx - y = 0.$ (2)

Substituting qx for y from (2) into (1),

$$x = \frac{pq+p}{q}.$$

Whence $y = pq + p.$

34. Let h represent the required number of hours.

Then $\frac{n}{3} + \frac{n}{4} = h,$

and $h = \frac{7n}{12}.$

35. Let h represent the required number of hours for traveling.

Then $3h =$ A's distance,

and $5h =$ B's distance.

Hence $3h + 5h = k,$

and $h = \frac{k}{8}.$

Whence $3h = \frac{3k}{8},$

and $5h = \frac{5k}{8}.$

36. Then $\frac{n}{p} + \frac{n}{q} = h,$

and $h = \frac{nq+np}{pq}.$

37. Let x represent the number of hours which must pass.

Then

$$3(x-h)+5x=k,$$

and $$x=\frac{k+3h}{8}.$$

Whence

$$\text{A's distance}=\frac{3k-15h}{8}\text{ miles,}$$

and $$\text{B's distance}=\frac{5k+15h}{8}\text{ miles.}$$

38. Let n represent the required number of hours.

Then $pn+qn=c$.

Whence $$n=\frac{c}{p+q}.$$

39. A travels $\frac{pc}{p+q}$ and B travels $\frac{qc}{p+q}$.

40. Let r represent the rate of carriage in miles per hour.

Then $$\frac{d}{r}+h=\frac{d}{3}.$$

Whence $$r=\frac{3d}{d-3h}.$$

41. Then $$\frac{d}{r}+h=\frac{d}{c}.$$

Whence $$r=\frac{cd}{d-ch}.$$

42. Let r represent the rate of riding in miles per hour.

Then $$\frac{p}{r}+\frac{p}{q}=t.$$

Whence $$r=\frac{pq}{qt-p}.$$

43. Let a = the rate of A in miles per hour,

and b = the rate of B in miles per hour.

Then $$an+bn=50, \quad (1)$$

and $$a-b=2. \quad (2)$$

$[(1)+(2)\cdot n]\div 2n$,

$$a=\frac{25+n}{n}.$$

Whence $$b=\frac{25-n}{n}.$$

44. Then

$$an+bn=h, \quad (1)$$

and $$a-b=k. \quad (2)$$

$[(1)+(2)\cdot n]\div 2n$,

$$a=\frac{h+kn}{2n}.$$

Whence $$b=\frac{h-kn}{2n}.$$

45. Let d represent the distance in miles.

Then $$\frac{d}{p}+\frac{d}{q}=t.$$

Whence $$d=\frac{tpq}{p+q}.$$

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1.

$$m+n-2p=13, \quad (1)$$
$$m-3n-p=-3, \quad (2)$$
$$m-n+4p=-17. \quad (3)$$

$(1)-(2)$, $\quad 4n-p=16. \quad (4)$

$(1)-(3)$, $\quad 2n-6p=30. \quad (5)$

$[(4)-(5)\cdot 2]\div 11$, $\quad p=-4$.

Whence $\quad n=3$,

and $\quad m=2$.

2.

$$x+y+3z=\tfrac{7}{2}, \quad (1)$$
$$x-2y+4z=7, \quad (2)$$
$$2x-11y-24z=5. \quad (3)$$

$(1)-(2)$, $\quad 3y-z=-\tfrac{7}{2}. \quad (4)$

$(1)\cdot 2-(3)$, $\quad 13y+30z=2. \quad (5)$

$[(4)\cdot 30+(5)]\div 103$, $\quad y=-1$.

Whence $\quad z=\tfrac{1}{2}$,

and $\quad x=3$.

3.

$$x + y + z = -1, \quad (1)$$
$$3x - y - 5z = 13, \quad (2)$$
$$5x + 3y + 2z = 1. \quad (3)$$

(1) + (2), $4x - 4z = 12. \quad (4)$

(1) · 3 − (3), $-2x + z = -4. \quad (5)$

[(4) + (5) · 2] ÷ − 2, $z = -2.$

Whence $x = 1,$

and $y = 0.$

4.

$$2h + 3k - 4l = -26, \quad (1)$$
$$3h - k + 27l = 87\tfrac{1}{2}, \quad (2)$$
$$h + 5k + 33l = 74\tfrac{1}{2}. \quad (3)$$

[(1) − (3) · 2] ÷ − 7,

$$k + 10l = 25. \quad (4)$$

[(2) · 1 − (3) · 3] ÷ − 8,

$$2k + 9l = 17. \quad (5)$$

[(4) · 2 − (5)] ÷ 11, $l = 3.$

Whence $k = -5,$

and $h = \tfrac{1}{2}.$

5.

$$2m + 3n - 4p = -3, \quad (1)$$
$$m + n + 3p = -9, \quad (2)$$
$$m + 2n - 7p = 6. \quad (3)$$
$$(1) - (2) = (3).$$

Hence the system has an infinite number of sets of roots.

6.

$$x + 8y + 5z = 1, \quad (1)$$
$$3x + 10z + 4y = -5, \quad (2)$$
$$x + 4z = 0. \quad (3)$$

[(1) − (2) · 2], $5x + 15z = -11. \quad (4)$

[(4) − (3) · 5] ÷ − 5, $z = 2\tfrac{1}{5}.$

Whence $x = -8\tfrac{4}{5},$

and $y = -\tfrac{3}{20}.$

7.

$$2h - 3l + 4k - 2 = 0, \quad (1)$$
$$3h - 3l - 15 = 0, \quad (2)$$
$$7h - 4k - 31 = 0. \quad (3)$$

[(1) − (2)] ÷ − 1,

$$h - 4k = 13. \quad (4)$$

[(3) − (4)] ÷ 6, $h = 3.$

Whence $k = -\tfrac{5}{2},$

and $l = -2.$

8.

$$4r - 10s = 5, \quad (1)$$
$$6r - t = 3, \quad (2)$$
$$5s + 2t = -\tfrac{3}{2}. \quad (3)$$

[(1) · 3 − (2) · 2],

$$-30s + 2t = 9. \quad (4)$$

[(4) − (3)] ÷ − 35, $s = -\tfrac{3}{10}.$

Whence $r = \tfrac{1}{2},$

and $t = 0.$

9.

$$2a_1 - 3a_2 = 4, \quad (1)$$
$$3a_1 + a_3 = 5, \quad (2)$$
$$a_2 - 2a_3 = 2. \quad (3)$$

[(1) · 3 − (2) · 2] ÷ − 1,

$$9a_2 + 2a_3 = -2. \quad (4)$$

[(4) + (3)] ÷ 10, $a_2 = 0.$

Whence $a_1 = 2,$

and $a_3 = -1.$

10.

$$3r_1 + 5r_2 = 74, \quad (1)$$
$$r_1 - 2r_3 = -16, \quad (2)$$
$$7r_3 - 4r_2 = 44. \quad (3)$$

[(1) − (2) · 3],

$$5r_2 + 6r_3 = 122. \quad (4)$$

[(3) · 5 + (4) · 4] ÷ 59,

$$r_3 = 12.$$

Whence $r_2 = 10,$

and $r_1 = 8.$

11.

$$\frac{1}{m} + \frac{1}{n} - \frac{1}{p} = 1, \quad (1)$$
$$\frac{1}{m} + \frac{1}{n} + \frac{1}{p} = \frac{2}{3}, \quad (2)$$
$$\frac{1}{m} - \frac{1}{n} + \frac{1}{p} = 0. \quad (3)$$

[(1) + (2)] ÷ 2,

$$\frac{1}{m} + \frac{1}{n} = \frac{5}{6}.$$

[(2) − (3)] ÷ 2, $\dfrac{1}{n} = \dfrac{1}{3}.$

Whence $n = 3.$

Then $m = 2,$

and $p = -6.$

12. $$\frac{2}{m}-\frac{3}{n}+\frac{10}{p}=-3, \quad (1)$$

$$\frac{4}{m}+\frac{5}{p}+\frac{6}{n}=15, \quad (2)$$

$$\frac{1}{m}-\frac{1}{n}+\frac{5}{p}=-\frac{1}{2}. \quad (3)$$

$[(1)-(2)\cdot 2]\div -3,$

$$\frac{2}{m}+\frac{5}{n}=11. \quad (4)$$

$(2)-(3),$ $$\frac{3}{m}+\frac{7}{n}=\frac{31}{2}. \quad (5)$$

$[(4)\cdot 3-(5)\cdot 2],$ $$\frac{1}{n}=2.$$

Whence $n=\frac{1}{2}.$

Then $m=2,$

and $p=5.$

13. $$\frac{1}{A}+\frac{1}{B}=2, \quad (1)$$

$$\frac{1}{A}+\frac{1}{C}=3, \quad (2)$$

$$\frac{1}{B}+\frac{1}{C}=4. \quad (3)$$

$[(1)-(2)]+(3),$ $$\frac{2}{B}=3.$$

Whence $B=\frac{2}{3}.$

Then $A=2$, and $C=\frac{2}{5}.$

14. $$r+s+t+u=2.8, \quad (1)$$

$$r-s+t-u=7.2, \quad (2)$$

$$r+2s+3t-5u=7, \quad (3)$$

$$r+s-8t+u=-1.7. \quad (4)$$

$(3)-(1),$ $s+2t-6u=4.2. \quad (5)$

$(3)-(2),$

$$3s+2t-4u=-.2. \quad (6)$$

$(4)-(2),$

$$2s-9t+2u=-8.9. \quad (7)$$

$[(7)-(5)\cdot 2],$

$$-13t+14u=-17.3. \quad (8)$$

$[(6)-(5)\cdot 3],$

$$-4t+14u=-12.8. \quad (9)$$

$[(9)-(8)]\div 9,$ $t=\frac{1}{2}.$

Whence $u=-\frac{54}{70},$

$s=-\frac{10}{7},$

and $r=\frac{9}{2}.$

15. $$\frac{2}{x}+\frac{3}{y}=26, \quad (1)$$

$$\frac{4}{y}-\frac{10}{z}=3, \quad (2)$$

$$\frac{1}{x}+\frac{5}{z}=\frac{25}{2}. \quad (3)$$

$[(3)\cdot 2+(2)],$ $$\frac{2}{x}+\frac{4}{y}=28. \quad (4)$$

$(4)-(1),$ $$\frac{1}{y}=2.$$

Whence $y=\frac{1}{2}.$

Then $x=\frac{1}{10},$

and $z=2.$

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1. Let x, y, and z represent the first, the second, and the third numbers respectively.

Then $x+y=76, \quad (1)$

$y+z=54, \quad (2)$

and $x+z=58. \quad (3)$

$[(1)-(2)+(3)]\div 2,$

$x=40.$

Whence $y=36,$

and $z=18.$

2. Let x, y, and z represent the first, the second, and the third numbers respectively.

Then $x+y+z=58, \quad (1)$

$x+y=24, \quad (2)$

and $\frac{x}{y}=2. \quad (3)$

$(1)-(2),$ $z=34.$

$[(2)-(3)\cdot y]\div 3,$ $y=8.$

Then $x=16.$

3. Let a, b, and c represent the sides of the triangle in feet.

Then $a + b + c = 64$, (1)

$a - b = 0$, (2)

and $a - c = -10$. (3)

$[(1) + (2) + (3)] \div 3$,

$a = 18$.

Whence $b = 18$,

and $c = 28$.

4. Let a, b, and c represent the sides of the triangle in feet.

Then $a + b = 52$, (1)

$a - b = 12$, (2)

and $a + b + c = 93$. (3)

$[(1) + (2)] \div 2$, $a = 32$.

Whence $b = 20$,

and $c = 41$.

5. Let a, b, and c represent the sides of the triangle in feet.

Then $a + b = 41$, (1)

$a + c = 48$, (2)

and $b + c = 43$. (3)

$[(1) - (2) + (3)] \div 2$,

$b = 18$.

Whence $a = 23$,

and $c = 25$.

6. Let x, y, and z represent the numbers.

Then $x + y + z = 26$, (1)

$$\frac{x}{y} = 9,$$

or $x - 9y = 0$. (2)

Also $$\frac{x + y}{z} = \frac{10}{3},$$

$3x + 3y - 10z = 0$. (3)

$[(1) \cdot 3 - (3) \cdot 1] \div 13$,

$z = 6$.

Whence $x = 18$,

and $y = 2$.

7. Let x, y, and z represent the angles respectively in degrees.

Then $x - y = 0$, (1)

$x + y - z = 0$, (2)

$x + y + z = 180$. (3)

$[(2) + (3) + (1) \cdot 2] \div 4$,

$x = 45$.

Whence $y = 45$,

and $z = 90$.

8. Let x, y, and z represent the angles respectively in degrees.

Then $x - y = 0$, (1)

$2x + 2y - z = 0$, (2)

$x + y + z = 180$. (3)

$[(2) + (3) + (1) \cdot 3] \div 6$,

$x = 30$.

Whence $y = 30$,

and $z = 120$.

9. Let x, y, and z represent the number of degrees in A, B, and C respectively.

Then $x - y = 17$, (1)

$y - z = 20$, (2)

and $x + y + z = 180$. (3)

$(1) + (2)$, $x - z = 37$. (4)

$(3) - (2)$, $x + 2z = 160$. (5)

$[(5) - (4)] \div 3$, $z = 41$.

Whence $x = 78$,

and $y = 61$.

10. Let x, y, and z represent the angles respectively in degrees.

Then $x + y - z = 36$, (1)

$6x - 6y - z = 0$, (2)

and $x + y + z = 180$. (3)

$(1) + (3)$, $2x + 2y = 216$. (4)

$(2) + (3)$, $7x - 5y = 180$. (5)

$[(4) \cdot 5 + (5) \cdot 2] \div 24$,

$x = 60$.

Whence $y = 48$,

and $z = 72$.

11. Let x, y, and z represent the time in hours required by A, B, and C respectively.

Then $\frac{1}{x}+\frac{1}{y}=\frac{1}{2}$, (1)

$\frac{1}{x}+\frac{1}{z}=\frac{1}{3}$, (2)

and $\frac{1}{y}+\frac{1}{z}=\frac{1}{4}$. (3)

(1) − (2) + (3), $\frac{2}{y}=\frac{5}{12}$.

Whence $y=\frac{24}{5}$.

Then $x=3\frac{3}{7}$,

and $z=24$.

12. Let x, y, and z represent the number of hours required by the pumps respectively.

Then $\frac{1}{x}+\frac{1}{y}=\frac{1}{4}$, (1)

$\frac{1}{x}+\frac{1}{z}=\frac{1}{6}$, (2)

and $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{13}{48}$. (3)

(3) − (1), $\frac{1}{z}=\frac{1}{48}$,

or $z=48$.

Then $y=9\frac{3}{5}$,

and $x=6\frac{6}{7}$.

13. Let $n_1=$ the first numerator,

$n_2=$ the second numerator,

and $d=$ each denominator.

Then $\frac{n_1}{d}+\frac{n_2}{d}=6$,

or $n_1+n_2-6d=0$; (1)

$\frac{n_1+1}{d}-\frac{n_2-1}{d}=0$,

or $n_1-n_2=-2$, (2)

and $\frac{n_1}{d+22}+\frac{n_2}{d+22}=\frac{1}{2}$,

or $2n_1+2n_2-d=22$. (3)

$[(3)-(1)\cdot 2]\div 11$, $d=2$.

Whence $n_1=5$,

and $n_2=7$.

Therefore the fractions are $\frac{5}{2}$ and $\frac{7}{2}$.

14. Let x, y, and z be the first, second, and third digits respectively.

Whence $x+y+z=15$, (1)

$z-y=5$, (2)

$100x+10y+z+396=100z+10y+x$,

or $x-z=-4$. (3)

$[(1)+(2)-(3)]\div 3$, $z=8$.

Whence $y=3$,

and $x=4$.

Therefore the number is 438.

15. Let x, y, and z be the hundreds', tens', and units' digits respectively.

Then $100x+10z+y=100x+10y+z+54$,

or $z-y=6$; (1)

$100y+10x+z=100x+10y+z-360$,

$x-y=4$, (2)

or $x+y+z=10$. (3)

(3) − (2), $2y+z=6$. (4)

$[(4)-(1)]\div 3$, $y=0$.

Whence $z = 6,$

and $x = 4.$

Therefore the number is 406.

16. Let a, b, c, and d be the thousands', hundreds', tens', and units' digits respectively.

Then $1000a + 100b + 10c + d + 2997 = 1000d + 100c + 10b + a,$

or $111a + 10b - 10c - 111d = -333;$ (1)

$2a - d = 0,$ (2)

$b - c = 0,$ (3)

and $a + b + c + d = 9.$ (4)

$[(1) - (3) \cdot 10] \div 111,$ $a - d = -3.$ (5)

$(2) - (5),$ $a = 3.$

Whence $b = 0,$

$c = 0,$

and $d = 6.$

Therefore the number is 3006.

17. Let a, b, c, and d be the number of degrees in the angles respectively.

Then $a + b + c + d = 360,$ (1)

$a + b = 200,$ (2)

$b + c = 180,$ (3)

and $b + c + d = 255.$ (4)

$(1) - (4),$ $a = 105.$

Then $b = 95,$

$c = 85,$

and $d = 75.$

18. Let a, b, c, and d be the number of degrees in the angles respectively.

Then $a + c = 180,$ (1)

$a - c = 30,$ (2)

$b - d = 36,$ (3)

and $a + b + c + d = 360.$ (4)

$(4) - (1),$ $b + d = 180.$ (5)

$[(5) + (3)] \div 2,$ $b = 108.$

Then $d = 72,$

$a = 105,$

and $c = 75.$

19. Let a, b, c, and d be the sides taken in order.

Then $a + c = 30,$ (1)

$b + d = 35,$ (2)

$a - b = 0,$ (3)

and $a + b - c - d + 17 = 0.$ (4)

$(1) + (2) + (3) + (4),$

$3a + b = 48.$ (5)

$[(5) + (3)] \div 4,$ $a = 12.$

Then $b = 12,$

$c = 18,$

and $d = 23.$

20. Let x, y, and z be the number of dollars A, B, and C have respectively.

Then $2(x - y - z) = 100,$ (1)

$y + y - (x - y - z) - 2z = 100,$ (2)

$2(z + z) = 100.$ (3)

$(1) \div 2,$ $x - y - z = 50.$ (4)

From (2), $-x + 3y - z = 100.$ (5)

From (3), $z = 25.$

Whence $x = 175,$

and $y = 100.$

21. Let a, b, and c be the number of dollars A, B, and C had at first respectively.

$$\begin{array}{llr}
\text{Then} & a-(b+c)+(a-b-c)+(2a-2b-2c)=64, & \\
\text{or} & a-b-c=16; & (1) \\
 & b+b-(2c+a-b-c)+(3b-a-c)=64, & \\
\text{or} & 3b-a-c=32; & (2) \\
 & c+c+2c-(2a-2b-2c)-(3b-a-c)=64, & \\
\text{or} & 7c-a-b=64. & (3) \\
[(1)+(2)]\div 2, & b-c=24. & (4) \\
[(1)+(3)]\div 2, & -b+3c=40. & (5) \\
[(4)+(5)]\div 2, & c=32. & \\
\text{Then} & b=56, & \\
\text{and} & a=104. &
\end{array}$$

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1.

$$\begin{array}{lll}
 & a^4+2a^3+3a^2+2a+1 & \underline{|\,a^2+a+1} \\
(a^2)^2= & a^4 & \\
2(a^2)=2a^2 & 2a^3+3a^2 & \\
2a^2+a & 2a^3+a^2 & =(2a^2+a)a \\
2(a^2+a)=2a^2+2a & 2a^2+2a+1 & \\
2a^2+2a+1 & 2a^2+2a+1 & =(2a^2+a+1)\cdot 1
\end{array}$$

Therefore the square roots are $\pm(a^2+a+1)$.

2.

$$\begin{array}{lll}
 & x^4-8x^3+24x^2-32x+16 & \underline{|\,x^2-4x+4} \\
(x^2)^2= & x^4 & \\
2(x^2)=2x^2 & -8x^3+24x^2 & \\
2x^2-4x & -8x^3+16x^2 & =(2x^2-4x)(-4x) \\
2(x^2-4x)=2x^2-8x & 8x^2-32x+16 & \\
2x^2-8x+4 & 8x^2-32x+16 & =(2x^2-8x+4)\cdot 4
\end{array}$$

Therefore the square roots are $\pm(x^2-4x+4)$.

3.

$$\begin{array}{lll}
 & c^4-10c^3+21c^2+20c+4 & \underline{|\,c^2-5c-2} \\
(c^2)^2= & c^4 & \\
2c^2 & -10c^3+21c^2 & \\
2c^2-5c & -10c^3+25c^2 & =(2c^2-5c)(-5c) \\
2(c^2-5c)=2c^2-10c & -4c^2+20c+4 & \\
2c^2-10c-2 & -4c^2+20c+4 & =(2c^2-10c-2)(-2)
\end{array}$$

Therefore the square roots are $\pm(c^2-5c-2)$.

4. $$n^6 - 6n^4 + 10n^3 + 9n^2 - 30n + 25 \,\underline{|\, n^3 - 3n + 5}$$

$$\begin{array}{l|l}
(n^3)^2 = n^6 & \\
2n^3 & -6n^4 + 10n^3 + 9n^2 \\
2n^3 - 3n & -6n^4 \qquad\qquad + 9n^2 = (2n^3 - 3n)(-3n) \\
2(n^3 - 3n) = 2n^3 - 6n & 10n^3 \qquad - 30n + 25 \\
2n^3 - 6n + 5 & 10n^3 \qquad - 30n + 25 = (2n^3 - 6n + 5)\,5
\end{array}$$

Therefore the square roots are $\pm (n^3 - 3n + 5)$.

5. $$4a^6 + 4a^5 - 11a^4 + 14a^3 + 19a^2 - 30a + 25 \,\underline{|\, 2a^3 + a^2 - 3a + 5}$$

$$\begin{array}{l|l}
(2a^3)^2 = 4a^6 & \\
4a^3 & 4a^5 - 11a^4 \\
4a^3 + a^2 & 4a^5 + a^4 = (4a^3 + a^2)\,a^2 \\
4a^3 + 2a^2 & -12a^4 + 14a^3 + 19a^2 \\
4a^3 + 2a^2 - 3a & -12a^4 - 6a^3 + 9a^2 = (4a^3 + 2a^2 - 3a)(-3a) \\
4a^3 + 2a^2 - 6a & 20a^3 + 10a^2 - 30a + 25 \\
4a^3 + 2a^2 - 6a + 5 & 20a^3 + 10a^2 - 30a + 25 = (4a^3 + 2a^2 - 6a + 5)\,5
\end{array}$$

Therefore the square roots are $\pm (2a^3 + a^2 - 3a + 5)$.

6. $$c^4 - 4c^3d + 6c^2d^2 - 4cd^3 + d^4 \,\underline{|\, c^2 - 2cd + d^2}$$

$$\begin{array}{l|l}
(c^2)^2 = c^4 & \\
2c^2 & -4c^3d + 6c^2d^2 \\
2c^2 - 2cd & -4c^3d + 4c^2d^2 \\
2c^2 - 4cd & 2c^2d^2 - 4cd^3 + d^4 \\
2c^2 - 4cd + d^2 & 2c^2d^2 - 4cd^3 + d^4
\end{array}$$

Therefore the square roots are $\pm (c^2 - 2cd + d^2)$.

7. $$25y^4 + 30xy^3 - 11x^2y^2 - 12x^3y + 4x^4 \,\underline{|\, 5y^2 + 3xy - 2x^2}$$

$$\begin{array}{l|l}
(5y^2)^2 = 25y^4 & \\
10y^2 & 30xy^3 - 11x^2y^2 \\
10y^2 + 3xy & 30xy^3 + 9x^2y^2 \\
10y^2 + 6xy & -20x^2y^2 - 12x^3y + 4x^4 \\
10y^2 + 6xy - 2x^2 & -20x^2y^2 - 12x^3y + 4x^4
\end{array}$$

Therefore the square roots are $\pm (5y^2 + 3xy - 2x^2)$.

8. $$9a^6 - 36a^4x - 24a^3x^2 + 36a^2x^2 + 48ax^3 + 16x^4 \,\underline{|\, 3a^3 - 6ax - 4x^2}$$

$$\begin{array}{l|l}
(3a^3)^2 = 9a^6 & \\
6a^3 & -36a^4x - 24a^3x^2 + 36a^2x^2 \\
6a^3 - 6ax & -36a^4x \qquad\qquad + 36a^2x^2 \\
6a^3 - 12ax & -24a^3x^2 \qquad + 48ax^3 + 16x^4 \\
6a^3 - 12ax - 4x^2 & -24a^3x^2 \qquad + 48ax^3 + 16x^4
\end{array}$$

Therefore the square roots are $\pm (3a^3 - 6ax - 4x^2)$.

9.
$$\begin{array}{l|l}
 & 9c^4 - 12abc^3 - 2a^2b^2c^2 + 4a^3b^3c + a^4b^4 \;\underline{|\,3c^2 - 2abc - a^2b^2} \\
(3c^2)^2 = 9c^4 & \\
6c^2 & -12abc^3 - 2a^2b^2c^2 \\
6c^2 - 2abc & -12abc^3 + 4a^2b^2c^2 \\
6c^2 - 4abc & -6a^2b^2c^2 + 4a^3b^3c + a^4b^4 \\
6c^2 - 4abc - a^2b^2 & -6a^2b^2c^2 + 4a^3b^3c + a^4b^4
\end{array}$$

Therefore the square roots are $\pm(3c^2 - 2abc - a^2b^2)$.

10.
$$\begin{array}{l|l}
 & c^6 - 4xc^3 + 4x^2 + 2a^2xc^3 - 4a^2x^2 + a^4x^2 \;\underline{|\,c^3 - 2x + a^2x} \\
(c^3)^2 = c^6 & \\
2c^3 & -4xc^3 + 4x^2 \\
2c^3 - 2x & -4xc^3 + 4x^2 \\
2c^3 - 4x & 2a^2xc^3 - 4a^2x^2 + a^4x^2 \\
2c^3 - 4x + a^2x & 2a^2xc^3 - 4a^2x^2 + a^4x^2
\end{array}$$

Therefore the square roots are $\pm(c^3 - 2x + a^2x)$.

11.
$$\begin{array}{l|l}
 & 4 - \frac{4c^2}{5} + \frac{c^4}{25} \;\underline{|\,2 - \frac{c^2}{5}} \\
2^2 = 4 & \\
4 & -\frac{4c^2}{5} + \frac{c^4}{25} \\
4 - \frac{c^2}{5} & -\frac{4c^2}{5} + \frac{c^4}{25}
\end{array}$$

Therefore the square roots are $\pm\left(2 - \frac{c^2}{5}\right)$.

12.
$$\begin{array}{l|l}
 & \frac{9}{a^2} - 3 + \frac{a^2}{4} \;\underline{|\,\frac{3}{a} - \frac{a}{2}} \\
\left(\frac{3}{a}\right)^2 = \frac{9}{a^2} & \\
\frac{6}{a} - \frac{a}{2} & -3 + \frac{a^2}{4} \\
 & -3 + \frac{a^2}{4}
\end{array}$$

Therefore the square roots are $\pm\left(\frac{3}{a} - \frac{a}{2}\right)$.

13.

$$
\begin{array}{l|l|l}
 & x^4 - 4x^3 + 5x^2 - 2x + \frac{1}{4} & x^2 - 2x + \frac{1}{2} \\
 & x^4 & \\ \hline
2x^2 & -4x^3 + 5x^2 & \\
2x^2 - 2x & -4x^3 + 4x^2 & \\ \hline
2x^2 - 4x & x^2 - 2x + \frac{1}{4} & \\
2x^2 - 4x + \frac{1}{2} & x^2 - 2x + \frac{1}{4} & \\ \hline
\end{array}
$$

Therefore the square roots are $\pm (x^2 - 2x + \frac{1}{2})$.

15.

$$
\begin{array}{l|l|l}
 & x^4 + 6x^3 + \dfrac{29x^2}{3} + 2x + \dfrac{1}{9} & x^2 + 3x + \dfrac{1}{3} \\
(x^2)^2 = & x^4 & \\ \hline
2x^2 & 6x^3 + \dfrac{29x^2}{3} & \\
2x^2 + 3x & 6x^3 + 9x^2 & \\ \hline
2x^2 + 6x & \dfrac{2x^2}{3} + 2x + \dfrac{1}{9} & \\
2x^2 + 6x + \dfrac{1}{3} & \dfrac{2x^2}{3} + 2x + \dfrac{1}{9} & \\ \hline
\end{array}
$$

Therefore the square roots are $\pm (x^2 + 3x + \frac{1}{3})$.

16.

$$
\begin{array}{l|l|l}
 & 4a^4 + \dfrac{4a^3}{3} - \dfrac{35a^2}{9} - \dfrac{2a}{3} + 1 & 2a^2 + \dfrac{a}{3} - 1 \\
(2a^2)^2 = & 4a^4 & \\ \hline
4a^2 & \dfrac{4a^3}{3} - \dfrac{35a^2}{9} & \\
4a^2 + \dfrac{a}{3} & \dfrac{4a^3}{3} + \dfrac{a^2}{9} & \\ \hline
4a^2 + \dfrac{2a}{3} & -4a^2 - \dfrac{2a}{3} + 1 & \\
4a^2 + \dfrac{2a}{3} - 1 & -4a^2 - \dfrac{2a}{3} + 1 & \\ \hline
\end{array}
$$

Therefore the square roots are $\pm \left(2a^2 + \dfrac{a}{3} - 1\right)$.

17.

$$
\begin{array}{ll|l}
 & \frac{25m^4}{4}+\frac{10m^3}{3}-\frac{127m^2}{18}-2m+\frac{9}{4} & \frac{5m^2}{2}+\frac{2m}{3}-\frac{3}{2} \\
 & \frac{25m^4}{4} & \\ \hline
5m^2 & \frac{10m^3}{3}-\frac{127m^2}{18} & \\
5m^2+\frac{2m}{3} & \frac{10m^3}{3}+\frac{4m^2}{9} & \\ \hline
5m^2+\frac{4m}{3} & -\frac{15m^2}{2}-2m+\frac{9}{4} & \\
5m^2+\frac{4m}{3}-\frac{3}{2} & -\frac{15m^2}{2}-2m+\frac{9}{4} & \\ \hline
\end{array}
$$

Therefore the square roots are $\pm\left(\frac{5m^2}{2}+\frac{2m}{3}-\frac{3}{2}\right)$.

18.

$$
\begin{array}{ll|l}
 & 9c^4-12c^3+4c^2+6-\frac{4}{c}+\frac{1}{c^4} & 3c^2-2c+\frac{1}{c^2} \\
 & 9c^4 & \\ \hline
6c^2 & -12c^3+4c^2 & \\
6c^2-2c & -12c^3+4c^2 & \\ \hline
6c^2-4c & +6-\frac{4}{c}+\frac{1}{c^4} & \\
6c^2-4c+\frac{1}{c^2} & +6-\frac{4}{c}+\frac{1}{c^4} & \\ \hline
\end{array}
$$

Therefore the square roots are $\pm\left(3c^2-2c+\frac{1}{c^2}\right)$.

19.

$$
\begin{array}{ll|l}
 & \frac{m^2}{n^2}-\frac{5m}{n}+\frac{17}{4}+\frac{5n}{m}+\frac{n^2}{m^2} & \frac{m}{n}-\frac{5}{2}-\frac{n}{m} \\
 & \frac{m^2}{n^2} & \\ \hline
\frac{2m}{n} & -\frac{5m}{n}+\frac{17}{4} & \\
\frac{2m}{n}-\frac{5}{2} & -\frac{5m}{n}+\frac{25}{4} & \\ \hline
\frac{2m}{n}-5 & -2+\frac{5n}{m}+\frac{n^2}{m^2} & \\
\frac{2m}{n}-5-\frac{n}{m} & -2+\frac{5n}{m}+\frac{n^2}{m^2} & \\ \hline
\end{array}
$$

Therefore the square roots are $\pm\left(\frac{m}{n}-\frac{5}{2}-\frac{n}{m}\right)$.

20.

$$
\begin{array}{l|l}
 & \dfrac{a^4}{25c^4}+\dfrac{2a^2}{c^3}+\dfrac{117}{5c^2}-\dfrac{40}{a^2c}+\dfrac{16}{a^4}\;\Big|\;\underline{\dfrac{a^2}{5c^2}+\dfrac{5}{c}-\dfrac{4}{a^2}} \\
 & \underline{\dfrac{a^4}{25c^4}} \\
\dfrac{2a^2}{5c^2} & \dfrac{2a^2}{c^3}+\dfrac{117}{5c^2} \\
\dfrac{2a^2}{5c^2}+\dfrac{5}{c} & \underline{\dfrac{2a^2}{c^3}+\dfrac{25}{c^2}} \\
\dfrac{2a^2}{5c^2}+\dfrac{10}{c} & -\dfrac{8}{5c^2}-\dfrac{40}{a^2c}+\dfrac{16}{a^4} \\
\dfrac{2a^2}{5c^2}+\dfrac{10}{c}-\dfrac{4}{a^2} & \underline{-\dfrac{8}{5c^2}-\dfrac{40}{a^2c}+\dfrac{16}{a^4}}
\end{array}
$$

Therefore the square roots are $\pm\left(\dfrac{a^2}{5c^2}+\dfrac{5}{c}-\dfrac{4}{a^2}\right)$.

21.

$$
\begin{array}{l|l}
 & \dfrac{a^2}{4c^4}-\dfrac{3a}{c^2}+9+\dfrac{2}{5ac}-\dfrac{12c}{5a^2}+\dfrac{4c^2}{25a^4}\;\Big|\;\underline{\dfrac{a}{2c^2}-3+\dfrac{2c}{5a^2}} \\
 & \underline{\dfrac{a^2}{4c^4}} \\
\dfrac{a}{c^2} & -\dfrac{3a}{c^2}+9 \\
\dfrac{a}{c^2}-3 & \underline{-\dfrac{3a}{c^2}+9} \\
\dfrac{a}{c^2}-6 & +\dfrac{2}{5ac}-\dfrac{12c}{5a^2}+\dfrac{4c^2}{25a^4} \\
\dfrac{a}{c^2}-6+\dfrac{2c}{5a^2} & \underline{+\dfrac{2}{5ac}-\dfrac{12c}{5a^2}+\dfrac{4c^2}{25a^4}}
\end{array}
$$

Therefore the square roots are $\pm\left(\dfrac{a}{2c^2}-3+\dfrac{2c}{5a^2}\right)$.

22.

$$
\begin{array}{l|l}
 & x^4-8x^3+24x^2-32x+16\;\big|\;\underline{x^2-4x+4} \\
 & \underline{x^4} \\
2x^2 & -8x^3+24x^2 \\
2x^2-4x & \underline{-8x^3+16x^2} \\
2x^2-8x & 8x^2-32x+16 \\
2x^2-8x+4 & \underline{8x^2-32x+16}
\end{array}
\qquad
\begin{array}{l|l}
 & x^2-4x+4\;\big|\;\underline{x-2} \\
 & \underline{x^2} \\
2x & -4x+4 \\
2x-2 & \underline{-4x+4}
\end{array}
$$

Therefore the fourth roots are $\pm(x-2)$.

$$
\begin{array}{l|l}
 & c^4 - 4c^3d + 6c^2d^2 - 4cd^3 + d^4 \;\underline{|\, c^2 - 2cd + d^2} \\
 & c^4 \\
2c^2 & -4c^3d + 6c^2d^2 \\
2c^2 - 2cd & -4c^3d + 4c^2d^2 \\
2c^2 - 4cd & \qquad 2c^2d^2 - 4cd^3 + d^4 \\
2c^2 - 4cd + d^2 & \qquad 2c^2d^2 - 4cd^3 + d^4
\end{array}
\qquad
\begin{array}{l|l}
 & c^2 - 2cd + d^2 \;\underline{|\, c - d} \\
 & c^2 \\
2c & -2cd + d^2 \\
2c - d & -2cd + d^2
\end{array}
$$

Therefore the fourth roots are $\pm (c - d)$.

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The square root of:

1. $6241 = \pm 79$.
2. $16129 = \pm 127$.
3. $223{,}729 = \pm 473$.
4. $2 = \pm 1.414+$.
5. $5 = \pm 2.236+$.
6. $7.135 = \pm 2.671+$.
7. $.6279 = \pm .792+$.
8. $.0451 = \pm .212+$.
9. $.0035 = \pm .059+$.
10. $\frac{11}{5} = \pm 1.483+$.
11. $1\frac{6}{7} = \pm 1.362+$.
12. $\frac{2}{21} = \pm .308+$.

13. $\sqrt{(84)^2 + (13)^2} = 85$, hypotenuse; $\dfrac{84 \cdot 13}{2} = 546$, area.

14. $\sqrt{(133)^2 + (156)^2} = 205$, hypotenuse; $\dfrac{133 \cdot 156}{2} = 10{,}374$, area.

15. $\sqrt{(645)^2 + (812)^2} = 1037$, hypotenuse; $\dfrac{645 \cdot 812}{2} = 261{,}870$, area.

16. $\sqrt{(65)^2 - (56)^2} = 33$, leg; $\dfrac{56 \cdot 33}{2} = 924$, area.

17. $\sqrt{(397)^2 - (325)^2} = 228$, leg; $\dfrac{325 \cdot 228}{2} = 37{,}050$, area.

18. $\sqrt{(143)^2 + (24)^2} = 145$ feet.

19. $\sqrt{(401)^2 - (399)^2} = 40$, side; $40 \cdot 399 = 15{,}960$, area.

20. $\sqrt{(677)^2 - (52)^2} = 675$, side. Then 1454 is the perimeter.

21. Let $s =$ one side in feet.
Then $s + 21 =$ the other side in feet.
Therefore $2s + 2(s + 21) = 102$.
Whence $s = 15$,
and $s + 21 = 36$.
$\sqrt{(15)^2 + (36)^2} = 39$, the diagonal in feet.

22. Let $s =$ the number of meters in the side.
Then $2s^2 = (74)^2$.
Whence $s = 52.325+$ meters.

23. $\sqrt{(52)^2 + (52)^2} = 73.539+$ inches.

24. Let $s =$ the width.
Then $2.4\,s =$ the length.
Therefore
$(s)^2 + (2.4\,s)^2 = (52)^2.$
Whence $s = 20,$
and $2.4\,s = 48$, the length.

25. Let $l =$ the length.

Then $\frac{3\,l}{4} =$ the width.

Whence

$$l^2 + \left(\frac{3\,l}{4}\right)^2 = (100)^2.$$

Then $l = 80,$

and $\frac{3\,l}{4} = 60.$

The area $= 60 \cdot 80 = 4800.$

26. Let $w =$ the width.
Then
$(2\,w)^2 - (w)^2 = (10)^2.$
Whence $w = 5.773+.$

27. $\sqrt{6^2 - 3^2} = 5.196+$, BD.
Then
the area $= 15.588+.$

28. $\sqrt{(10)^2 - (5)^2} = 8.660+$, BD.
Then the area $= 43.30+.$

29. $(AB)^2 - \left(\frac{AB}{2}\right)^2 = (10)^2.$

Whence $AB = 11.546+.$
Area $= 57.734+.$

30. Let $a =$ the altitude.
Then
$(12)^2 - (6)^2 = a^2,$
or $a = 10.392+.$

31. Let $s =$ the side in centimeters.

Then $s^2 - \frac{s^2}{4} = (25)^2,$

or $s = 28.86+.$

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1. $x^{\frac{4}{3}} = \sqrt[3]{x^4}.$
2. $x^{\frac{5}{2}} = \sqrt{x^5}.$
3. $(cd)^{\frac{3}{2}} = \sqrt{(cd)^3}.$
4. $(2\,x)^{\frac{1}{3}} = \sqrt[3]{2\,x}.$
5. $2\,x^{\frac{1}{3}} = 2\sqrt[3]{x}.$
6. $5\,a^{\frac{3}{4}} = 5\sqrt[4]{a^3}.$
7. $(5\,a)^{\frac{3}{4}} = \sqrt[4]{(5\,a)^3}.$
8. $3\,rx^{\frac{2}{3}} = 3\,r\sqrt[3]{x^2}.$
9. $h^{\frac{2}{3}}k^{\frac{2}{3}} = \sqrt[3]{h^2k^2}.$
10. $7\,s^{\frac{2}{3}}(t + w)^{\frac{1}{2}} = 7\sqrt[3]{s^2} \cdot \sqrt{t + w}.$
11. $3\,a^{\frac{5}{2}}(bc)^{\frac{1}{3}} = 3\sqrt{a^5} \cdot \sqrt[3]{bc}.$
12. $5^{\frac{1}{2}}x^{\frac{2}{3}}k^{\frac{5}{4}} = \sqrt{5} \cdot \sqrt[3]{x^2} \cdot \sqrt[4]{k^5}.$
13. $4^{\frac{1}{3}}t^{\frac{1}{a}} = \sqrt[3]{4} \cdot \sqrt[a]{t}.$
14. $2\,x^{\frac{1}{b}}y^{\frac{c}{d}} = 2\sqrt[b]{x} \cdot \sqrt[d]{y^c}.$
15. $25^{\frac{1}{2}} = 5.$
16. $27^{\frac{1}{3}} = 3.$
17. $16^{\frac{1}{4}} = 2.$
18. $4^{\frac{3}{2}} = 8.$
19. $64^{\frac{2}{3}} = 16.$
20. $125^{\frac{2}{3}} = 25.$
21. $(-8)^{\frac{4}{3}} = 16.$
22. $32^{\frac{1}{5}} = 2.$
23. $81^{\frac{3}{4}} = 27.$
24. $(-216)^{\frac{2}{3}} = 36.$
25. $(\frac{1}{16})^{\frac{1}{2}} = \frac{1}{4}.$
26. $(\frac{1}{25})^{\frac{1}{2}} = \frac{1}{5}.$
27. $(\frac{1}{16})^{\frac{1}{4}} = \frac{1}{2}.$
28. $25^{\frac{1}{2}} \cdot 4^{\frac{5}{2}} = 160.$

29. $4^{\frac{1}{2}} \cdot (\frac{1}{4})^{\frac{1}{2}} = 1.$

30. $(-32)^{\frac{4}{5}}(-64)^{\frac{1}{3}} = -64.$

31. $36^{\frac{1}{2}} \cdot (\frac{1}{9})^{\frac{1}{2}} = 2.$

32. $9^{\frac{1}{2}} \cdot (\frac{1}{27})^{\frac{1}{3}} = 1.$

33. $2(\frac{4}{9})^{\frac{1}{2}} \cdot (\frac{1}{8})^{\frac{1}{3}} = \frac{2}{3}.$

34. $\sqrt[5]{32} \cdot 4^{\frac{1}{2}} = 4.$

35. $\sqrt[3]{8^2} \cdot 25^{\frac{3}{2}} = 500.$

36. $121^{\frac{1}{2}} \cdot \sqrt{\frac{25}{121}} = 5.$

37. $(-343)^{\frac{1}{3}} \cdot \sqrt{\frac{1}{49}} = -1.$

38. $\frac{2}{3}(\frac{169}{144})^{\frac{1}{2}} \div (\frac{1}{216})^{\frac{1}{3}} = \frac{13}{3}.$

39. $\sqrt[2]{a^3} = a^{\frac{3}{2}}.$

40. $\sqrt[2]{ax^4} = a^{\frac{1}{2}}x^2.$

41. $3\sqrt{2x^5} = 3 \cdot 2^{\frac{1}{2}} x^{\frac{5}{2}}.$

42. $\sqrt{9x} = 3x^{\frac{1}{2}}.$

43. $5\sqrt{16ax^2} = 20a^{\frac{1}{2}}x.$

44. $\sqrt[3]{a^2} = a^{\frac{2}{3}}.$

45. $\sqrt[3]{ax^4} = a^{\frac{1}{3}}x^{\frac{4}{3}}.$

46. $2\sqrt[3]{2x^2} = 2^{\frac{4}{3}}x^{\frac{2}{3}}.$

47. $3\sqrt[3]{8x^4} = 6x^{\frac{4}{3}}.$

48. $4\sqrt[3]{27ax^3} = 12a^{\frac{1}{3}}x.$

49. $2\sqrt[4]{a^2x^3} = 2a^{\frac{1}{2}}x^{\frac{3}{4}}.$

50. $4\sqrt[4]{16x} = 8x^{\frac{1}{4}}.$

51. $7\sqrt[5]{rs^7} = 7r^{\frac{1}{5}}s^{\frac{7}{5}}.$

52. $5a\sqrt[5]{32m^2} = 10am^{\frac{2}{5}}.$

53. $12x^2\sqrt[3]{ax^3} = 12a^{\frac{1}{3}}x^3.$

54. $c\sqrt[2]{(de)^3} = cd^{\frac{3}{2}}e^{\frac{3}{2}}.$

55. $uv\sqrt[2]{(u+v)^5} = uv(u+v)^{\frac{5}{2}}.$

56. $3\sqrt[2]{a^3} \cdot \sqrt[3]{x^2} = 3a^{\frac{3}{2}}x^{\frac{2}{3}}.$

57. $2a\sqrt[5]{ax^2} \cdot \sqrt[3]{2m} = 2^{\frac{4}{3}}a^{\frac{6}{5}}x^{\frac{2}{5}}m^{\frac{1}{3}}.$

58. $\sqrt[n]{x^a} \cdot \sqrt[n]{y^b} = x^{\frac{a}{n}}y^{\frac{b}{n}}.$

59. $\sqrt[n]{x^a} \cdot \sqrt[n]{x^{3a}} = x^{\frac{4a}{n}}.$

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1. $\sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}.$

2. $\sqrt{32} = \sqrt{16 \cdot 2} = 4\sqrt{2}.$

3. $\sqrt{75} = \sqrt{25 \cdot 3} = 5\sqrt{3}.$

4. $2\sqrt{50} = 2\sqrt{25 \cdot 2} = 10\sqrt{2}.$

5. $\sqrt[3]{40} = \sqrt[3]{8 \cdot 5} = 2\sqrt[3]{5}.$

6. $\sqrt[3]{54} = \sqrt[3]{27 \cdot 2} = 3\sqrt[3]{2}.$

7. $\sqrt{20} = \sqrt{4 \cdot 5} = 2\sqrt{5}.$

8. $\sqrt[3]{3000} = \sqrt[3]{1000 \cdot 3} = 10\sqrt[3]{3}.$

9. $2\sqrt[3]{24} = 2\sqrt[3]{8 \cdot 3} = 4\sqrt[3]{3}.$

10. $\sqrt{\frac{1}{2}} = \sqrt{\frac{2}{4}} = \frac{1}{2}\sqrt{2}.$

11. $\sqrt{\frac{2}{5}} = \sqrt{\frac{10}{25}} = \frac{1}{5}\sqrt{10}.$

12. $\sqrt{\frac{3}{8}} = \sqrt{\frac{6}{16}} = \frac{1}{4}\sqrt{6}.$

13. $3\sqrt{\frac{9}{2}} = 9\sqrt{\frac{2}{4}} = \frac{9}{2}\sqrt{2}.$

14. $5\sqrt{\frac{3}{5}} = 5\sqrt{\frac{15}{25}} = \frac{5}{5}\sqrt{15} = \sqrt{15}.$

15. $\sqrt[3]{\frac{1}{2}} = \sqrt[3]{\frac{4}{8}} = \frac{1}{2}\sqrt[3]{4}.$

16. $4\sqrt[3]{\frac{1}{4}} = 4\sqrt[3]{\frac{2}{8}} = \frac{4}{2}\sqrt[3]{2} = 2\sqrt[3]{2}.$

17. $8\sqrt[3]{\frac{7}{4}} = 8\sqrt[3]{\frac{14}{8}} = \frac{8}{2}\sqrt[3]{14} = 4\sqrt[3]{14}.$

18. $\sqrt{1 - (\frac{1}{2})^2} = \sqrt{\frac{3}{4}} = \frac{1}{2}\sqrt{3}.$

19. $\sqrt{1 + (\frac{1}{3})^2} = \sqrt{\frac{10}{9}} = \frac{1}{3}\sqrt{10}.$

20. $\sqrt{2 + (\frac{3}{5})^2} = \sqrt{\frac{59}{25}} = \frac{1}{5}\sqrt{59}.$

21. $\sqrt[3]{1 - (\frac{1}{2})^2} = \sqrt[3]{\frac{3}{4}} = \sqrt[3]{\frac{6}{8}} = \frac{1}{2}\sqrt[3]{6}.$

22. $\sqrt[3]{4 + (\frac{2}{3})^2} = \sqrt[3]{\frac{40}{9}} = 2\sqrt[3]{\frac{5}{9}} = \frac{2}{3}\sqrt[3]{15}.$

23. $\sqrt[3]{1 + (\frac{1}{3})^3} = \sqrt[3]{\frac{28}{27}} = \frac{1}{3}\sqrt[3]{28}.$

24. $\sqrt[4]{25a^2} = \sqrt[4]{5^2 \cdot a^2} = \sqrt{5a}.$

25. $3\sqrt{4x^5} = 3\sqrt{4x^4} \cdot \sqrt{x} = 6x^2\sqrt{x}.$

26. $a\sqrt[6]{8x^3} = a\sqrt[6]{2^3x^3} = a\sqrt{2x}.$

27. $x\sqrt[4]{49a^2x^2} = x\sqrt[4]{(7ax)^2} = x\sqrt{7ax}.$

28. $\sqrt[6]{125x^{3a}y^{3m}} = \sqrt[6]{5^3 \cdot x^{3a} \cdot y^{3m}} = \sqrt{5x^ay^m}.$

29. $\sqrt{\dfrac{a^3}{27}} = \sqrt{3\,a \cdot \dfrac{a^2}{81}} = \dfrac{a}{9}\sqrt{3\,a}.$

30. $a\sqrt[3]{\dfrac{5}{2\,a^2}} = a\sqrt[3]{\dfrac{20\,a}{8\,a^3}} = \dfrac{a}{2\,a}\sqrt[3]{20\,a} = \dfrac{1}{2}\sqrt[3]{20\,a}.$

31. $\sqrt{R^2 - \left(\dfrac{R}{2}\right)^2} = \sqrt{\dfrac{3\,R^2}{4}} = \dfrac{R}{2}\sqrt{3}.$

32. $\sqrt{R^2 + \left(\dfrac{R}{2}\right)^2} = \sqrt{\dfrac{5\,R^2}{4}} = \dfrac{R}{2}\sqrt{5}.$

33. $\sqrt{\left(a + \dfrac{a}{3}\right)\left(a - \dfrac{a}{3}\right)} = \sqrt{a^2 - \dfrac{a^2}{9}} = \sqrt{\dfrac{8\,a^2}{9}} = \dfrac{2\,a}{3}\sqrt{2}.$

34. $\sqrt[3]{R^3 - \left(\dfrac{R}{2}\right)^3} = \sqrt[3]{\dfrac{7\,R^3}{8}} = \dfrac{R}{2}\sqrt[3]{7}.$

35. $(x-y)\sqrt{\dfrac{x+y}{x-y}} = (x-y)\sqrt{\dfrac{(x^2-y^2)}{(x-y)^2}} = \dfrac{(x-y)}{(x-y)}\sqrt{x^2-y^2} = \sqrt{x^2-y^2}.$

36. $\sqrt{4 + 4\sqrt{2}} = \sqrt{4(1 + \sqrt{2})} = 2\sqrt{1 + \sqrt{2}}.$

37. $\sqrt{8 - 4\sqrt{2}} = \sqrt{4\,(2 - \sqrt{2})} = 2\sqrt{2 - \sqrt{2}}.$

38. $\sqrt{18 + 9\sqrt{3}} = \sqrt{9\,(2 + \sqrt{3})} = 3\sqrt{2 + \sqrt{3}}.$

39. $\sqrt{25\sqrt{5} - 100} = \sqrt{25\,(\sqrt{5} - 4)} = 5\sqrt{\sqrt{5} - 4}.$

40. $\sqrt[3]{16 + 8\sqrt{2}} = \sqrt[3]{8\,(2 + \sqrt{2})} = 2\sqrt[3]{2 + \sqrt{2}}.$

41. $\sqrt[3]{81 + 3\sqrt[2]{243}} = \sqrt[3]{81 + 27\sqrt{3}} = \sqrt[3]{27\,(3 + \sqrt{3})} = 3\sqrt[3]{3 + \sqrt{3}}.$

42. $\sqrt{R^2 - 2\,R^2\sqrt{2}} = \sqrt{R^2\,(1 - 2\sqrt{2})} = R\sqrt{1 - 2\sqrt{2}}.$

43. $\sqrt[4]{32 - 64\sqrt{3}} = \sqrt[4]{16\,(2 - 4\sqrt{3})} = 2\sqrt[4]{2 - 4\sqrt{3}}.$

44. $\sqrt{\dfrac{R^2 + R^2\sqrt{3}}{2}} = \sqrt{\dfrac{2\,R^2\,(1 + \sqrt{3})}{4}} = \dfrac{R}{2}\sqrt{2 + 2\sqrt{3}}.$

45. $\sqrt{R^2 - \left(\dfrac{R}{2}\right)^2\sqrt{2}} = \sqrt{\dfrac{4\,R^2 - R^2\sqrt{2}}{4}} = \sqrt{\dfrac{R^2}{4}(4 - \sqrt{2})} = \dfrac{R}{2}\sqrt{4 - \sqrt{2}}.$

46. $\sqrt{R^2\sqrt{2} - \dfrac{R^2}{2}} = \sqrt{\dfrac{4\,R^2\sqrt{2} - 2\,R^2}{4}} = \dfrac{1}{2}\sqrt{R^2(4\sqrt{2} - 2)} = \dfrac{R}{2}\sqrt{4\sqrt{2} - 2}.$

47. $\sqrt{\dfrac{R^2}{3} - R^2\sqrt{3}} = \sqrt{\dfrac{3\,R^2 - 9\,R^2\sqrt{3}}{9}} = \dfrac{1}{3}\sqrt{R^2\,(3 - 9\sqrt{3})} = \dfrac{R}{3}\sqrt{3 - 9\sqrt{3}}.$

48. $2\sqrt{2} = \sqrt{4} \cdot \sqrt{2} = \sqrt{8}.$

49. $3\sqrt{5} = \sqrt{9} \cdot \sqrt{5} = \sqrt{45}.$

50. $3\sqrt[3]{4} = \sqrt[3]{27} \cdot \sqrt[3]{4} = \sqrt[3]{108}.$

51. $4\sqrt[3]{2} = \sqrt[3]{64}\,\sqrt[3]{2} = \sqrt[3]{128}.$

52. $3\sqrt[3]{\tfrac{1}{3}} = \sqrt[3]{27}\,\sqrt[3]{\tfrac{1}{3}} = \sqrt[3]{\tfrac{27}{3}} = \sqrt[3]{9}.$

53. $x^2\sqrt{x^3} = \sqrt{x^4}\,\sqrt{x^3} = \sqrt{x^7}.$

54. $2x^2\sqrt[3]{x^2} = \sqrt[3]{8x^6} \cdot \sqrt[3]{x^2} = \sqrt[3]{8x^8}.$

55. $\frac{a}{2}\sqrt[3]{\frac{4}{a^2}} = \sqrt[3]{\frac{a^3}{8}} \cdot \sqrt[3]{\frac{4}{a^2}} = \sqrt[3]{\frac{a}{2}}.$

56. $(a+2)\sqrt{\frac{1}{a^2-4}} = \sqrt{(a+2)^2}\sqrt{\frac{1}{a^2-4}} = \sqrt{\frac{a+2}{a-2}}.$

57. $\frac{x+3}{ax}\sqrt[3]{\frac{a^2x^2}{(x+3)^2}} = \sqrt[3]{\frac{(x+3)^3}{(ax)^3}}\sqrt[3]{\frac{a^2x^2}{(x+3)^2}} = \sqrt[3]{\frac{x+3}{ax}}.$

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1. $\sqrt{27} + \sqrt{12} = 3\sqrt{3} + 2\sqrt{3} = 5\sqrt{3}.$

2. $\sqrt{45} - \sqrt{20} = 3\sqrt{5} - 2\sqrt{5} = \sqrt{5}.$

3. $2\sqrt{200} - 3\sqrt{8} = 20\sqrt{2} - 6\sqrt{2} = 14\sqrt{2}.$

4. $\sqrt[3]{56} + 2\sqrt[3]{189} = 2\sqrt[3]{7} + 6\sqrt[3]{7} = 8\sqrt[3]{7}.$

5. $2\sqrt[3]{320} - \sqrt[2]{50} = 8\sqrt[3]{5} - 5\sqrt{2}.$

6. $\sqrt[4]{25} - \sqrt[2]{50} = \sqrt{5} - 5\sqrt{2}.$

7. $\sqrt[4]{32} + 5\sqrt[4]{162} = 2\sqrt[4]{2} + 15\sqrt[4]{2} = 17\sqrt[4]{2}.$

8. $5\sqrt{\tfrac{1}{2}} - \tfrac{3}{2}\sqrt{2} = \tfrac{5}{2}\sqrt{2} - \tfrac{3}{2}\sqrt{2} = \sqrt{2}.$

9. $\sqrt{\tfrac{1}{3}} + 2\sqrt{\tfrac{4}{3}} - 3\sqrt{\tfrac{2}{3}} = \tfrac{1}{3}\sqrt{3} + \tfrac{4}{3}\sqrt{3} - \sqrt{6} = \tfrac{5}{3}\sqrt{3} - \sqrt{6}.$

10. $\sqrt{\tfrac{3}{10}} - \sqrt{120} - 2\sqrt{\tfrac{6}{5}} = \tfrac{1}{10}\sqrt{30} - 2\sqrt{30} - \tfrac{2}{5}\sqrt{30} = -\tfrac{23}{10}\sqrt{30}.$

11. $\sqrt{\tfrac{5}{18}} + 2\sqrt{\tfrac{32}{5}} - \sqrt{\tfrac{10}{9}} = \tfrac{1}{6}\sqrt{10} + \tfrac{8}{5}\sqrt{10} - \tfrac{1}{3}\sqrt{10} = \tfrac{43}{30}\sqrt{10}.$

12. $\sqrt[3]{\tfrac{6}{8}} - \sqrt[3]{12} + \sqrt[3]{6} = \tfrac{1}{2}\sqrt[3]{6} - \sqrt[3]{12} + \sqrt[3]{6} = \tfrac{3}{2}\sqrt[3]{6} - \sqrt[3]{12}.$

13. $a\sqrt[4]{4} + \sqrt{8a^2} = a\sqrt{2} + 2a\sqrt{2} = 3a\sqrt{2}.$

14. $R - \sqrt{\frac{3R^2}{4}} = R - \frac{R}{2}\sqrt{3}.$

15. $2x\sqrt[3]{54x} - 3\sqrt[3]{16x^4} + \sqrt[6]{4x^2} = 6x\sqrt[3]{2x} - 6x\sqrt[3]{2x} + \sqrt[3]{2x} = \sqrt[3]{2x}.$

16. $\sqrt[3]{81x^7} + x\sqrt[3]{375x^4} - \sqrt[12]{16x^4} = 3x^2\sqrt[3]{3x} + 5x^2\sqrt[3]{3x} - \sqrt[3]{2x}$

$= 8x^2\sqrt[3]{3x} - \sqrt[3]{2x}.$

17. $\sqrt{a^3bc} - a\sqrt{abc} + ac\sqrt{\frac{b}{ac}} = a\sqrt{abc} - a\sqrt{abc} + \sqrt{abc} = \sqrt{abc}.$

18. $\sqrt{(m+n)^3} - n\sqrt[4]{(m+n)^2} = (m+n)\sqrt{m+n} - n\sqrt{(m+n)}$

$= m\sqrt{m+n}.$

19. $\sqrt[3]{(a+b)^4} - \sqrt[3]{8a^3(a+b)} + \sqrt[6]{a^2+2ab+b^2}$

$= (a+b)\sqrt[3]{a+b} - 2a\sqrt[3]{a+b} + \sqrt[3]{a+b}$

$= b\sqrt[3]{a+b} - a\sqrt[3]{a+b} + \sqrt[3]{a+b}$

$= (b-a+1)\sqrt[3]{a+b}.$

20. $\sqrt[2]{a^3+4a^2+4a}-\sqrt[2]{a^3}-\dfrac{2}{a^3}\sqrt{a^7}=(a+2)\sqrt{a}-a\sqrt{a}-2\sqrt{a}=0.$

21. $\sqrt{x^3y^3}-x^2y^2\sqrt{\dfrac{1}{xy}}+xy\sqrt{2+\dfrac{x^2+y^2}{xy}}$
$$=xy\sqrt{xy}-xy\sqrt{xy}+\sqrt{(x^2+2xy+y^2)xy}$$
$$=(x+y)\sqrt{xy}.$$

22. $rs\sqrt[3]{rs}+\sqrt[3]{\dfrac{1}{r^2s^2}}-2\sqrt[3]{r^4s^4}=rs\sqrt[3]{rs}+\dfrac{1}{rs}\sqrt[3]{rs}-2rs\sqrt[3]{rs}$
$$=\frac{1}{rs}\sqrt[3]{rs}-rs\sqrt[3]{rs}$$
$$=\sqrt{rs}\left(\frac{1}{rs}-rs\right)$$
$$=\frac{1-r^2s^2}{rs}\sqrt{rs}.$$

23. $2\sqrt{\dfrac{y}{x}}-\sqrt{\dfrac{x}{y}}+\sqrt{2+\dfrac{x^2+y^2}{xy}}=\dfrac{2}{x}\sqrt{xy}-\dfrac{1}{y}\sqrt{xy}+\dfrac{x+y}{xy}\sqrt{xy}$
$$=\sqrt{xy}\left(\frac{2}{x}-\frac{1}{y}+\frac{x+y}{xy}\right)=\frac{3}{x}\sqrt{xy}.$$

24. $\sqrt{3x^2-18x+27}-\sqrt{27(x^2+2x+1)}=(x-3)\sqrt{3}-(3x+3)\sqrt{3}$
$$=-2x\sqrt{3}-6\sqrt{3}=\sqrt{3}(-2x-6).$$

25. $\sqrt[3]{(a-3)^2(5a-15)}+\sqrt[3]{40}=(a-3)\sqrt[3]{5}+2\sqrt[3]{5}$
$$=a\sqrt[3]{5}-\sqrt[3]{5}=\sqrt[3]{5}(a-1).$$

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1. $\sqrt{2}\cdot\sqrt{8}=\sqrt{16}=4.$
2. $\sqrt{3}\cdot\sqrt{27}=\sqrt{81}=9.$
3. $5^{\frac{1}{2}}\cdot 20^{\frac{1}{2}}=100^{\frac{1}{2}}=10.$
4. $18^{\frac{1}{2}}\cdot 8^{\frac{1}{2}}=144^{\frac{1}{2}}=12.$
5. $\sqrt[3]{16}\cdot\sqrt[3]{4}=\sqrt[3]{64}=4.$
6. $\sqrt[3]{4}\cdot\sqrt[3]{12}=\sqrt[3]{48}=2\sqrt[3]{6}.$
7. $(100)^{\frac{1}{3}}(30)^{\frac{1}{3}}=3000^{\frac{1}{3}}=10\sqrt[3]{3}.$
8. $\sqrt[4]{8}\cdot\sqrt[4]{32}=\sqrt[4]{2^3\cdot 2^5}=\sqrt[4]{2^8}$
$$=2^2=4.$$
9. $\sqrt{\frac{12}{25}}\cdot\sqrt{75}=\sqrt{36}=6.$
10. $\sqrt{\frac{2}{3}}\cdot\sqrt{\frac{27}{8}}=\sqrt{\frac{9}{4}}=\frac{3}{2}.$
11. $\sqrt{11}\cdot\sqrt{\frac{1}{11}}=\sqrt{1}=1.$
12. $a^{\frac{1}{2}}\cdot(bc)^{\frac{1}{2}}=\sqrt{abc}.$
13. $2\sqrt{x}\cdot\sqrt[2]{4x^3}=2\sqrt{4x^4}=4x^2.$
14. $5\sqrt[3]{2a}\cdot 3\sqrt[3]{16a}=15\sqrt[3]{32a^2}$
$$=30\sqrt[3]{4a^2}.$$
15. $2\sqrt{Rs}\cdot 7\sqrt{r^3s^3t^2}=14\sqrt{r^3s^4t^2R}$
$$=14rs^2t\sqrt{Rr}.$$
16. $\sqrt{\dfrac{a}{x}}\cdot\sqrt{\dfrac{4x}{a}}=\sqrt{4}=2.$
17. $\sqrt{75a}\cdot(45a)^{\frac{1}{2}}=\sqrt{3\cdot 5^2\cdot 3^2\cdot 5a^2}=15a\sqrt{15}.$
18. $\sqrt{2u}\cdot\sqrt{4v}\cdot\sqrt{6uv}=\sqrt{48u^2v^2}=4uv\sqrt{3}.$

19. $5\sqrt{3m}\cdot 5\sqrt{3m} = 25\sqrt{9m^2} = 75m.$

20. $(3\sqrt{3x})^2 = 9\cdot 3x = 27x.$

21. $\sqrt[3]{2}\cdot\sqrt{2} = 2^{\frac{1}{3}}\cdot 2^{\frac{1}{2}} = 2^{\frac{5}{6}} = \sqrt[6]{2^5} = \sqrt[6]{32}.$

22. $\sqrt{3}\cdot\sqrt[3]{3} = 3^{\frac{1}{2}}\cdot 3^{\frac{1}{3}} = 3^{\frac{5}{6}} = \sqrt[6]{3^5} = \sqrt[6]{243}.$

23. $\sqrt[3]{2}\cdot\sqrt{3} = 2^{\frac{1}{3}}\cdot 3^{\frac{1}{2}} = 2^{\frac{2}{6}}\cdot 3^{\frac{3}{6}} = \sqrt[6]{2^2\cdot 3^3} = \sqrt[6]{108}.$

24. $\sqrt{2}\cdot\sqrt[3]{3} = 2^{\frac{1}{2}}\cdot 3^{\frac{1}{3}} = 2^{\frac{3}{6}}\cdot 3^{\frac{2}{6}} = \sqrt[6]{2^3\cdot 3^2} = \sqrt[6]{72}.$

25. $\sqrt[3]{4}\cdot\sqrt{2} = 2^{\frac{2}{3}}\cdot 2^{\frac{1}{2}} = 2^{\frac{7}{6}} = 2\sqrt[6]{2}.$

26. $\sqrt{8}\cdot\sqrt[3]{8} = 2^{\frac{3}{2}}\cdot 2 = 4\sqrt{2}.$

27. $\sqrt{3}\cdot\sqrt[3]{24} = 3^{\frac{1}{2}}\cdot 2\cdot 3^{\frac{1}{3}} = 2\cdot 3^{\frac{5}{6}} = 2\sqrt[6]{243}.$

28. $\sqrt{a}\cdot\sqrt[3]{a} = a^{\frac{1}{2}}\cdot a^{\frac{1}{3}} = a^{\frac{5}{6}} = \sqrt[6]{a^5}.$

29. $\sqrt{a^3}\cdot\sqrt[3]{a^2} = a^{\frac{3}{2}}\cdot a^{\frac{2}{3}} = a^{\frac{13}{6}} = a^2\sqrt[6]{a}.$

30. $\sqrt{2a}\cdot\sqrt[3]{2a} = (2a)^{\frac{1}{2}}(2a)^{\frac{1}{3}} = (2a)^{\frac{5}{6}} = \sqrt[6]{(2a)^5} = \sqrt[6]{32a^5}.$

31. $\sqrt[3]{4x^2}\cdot\sqrt{2x} = 2^{\frac{2}{3}}\cdot x^{\frac{2}{3}}\cdot 2^{\frac{1}{2}}\cdot x^{\frac{1}{2}} = 2^{\frac{4}{6}}\cdot x^{\frac{4}{6}}\cdot 2^{\frac{3}{6}}\cdot x^{\frac{3}{6}} = 2^{\frac{7}{6}}x^{\frac{7}{6}} = 2x\sqrt[6]{2x}.$

32. $$\sqrt[3]{\frac{5x}{a}}\cdot\sqrt[2]{\frac{a^3}{5x^2}} = \left(\frac{5x}{a}\right)^{\frac{1}{3}}\left(\frac{a}{5}\right)^{\frac{1}{2}}\cdot\frac{a}{x} = \frac{a}{x}\left(\frac{5x}{a}\right)^{\frac{2}{6}}\left(\frac{a}{5}\right)^{\frac{3}{6}} = \frac{a}{x}\sqrt[6]{\left(\frac{5x}{a}\right)^2\left(\frac{a}{5}\right)^3}$$
$$= \frac{a}{x}\sqrt[6]{\frac{x^2a}{5}} = \frac{a}{5x}\sqrt[6]{3125\,ax^2}.$$

33. $(x + a\sqrt{m})\sqrt{m^3} = x\sqrt{m^3} + a\sqrt{m^4} = mx\sqrt{m} + am^2.$

34. $(\sqrt{5a} - \sqrt{10a^2})(\sqrt{5a}) = 5a - \sqrt{50a^3} = 5a - 5a\sqrt{2a}.$

35. $(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3}) = 2 - 3 = -1.$

36.
$$\begin{array}{l}
5\sqrt{3} - 4 \\
\underline{3\sqrt{3} + 8} \\
15\cdot 3 - 12\sqrt{3} \\
\underline{\qquad\quad 40\sqrt{3} - 32} \\
\quad 45 + 28\sqrt{3} - 32 = 13 + 28\sqrt{3}.
\end{array}$$

37.
$$\begin{array}{lllll}
\sqrt{5} - \sqrt{3} & - \sqrt{2} & & & \\
\sqrt{5} + \sqrt{3} & + \sqrt{2} & & & \\
\hline
5 - \sqrt{15} & - \sqrt{10} & & & \\
\quad\ \sqrt{15} & & - 3 & - \sqrt{6} & \\
 & + \sqrt{10} & & - \sqrt{6} & - 2 \\
\hline
5 & & - 3 & - 2\sqrt{6} & - 2 = -2\sqrt{6}.
\end{array}$$

38.

$$\begin{array}{l}
3\sqrt{2}+2\sqrt{3}+\sqrt{30} \\
\underline{\sqrt{2}+\sqrt{3}-\sqrt{5}\qquad\qquad} \\
3\cdot 2+2\sqrt{6}+2\sqrt{15} \\
\qquad 3\sqrt{6} \qquad\qquad +2\cdot 3+3\sqrt{10} \\
\underline{\qquad -5\sqrt{6}-2\sqrt{15} \qquad -3\sqrt{10}} \\
6 \qquad\qquad\qquad +6 \qquad\qquad = 12.
\end{array}$$

39.

$$\frac{6a+b\sqrt{2}}{2}\cdot\frac{6a+b\sqrt{2}}{2}=\frac{36a^2+12ab\sqrt{2}+2b^2}{4}$$
$$=9a^2+3ab\sqrt{2}+\frac{b^2}{2}.$$

40.

$$\begin{array}{l}
R-\frac{R}{2}\sqrt{3} \\
\underline{2R+\frac{3R}{2}\sqrt{3}} \\
2R^2-R^2\sqrt{3} \\
\qquad \underline{\frac{3R^2}{2}\sqrt{3}-\frac{3R^2}{4}\cdot 3} \\
2R^2+\frac{R^2}{2}\sqrt{3}-\frac{9R^2}{4}=\frac{R^2}{2}\sqrt{3}-\frac{R^2}{4}=\frac{R^2}{2}(\sqrt{3}-\tfrac{1}{2}).
\end{array}$$

41. $\sqrt{a+b}\cdot\sqrt{a-b}\cdot\sqrt{2a^2-2b^2}=(a^2-b^2)\sqrt{2}.$

42.

$$\begin{array}{l}
\quad 2\sqrt{x-a}+\sqrt{a} \\
\underline{-3\sqrt{x-a}-5\sqrt{a}} \\
-6(x-a)-3\sqrt{xa-a^2} \\
\underline{\qquad\qquad -10\sqrt{xa-a^2}-5a} \\
-6x+6a-13\sqrt{ax-a^2}-5a=a-6x-13\sqrt{ax-a^2}.
\end{array}$$

43. $(3\sqrt[3]{x-a})(-5\sqrt[3]{x-a})=-15\sqrt[3]{x^2-2ax+a^2}.$

44. $\sqrt{2-\sqrt{2}}\cdot\sqrt{2+\sqrt{2}}=\sqrt{4-2}=\sqrt{2}.$

45. $\sqrt{a-\sqrt{b}}\cdot\sqrt{2a-2\sqrt{b}}=\sqrt{a-\sqrt{b}}\cdot\sqrt{2(a-\sqrt{b})}=(a-\sqrt{b})\sqrt{2}.$

46. $\sqrt{\frac{R}{2}\sqrt{5}+R}\cdot\sqrt{\frac{R}{2}\sqrt{5}-R}=\sqrt{\frac{R^2\cdot 5}{4}-R^2}=\sqrt{\frac{R^2}{4}}=\frac{R}{2}.$

47. $(\sqrt[3]{2})^2=\sqrt[3]{4}.$

48. $(2\sqrt[3]{3})^2=4\sqrt[3]{9}.$

49. $(2\sqrt[3]{12})^2=4\sqrt[3]{144}=8\sqrt[3]{18}.$

50. $\left(\sqrt{2-\sqrt{2}}\right)^2=2-\sqrt{2}.$

51. $\left(3\sqrt{x+\sqrt{3}}\right)^2=9(x+\sqrt{3})=9x+9\sqrt{3}.$

52. $[(2\sqrt[3]{3})^2]^2=(4\sqrt[3]{9})^2=16\sqrt[3]{81}=48\sqrt[3]{3}.$

53. $(2a\sqrt[3]{8x})^2 = 4a^2\sqrt[3]{64x^2} = 16a^2\sqrt[3]{x^2}.$

54. $(\sqrt[3]{2}+\sqrt[3]{3})^2 = \sqrt[3]{4} + 2\sqrt[3]{6} + \sqrt[3]{9}.$

55. $\left(\sqrt[3]{4+4\sqrt{3}}\right)^2 = \sqrt[3]{16+32\sqrt{3}+48} = \sqrt[3]{8(8+4\sqrt{3})} = 2\sqrt[3]{8+4\sqrt{3}}.$

56. $\left(\frac{1}{3}\sqrt[3]{9-9\sqrt{2}}\right)^2 = \frac{1}{9}\sqrt[3]{81-162\sqrt{2}+162} = \frac{1}{9}\sqrt[3]{27(9-6\sqrt{2})}$
$$= \tfrac{1}{3}\sqrt[3]{9-6\sqrt{2}}.$$

57. $(\sqrt{2}+\sqrt[3]{3})^2 = 2 + 2\sqrt{2}\sqrt[3]{3} + \sqrt[3]{9} = 2 + 2\sqrt[6]{72} + \sqrt[3]{9}.$

58. $\left(2\cdot\sqrt[4]{3-\sqrt{2}}\right)^2 = 4\sqrt[2]{3-\sqrt{2}}.$

59. $(2\sqrt{3})^3 = 8\sqrt{27} = 24\sqrt{3}.$

60. $(3\sqrt{2})^3 = 27\sqrt{8} = 54\sqrt{2}.$

61. $[(\sqrt{2})^2]^3 = (2)^3 = 8.$

62. $[(\sqrt[2]{3})^3]^3 = (3^{\frac{1}{2}})^9 = 3^{\frac{9}{2}} = 81\sqrt{3}.$

63. $(\sqrt{3}-\sqrt{6})^3 = (\sqrt{3})^3 - 3(\sqrt{3})^2\sqrt{6} + 3(\sqrt{3})(\sqrt{6})^2 - (\sqrt{6})^3$
$$= 21\sqrt{3} - 15\sqrt{6}.$$

64. $(2\sqrt{2}+\sqrt{3})^3 = (2\sqrt{2})^3 + 3(2\sqrt{2})^2\sqrt{3} + 3(2\sqrt{2})(\sqrt{3})^2 + (\sqrt{3})^3$
$$= 34\sqrt{2} + 27\sqrt{3}.$$

65. $(\sqrt[3]{3}-\sqrt[3]{2})^3 = (\sqrt[3]{3})^3 - 3(\sqrt[3]{3})^2\cdot\sqrt[3]{2} + 3(\sqrt[3]{3})(\sqrt[3]{2})^2 - (\sqrt[3]{2})^3$
$$= 1 - 3\sqrt[3]{18} + 3\sqrt[3]{12}.$$

66. $\left(\sqrt{2+\sqrt{2}}\right)^3 = (2+\sqrt{2})\sqrt{2+\sqrt{2}}.$

67. $(\sqrt[3]{2}-\sqrt{3})^3 = (\sqrt[3]{2})^3 - 3(\sqrt[3]{2})^2\sqrt{3} + 3(\sqrt[3]{2})(\sqrt{3})^2 - (\sqrt{3})^3$
$$= 2 - 3\sqrt[6]{432} + 9\sqrt[3]{2} - 3\sqrt{3}.$$

68. $R^2 + \left(\frac{R}{2}\sqrt{2-\sqrt{2}}\right)^2 = R^2 + \left[\frac{R^2}{4}(2-\sqrt{2})\right]$
$$= R^2 + \frac{R^2}{2} - \frac{R^2\sqrt{2}}{4}$$
$$= \frac{3R^2}{2} - \frac{R^2\sqrt{2}}{4}$$
$$= \frac{R^2}{4}(6-\sqrt{2}).$$

69. $\sqrt{R^2 - \left(\frac{R}{2}\sqrt{5} - \frac{R}{2}\right)^2} = \sqrt{R^2 - \left(\frac{R^2\cdot 5}{4} - \frac{2R^2\sqrt{5}}{4} + \frac{R^2}{4}\right)}$
$$= \sqrt{\frac{4R^2 - 6R^2 + 2R^2\sqrt{5}}{4}}$$
$$= \sqrt{\frac{R^2}{4}(2\sqrt{5}-2)}$$
$$= \frac{R}{2}\sqrt{2\sqrt{5}-2}.$$

70. $$\left[R^2-\left(\frac{R\sqrt{5}-R}{4}\right)^2\right]^{\frac{1}{2}}=\left[R^2-\left(\frac{R^2\cdot 5-2R^2\sqrt{5}+R^2}{16}\right)\right]^{\frac{1}{2}}$$
$$=\tfrac{1}{4}\left[16R^2-6R^2+2R^2\sqrt{5}\right]^{\frac{1}{2}}$$
$$=\tfrac{1}{4}\left[R^2(10+2\sqrt{5})\right]^{\frac{1}{2}}$$
$$=\frac{R}{4}\sqrt{10+2\sqrt{5}}.$$

71. $$\left[\left(R-\frac{R}{2}\sqrt{3}\right)^2+\left(\frac{R}{2}\right)^2\right]^{\frac{1}{2}}=\left[\left(R^2-R^2\sqrt{3}+\frac{3R^2}{4}\right)+\frac{R^2}{4}\right]^{\frac{1}{2}}$$
$$=\left[R^2(2-\sqrt{3})\right]^{\frac{1}{2}}$$
$$=R\sqrt{2-\sqrt{3}}.$$

72. $$\left[R^2-\left(\frac{R\sqrt{2-\sqrt{3}}}{2}\right)^2\right]^{\frac{1}{2}}=\left[R^2-\left(\frac{R^2(2-\sqrt{3})}{4}\right)\right]^{\frac{1}{2}}$$
$$=\tfrac{1}{2}\left[4R^2-2R^2+R^2\sqrt{3}\right]^{\frac{1}{2}}$$
$$=\tfrac{1}{2}\left[R^2(2+\sqrt{3})\right]^{\frac{1}{2}}$$
$$=\frac{R}{2}\sqrt{2+\sqrt{3}}.$$

73. $$\sqrt{R^2-\left(\frac{R\sqrt{2-\sqrt{2}}}{2}\right)^2}=\left[R^2-\frac{R^2(2-\sqrt{2})}{4}\right]^{\frac{1}{2}}$$
$$=\left[R^2-\frac{2R^2}{4}+\frac{R^2\sqrt{2}}{4}\right]^{\frac{1}{2}}$$
$$=\left[\frac{2R^2+R^2\sqrt{2}}{4}\right]^{\frac{1}{2}}$$
$$=\frac{R}{2}\sqrt{2+\sqrt{2}}.$$

74. $$\sqrt{\left(\frac{R\sqrt{2}}{2}\right)^2+\left(R-\frac{R}{2}\sqrt{2}\right)^2}=\sqrt{\frac{2R^2}{4}+\left(R^2-R^2\sqrt{2}+\frac{2R^2}{4}\right)}$$
$$=\sqrt{2R^2-R^2\sqrt{2}}$$
$$=R\sqrt{2-\sqrt{2}}.$$

75. $$\left(\frac{R}{2}\sqrt{2+\sqrt{2}}\right)\left(\frac{R\sqrt{2-\sqrt{2}}}{2}\right)4=\left(\frac{R^2}{4}\sqrt{4-2}\right)4=R^2\sqrt{2}.$$

76. When $x=-2+\sqrt{3}$,
$$x^2+4x+1=(4-4\sqrt{3}+3)+(-8+4\sqrt{3})+1=0.$$

77. When $x=2+\sqrt{5}$,
$$x^2-2x-3=(4+4\sqrt{5}+5)-(4+2\sqrt{5})-3=2+2\sqrt{5}.$$

78. If $x = 2 + \sqrt{5}$,

$$x^2 - 4x - 1 = (4 + 4\sqrt{5} + 5) - (8 + 4\sqrt{5}) - 1 = 0.$$

If $x = 2 - \sqrt{5}$,

$$x^2 - 4x - 1 = (4 - 4\sqrt{5} + 5) - (8 - 4\sqrt{5}) - 1 = 0.$$

79. If $x = \frac{1}{2} + \frac{1}{2}\sqrt{13}$,

$$\begin{aligned} 3x^2 + 3x - 5 &= 3\left(\tfrac{1}{4} + \tfrac{1}{2}\sqrt{13} + \tfrac{13}{4}\right) + \left(\tfrac{3}{2} + \tfrac{3}{2}\sqrt{13}\right) - 5 \\ &= \tfrac{21}{2} + \tfrac{3}{2}\sqrt{13} + \tfrac{3}{2} + \tfrac{3}{2}\sqrt{13} - 5 \\ &= 7 + 3\sqrt{13}. \end{aligned}$$

If $x = \frac{1}{2} - \frac{1}{2}\sqrt{13}$,

$$\begin{aligned} 3x^2 + 3x - 5 &= \left(\tfrac{3}{4} - \tfrac{3}{2}\sqrt{13} + \tfrac{39}{4}\right) + \left(\tfrac{3}{2} - \tfrac{3}{2}\sqrt{13}\right) - 5 \\ &= 7 - 3\sqrt{13}. \end{aligned}$$

80. If $x = 2 + \sqrt{3}$, $4 + 4\sqrt{3} + 3 - 8 - 4\sqrt{3} + 1 = 0$. Yes.

If $x = 2 - \sqrt{3}$, $4 - 4\sqrt{3} + 3 - 8 + 4\sqrt{3} + 1 = 0$. Yes.

81. If $x = -4 + \sqrt{5}$, $16 - 8\sqrt{5} + 5 - 32 + 8\sqrt{5} + 11 = 0$. Yes.

If $x = -4 - \sqrt{5}$, $16 + 8\sqrt{5} + 5 - 32 - 8\sqrt{5} + 11 = 0$. Yes.

82. That $2 \pm \sqrt{3}$ are roots of $x^2 - 4x + 1 = 0$, and that $-4 \pm \sqrt{5}$ are roots of $x^2 + 8x + 11 = 0$.

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A rationalizing factor of:

1. $\sqrt{5}$ is $\sqrt{5}$, $\sqrt{5} \cdot \sqrt{5} = 5$.
2. $3\sqrt{6}$ is $\sqrt{6}$, $3\sqrt{6} \cdot \sqrt{6} = 18$.
3. $2\sqrt{7}$ is $\sqrt{7}$, $2\sqrt{7} \cdot \sqrt{7} = 14$.
4. $\sqrt{8}$ is $\sqrt{2}$, $\sqrt{8} \cdot \sqrt{2} = 4$.
5. $\sqrt{32}$ is $\sqrt{2}$, $\sqrt{32} \cdot \sqrt{2} = 8$.
6. $\sqrt{27}$ is $\sqrt{3}$, $\sqrt{27} \cdot \sqrt{3} = 9$.
7. $\sqrt[3]{2}$ is $\sqrt[3]{4}$, $\sqrt[3]{2} \cdot \sqrt[3]{4} = 2$.
8. $\sqrt[3]{3}$ is $\sqrt[3]{9}$, $\sqrt[3]{3} \cdot \sqrt[3]{9} = 3$.
9. $2\sqrt[3]{5}$ is $\sqrt[3]{25}$, $2\sqrt[3]{5} \cdot \sqrt[3]{25} = 10$.
10. $\sqrt[3]{16}$ is $\sqrt[3]{4}$, $\sqrt[3]{16} \cdot \sqrt[3]{4} = 4$.
11. $\sqrt[3]{25}$ is $\sqrt[3]{5}$, $\sqrt[3]{25} \cdot \sqrt[3]{5} = 5$.
12. $\sqrt[3]{36}$ is $\sqrt[3]{6}$, $\sqrt[3]{36} \cdot \sqrt[3]{6} = 6$.
13. $\sqrt[3]{49}$ is $\sqrt[3]{7}$, $\sqrt[3]{49} \cdot \sqrt[3]{7} = 7$.
14. $\sqrt{2} + 3$ is $\sqrt{2} - 3$, $(\sqrt{2} + 3)(\sqrt{2} - 3) = -7$.
15. $\sqrt{3} - \sqrt{2}$ is $\sqrt{3} + \sqrt{2}$, $(\sqrt{3} - \sqrt{2})(\sqrt{3} + 2) = 1$.
16. $3 + \sqrt{7}$ is $3 - \sqrt{7}$, $(3 + \sqrt{7})(3 - \sqrt{7}) = 2$.
17. $3\sqrt{2} - 5$ is $3\sqrt{2} + 5$, $(3\sqrt{2} - 5)(3\sqrt{2} + 5) = -7$.
18. $4\sqrt{3} - \sqrt{2}$ is $4\sqrt{3} + \sqrt{2}$, $(4\sqrt{3} - \sqrt{2})(4\sqrt{3} + \sqrt{2}) = 46$.
19. $2\sqrt{5} + 7\sqrt{6}$ is $2\sqrt{5} - 7\sqrt{6}$,

    $(2\sqrt{5} + 7\sqrt{6})(2\sqrt{5} - 7\sqrt{6}) = -274$.
20. $\sqrt{x} - \sqrt{a}$ is $\sqrt{x} + \sqrt{a}$, $(\sqrt{x} - \sqrt{a})(\sqrt{x} + \sqrt{a}) = x - a$.
21. $\sqrt{3a} + \sqrt{x}$ is $\sqrt{3a} - \sqrt{x}$, $(\sqrt{3a} + \sqrt{x})(\sqrt{3a} - \sqrt{x}) = 3a - x$.

22. $3\sqrt{x} - a\sqrt{2}$ is $3\sqrt{x} + a\sqrt{2}$, $(3\sqrt{x} - a\sqrt{2})(3\sqrt{x} + a\sqrt{2})$
$= 9x - 2a^2.$

23. $\sqrt{x+a} + \sqrt{x}$ is $\sqrt{x+a} - \sqrt{x}$, $(\sqrt{x+a} + \sqrt{x})(\sqrt{x+a} - \sqrt{x}) = a.$

24. $\sqrt{x} - \sqrt{a-x}$ is $\sqrt{x} + \sqrt{a-x}$, $(\sqrt{x} - \sqrt{a-x})(\sqrt{x} + \sqrt{a-x})$
$= 2x - a.$

25. $\sqrt{2ax - x^2} - \sqrt{ax}$ is $\sqrt{2ax - x^2} + \sqrt{ax}$,
$(\sqrt{2ax - x^2} - \sqrt{ax})(\sqrt{2ax - x^2} + \sqrt{ax}) = ax - x^2.$

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1. $\dfrac{\sqrt{10}}{\sqrt{2}} = \sqrt{5}.$

2. $\dfrac{(18)^{\frac{1}{2}}}{(3)^{\frac{1}{2}}} = 6^{\frac{1}{2}}.$

3. $\dfrac{\sqrt{6}}{\sqrt{18}} = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{\sqrt{9}} = \dfrac{1}{3}\sqrt{3}.$

4. $\dfrac{6}{2\sqrt{2}} = \dfrac{3\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}} = \dfrac{3}{2}\sqrt{2}.$

5. $\dfrac{3\sqrt{12}}{\sqrt{6}} = 3\sqrt{2}.$

6. $\dfrac{8\sqrt{15}}{4\sqrt{5}} = 2\sqrt{3}.$

7. $\dfrac{10}{2\sqrt{5}} = \dfrac{5\cdot\sqrt{5}}{\sqrt{5}\cdot\sqrt{5}} = \sqrt{5}.$

8. $\dfrac{4\sqrt{10}}{8\sqrt{5}} = \dfrac{\sqrt{2}}{2}.$

9. $\dfrac{8}{4\sqrt{3}} = \dfrac{2\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = \dfrac{2}{3}\sqrt{3}.$

10. $\dfrac{3\sqrt{2}}{15\sqrt{8}} = \dfrac{1}{5\sqrt{4}} = \dfrac{1}{10}.$

11. $\dfrac{\sqrt{6} + \sqrt{18}}{3\sqrt{2}} = \dfrac{\sqrt{3} + \sqrt{9}}{3} = \dfrac{\sqrt{3} + 3}{3} = \dfrac{1}{3}\sqrt{3} + 1.$

12. $\dfrac{\sqrt{12} - \sqrt{24}}{2\sqrt{3}} = \dfrac{\sqrt{4} - \sqrt{8}}{2} = \dfrac{2 - 2\sqrt{2}}{2} = 1 - \sqrt{2}.$

13. $\dfrac{6\sqrt{10} + 4\sqrt{15} - \sqrt{20}}{2\sqrt{5}} = 3\sqrt{2} + 2\sqrt{3} - \dfrac{\sqrt{4}}{2} = 3\sqrt{2} + 2\sqrt{3} - 1.$

14. $\dfrac{12 + \sqrt{3} + \sqrt{5}}{\sqrt{6}} = \dfrac{12\sqrt{6} + \sqrt{18} + \sqrt{30}}{6} = 2\sqrt{6} + \dfrac{1}{2}\sqrt{2} + \dfrac{1}{6}\sqrt{30}.$

15. $\dfrac{\sqrt{6} - \sqrt{9} + 18}{2\sqrt{2}} = \dfrac{\sqrt{12} - \sqrt{18} + 18\sqrt{2}}{2\cdot 2} = \dfrac{2\sqrt{3} - 3\sqrt{2} + 18\sqrt{2}}{4}$
$= \dfrac{2\sqrt{3} + 15\sqrt{2}}{4}.$

16. $\dfrac{\sqrt{5} + 2}{\sqrt{125}} = \dfrac{\sqrt{25} + 2\sqrt{5}}{\sqrt{625}} = \dfrac{5}{25} + \dfrac{2}{25}\sqrt{5} = \dfrac{1}{5} + \dfrac{2}{25}\sqrt{5}.$

17. $\dfrac{\sqrt{\frac{32}{10}}}{\sqrt{\frac{4}{5}}} = \sqrt{\frac{32}{10}\cdot\frac{5}{4}} = \sqrt{4} = 2.$

18. $\dfrac{(xy)^{\frac{1}{2}}}{x^{\frac{1}{2}}} = y^{\frac{1}{2}}.$

19. $\dfrac{a\sqrt{bc}}{d\sqrt{c}} = \dfrac{a\sqrt{b}}{d}.$

20. $\dfrac{a^2\sqrt{c}}{a\sqrt{bc}} = \dfrac{a}{\sqrt{b}} = \dfrac{a\sqrt{b}}{\sqrt{b^2}} = \dfrac{a}{b}\sqrt{b}.$

21. $\dfrac{\sqrt[3]{16}}{\sqrt[3]{4}} = \sqrt[3]{4}.$

22. $\dfrac{8\sqrt[3]{125}}{4\sqrt[3]{25}} = 2\sqrt[3]{5}.$

23. $\dfrac{2\sqrt[3]{5}}{3\sqrt[3]{4}} = \dfrac{2\sqrt[3]{5}\cdot\sqrt[3]{2}}{3\sqrt[3]{8}}$

$= \dfrac{2\sqrt[3]{10}}{6} = \dfrac{1}{3}\sqrt[3]{10}.$

24. $\dfrac{a\sqrt[3]{9}}{b\sqrt[3]{8}} = \dfrac{a\sqrt[3]{9}}{2b}.$

25. $\dfrac{\sqrt[3]{\frac{1}{8}}}{\sqrt[3]{\frac{1}{4}}} = \sqrt[3]{\frac{1}{8}\cdot\frac{4}{1}} = \sqrt[3]{\frac{1}{2}} = \frac{1}{2}\sqrt[3]{4}.$

26. $\dfrac{\sqrt[3]{4}}{\sqrt{2}} = \dfrac{\sqrt[6]{16}}{\sqrt[6]{8}} = \sqrt[6]{2}.$

27. $\dfrac{\sqrt{2}}{\sqrt[3]{4}} = \dfrac{\sqrt{2}\cdot\sqrt[3]{2}}{\sqrt[3]{8}} = \dfrac{2^{\frac{1}{2}}\cdot 2^{\frac{1}{3}}}{2} = \dfrac{1}{2}\sqrt[6]{2^5} = \dfrac{1}{2}\sqrt[6]{32}.$

28. $\dfrac{4^{\frac{1}{3}}}{6^{\frac{1}{2}}} = \dfrac{\sqrt[3]{4}}{\sqrt{6}} = \dfrac{\sqrt[3]{4}\cdot\sqrt{6}}{\sqrt{6}\cdot\sqrt{6}} = \dfrac{2^{\frac{2}{3}}\cdot 2^{\frac{1}{2}}\cdot 3^{\frac{1}{2}}}{6} = \dfrac{2^{\frac{7}{6}}\cdot 3^{\frac{3}{6}}}{6} = \dfrac{2}{6}\sqrt[6]{2\cdot 3^3} = \dfrac{1}{3}\sqrt[6]{54}.$

29. $\dfrac{6^{\frac{1}{2}}}{4^{\frac{1}{3}}} = \dfrac{\sqrt{6}}{\sqrt[3]{4}} = \dfrac{\sqrt{6}\cdot\sqrt[3]{2}}{\sqrt[3]{4}\cdot\sqrt[3]{2}} = \dfrac{2^{\frac{1}{2}}\cdot 3^{\frac{1}{2}}\cdot 2^{\frac{1}{3}}}{2} = \dfrac{2^{\frac{5}{6}}\cdot 3^{\frac{3}{6}}}{2} = \dfrac{1}{2}\sqrt[6]{2^5\cdot 3^3} = \dfrac{1}{2}\sqrt[6]{864}.$

30. $\dfrac{\sqrt[5]{27}}{\sqrt{27}} = \dfrac{\sqrt[5]{27}\cdot\sqrt{3}}{\sqrt{27}\cdot\sqrt{3}} = \dfrac{3^{\frac{3}{5}}\cdot 3^{\frac{1}{2}}}{9} = \dfrac{3^{\frac{11}{10}}}{9} = \dfrac{1}{3}\sqrt[10]{3}.$

31. $\dfrac{\sqrt[4]{\frac{1}{8}}}{\sqrt[3]{\frac{1}{2}}} = \dfrac{\sqrt[4]{\frac{1}{8}}\cdot\sqrt[3]{\frac{1}{4}}}{\sqrt[3]{\frac{1}{2}}\cdot\sqrt[3]{\frac{1}{4}}} = \dfrac{(\frac{1}{2})^{\frac{3}{4}}(\frac{1}{2})^{\frac{2}{3}}}{\frac{1}{2}} = \dfrac{(\frac{1}{2})^{\frac{17}{12}}}{\frac{1}{2}} = \left(\dfrac{1}{2}\right)^{\frac{5}{12}} = \sqrt[12]{\dfrac{1}{32}} = \sqrt[12]{\dfrac{128}{2^{12}}} = \dfrac{1}{2}\sqrt[12]{128}.$

32. $\dfrac{\sqrt[6]{\dfrac{a^2}{72a}}}{\sqrt[3]{\dfrac{a}{81}}} = \dfrac{\sqrt[6]{\dfrac{a^2}{72a}}}{\sqrt[6]{\dfrac{a^2}{81\cdot 81}}} = \sqrt[6]{\dfrac{81\cdot 81}{72a}} = \sqrt[6]{\dfrac{729}{8a}} = 3\sqrt[6]{\dfrac{8a^5}{64a^6}} = \dfrac{3}{2a}\sqrt[6]{8a^5}.$

33. $\dfrac{5}{\sqrt{5}+2} = \dfrac{5(\sqrt{5}-2)}{(\sqrt{5}+2)(\sqrt{5}-2)} = \dfrac{5\sqrt{5}-10}{5-4} = 5\sqrt{5}-10.$

34. $\dfrac{2}{2-\sqrt{3}} = \dfrac{2(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} = \dfrac{4+2\sqrt{3}}{4-3} = 4+2\sqrt{3}.$

35. $\dfrac{4}{\sqrt{3}-\sqrt{2}} = \dfrac{4(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})} = \dfrac{4\sqrt{3}+4\sqrt{2}}{3-2} = 4\sqrt{3}+4\sqrt{2}.$

36. $\dfrac{\sqrt{5}}{3\sqrt{2}-\sqrt{5}} = \dfrac{\sqrt{5}(3\sqrt{2}+\sqrt{5})}{(3\sqrt{2}-\sqrt{5})(3\sqrt{2}+\sqrt{5})} = \dfrac{3\sqrt{10}+5}{18-5} = \dfrac{3}{13}\sqrt{10}+\dfrac{5}{13}.$

37. $\dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}=\dfrac{(\sqrt{5}-\sqrt{3})(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}=\dfrac{5-2\sqrt{15}+3}{5-3}=4-\sqrt{15}.$

38. $\dfrac{4\sqrt{3}+2\sqrt{2}}{4\sqrt{3}-2\sqrt{2}}=\dfrac{(2\sqrt{3}+\sqrt{2})(2\sqrt{3}+\sqrt{2})}{(2\sqrt{3}-\sqrt{2})(2\sqrt{3}+\sqrt{2})}=\dfrac{12+4\sqrt{6}+2}{12-2}=\dfrac{7}{5}+\dfrac{2}{5}\sqrt{6}.$

39. $\dfrac{2\sqrt{5}-3\sqrt{3}}{5\sqrt{5}-5}=\dfrac{(2\sqrt{5}-3\sqrt{3})(\sqrt{5}+1)}{5(\sqrt{5}-1)(\sqrt{5}+1)}=\dfrac{2\cdot 5-3\sqrt{15}+2\sqrt{5}-3\sqrt{3}}{5(5-1)}$

$=\frac{1}{2}-\frac{3}{20}\sqrt{15}+\frac{1}{10}\sqrt{5}-\frac{3}{20}\sqrt{3}.$

40. $3+\sqrt{2}=3+1.414=4.414.$

41. $14-5\sqrt{7}=14-13.2285=.771+.$

42. $6+2\sqrt{5}=6+2\cdot 2.236=6+4.472=10.472.$

$6-2\sqrt{5}=6-4.472+=1.527.$

43. $\dfrac{7\pm\sqrt{6}}{3}=\dfrac{7\pm 2.449}{3}=\dfrac{9.449}{3}$ or $\dfrac{4.550}{3}$ which equal 3.149 or 1.516 respectively.

44. $\dfrac{3\sqrt{6}}{2\sqrt{5}}=\dfrac{3\sqrt{30}}{10}=\dfrac{16.431}{10}=1.643.$

45. $\dfrac{2\sqrt{5}+1}{3\sqrt{5}-\sqrt{3}}=\dfrac{(2\sqrt{5}+1)(3\sqrt{5}+\sqrt{3})}{(3\sqrt{5}-\sqrt{3})(3\sqrt{5}+\sqrt{3})}=\dfrac{6\cdot 5+3\sqrt{5}+2\sqrt{15}+\sqrt{3}}{9.5-3}$

$=\dfrac{30+3\sqrt{5}+2\sqrt{15}+\sqrt{3}}{42}=\dfrac{30+6.708+7.745+1.732}{42}=1.099.$

46. $\sqrt{2-\sqrt{3}}=\sqrt{2-1.732}=\sqrt{.267}=.517.$

47. $\dfrac{\sqrt{3}}{\sqrt{3}+\sqrt[3]{4}+\sqrt[4]{5}}=\dfrac{1.732}{1.732+1.587+1.495}=\dfrac{1.732}{4.814}=.359.$

48. $\dfrac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}=\dfrac{(\sqrt{a}+\sqrt{b})(\sqrt{a}+\sqrt{b})}{(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})}=\dfrac{a+2\sqrt{ab}+b}{a-b}.$

49. $\dfrac{2\sqrt{x}-\sqrt{a}}{\sqrt{x}+3\sqrt{a}}=\dfrac{(2\sqrt{x}-\sqrt{a})(\sqrt{x}-3\sqrt{a})}{(\sqrt{x}+3\sqrt{a})(\sqrt{x}-3\sqrt{a})}=\dfrac{2x-7\sqrt{ax}+3a}{x-9a}.$

50. $\dfrac{r\sqrt{3}+\sqrt{r}}{\sqrt{3}-\sqrt{r}}=\dfrac{(r\sqrt{3}+\sqrt{r})(\sqrt{3}+\sqrt{r})}{(\sqrt{3}-\sqrt{r})(\sqrt{3}+\sqrt{r})}=\dfrac{3r+\sqrt{3r}+r\sqrt{3r}+r}{3-r}$

$=\dfrac{4r+\sqrt{3r}+r\sqrt{3r}}{3-r}.$

51. $\dfrac{m\sqrt{n}+a\sqrt{b}}{m\sqrt{n}-a\sqrt{b}}=\dfrac{(m\sqrt{n}+a\sqrt{b})(m\sqrt{n}+a\sqrt{b})}{(m\sqrt{n}-a\sqrt{b})(m\sqrt{n}+a\sqrt{b})}$

$=\dfrac{m^2n+2am\sqrt{bn}+a^2b}{m^2n-a^2b}.$

52. $$\frac{\sqrt{a-2}-2}{\sqrt{a-2}+2}=\frac{(\sqrt{a-2}-2)(\sqrt{a-2}-2)}{(\sqrt{a-2}+2)(\sqrt{a-2}-2)}=\frac{a-2-4\sqrt{a-2}+4}{a-2-4}$$
$$=\frac{a-4\sqrt{a-2}+2}{a-6}.$$

53. $$\frac{2}{\sqrt[4]{2}+\sqrt{2}}=\frac{2\sqrt[4]{2}-\sqrt{2}}{\sqrt[4]{2}}=\frac{2\sqrt{16-8\sqrt{2}}}{2}=\sqrt[4]{16-8\sqrt{2}}.$$

54. $$\frac{x-\sqrt{a+b}}{x+\sqrt{a+b}}=\frac{(x-\sqrt{a+b})(x-\sqrt{a+b})}{(x+\sqrt{a+b})(x-\sqrt{a+b})}=\frac{x^2-2x\sqrt{a+b}+a+b}{x^2-a-b}.$$

55. $$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{c}+\sqrt{d}}=\frac{(\sqrt{a}+\sqrt{b})(\sqrt{c}-\sqrt{d})}{(\sqrt{c}+\sqrt{d})(\sqrt{c}-\sqrt{d})}=\frac{\sqrt{ac}+\sqrt{bc}-\sqrt{ad}-\sqrt{bd}}{c-d}.$$

56. No.

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1. $10^2+5^2=h^2.$
$h^2=125.$
$h=\sqrt{125}=5\sqrt{5},$
the hypotenuse.

2. $10^2-5^2=l^2.$
$l^2=75.$
$l=\sqrt{75}=5\sqrt{3},$
the leg.
$$\text{Area}=\frac{5\cdot 5\sqrt{3}}{2}=12\tfrac{1}{2}\sqrt{3}.$$

3. $R^2-\frac{R^2}{4}=l^2.$
$l^2=\frac{3R^2}{4}.$
$l=\sqrt{\frac{3R^2}{4}}=\frac{R}{2}\sqrt{3},$
the leg.
$$\text{Area}=\frac{\frac{R}{2}\cdot\frac{R}{2}\sqrt{3}}{2}=\frac{R^2}{8}\sqrt{3}.$$

4. $10^2+10^2=D^2.$
$D^2=200.$
$D=\sqrt{200}=10\sqrt{2},$
the diagonal.

5. $10^2=2S^2.$
$S^2=50.$
$S=\sqrt{50}=5\sqrt{2},$
the side.
Area $=S^2=50.$

6. $4R^2=2S^2.$
$S^2=2R^2.$
$S=\sqrt{2R^2}=R\sqrt{2},$
the side.
Area $=S^2=2R^2.$

7. $A^2=12^2-6^2.$
$A^2=108.$
$A=\sqrt{108}=6\sqrt{3},$
the altitude.
$$\text{Area}=\frac{12\cdot 6\sqrt{3}}{2}=36\sqrt{3}.$$

8. $A^2=S^2-\frac{S^2}{4}.$
$A^2=\frac{3S^2}{4}.$
$A=\sqrt{\frac{3S^2}{4}}=\frac{S}{2}\sqrt{3},$
the altitude.
$$\text{Area}=\frac{S\cdot S\sqrt{3}}{2\cdot 2}=\frac{S^2}{4}\sqrt{3}.$$

9. $S^2 - \frac{S^2}{4} = 10^2.$

$\frac{3S^2}{4} = 100.$

$S^2 = \frac{400}{3}.$

$S = \sqrt{\frac{400}{3}} = 20\sqrt{\frac{3}{9}}$

$= 6\frac{2}{3}\sqrt{3}$, the side.

$\text{Area} = \frac{\left(\frac{20}{3}\sqrt{3}\right)^2}{4}\sqrt{3}$

$= \frac{100}{3}\sqrt{3}.$

10. $2l^2 = 20^2.$

$l^2 = 200.$

$l = \sqrt{200} = 10\sqrt{2}$, the leg.

$\text{Area} = \frac{10\sqrt{2}\cdot 10\sqrt{2}}{2}$

$= 100.$

11. $\frac{l^2}{2} = 32.$

$l^2 = 64.$

$l = 8$, the leg.

$H^2 = 8^2 + 8^2 = 128.$

$H = \sqrt{128} = 8\sqrt{2}$, the hypotenuse.

12. $H^2 = \frac{R^2}{4} + \frac{9R^2}{4}.$

$H = \sqrt{\frac{10R^2}{4}} = \frac{R}{2}\sqrt{10}$, the hypotenuse.

13. $l^2 = R^2 - \frac{R^2}{25}.$

$l^2 = \frac{24R^2}{25}.$

$l = \sqrt{\frac{24R^2}{25}} = \frac{2R}{5}\sqrt{6}$, the leg.

14. $H^2 = R^2 + \frac{R^2}{4}(5 - 2\sqrt{5} + 1)$

$= \frac{4R^2 + 6R^2 - 2R^2\sqrt{5}}{4}$

$= \frac{R^2}{4}(10 - 2\sqrt{5}).$

$H = \frac{R}{2}\sqrt{10 - 2\sqrt{5}}$, the hypotenuse.

15. $H^2 = \frac{R^2}{4} + \left(R^2 - R^2\sqrt{3} + \frac{3R^2}{4}\right)$

$= 2R^2 - R^2\sqrt{3}.$

$H = R\sqrt{2 - \sqrt{3}}$, the hypotenuse.

16. $\text{Area} = 5 \cdot \frac{2R}{3}\sqrt{4 - \sqrt{3}} \cdot 3 \cdot \frac{9R\sqrt{4 + \sqrt{3}}}{10} = 9R^2\sqrt{16 - 3} = 9R^2\sqrt{13}.$

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1. $x^2 - 6 = (x + \sqrt{6})(x - \sqrt{6}).$
2. $3x^2 - 4 = (x\sqrt{3} + 2)(x\sqrt{3} - 2).$
3. $5x^2 - 1 = (x\sqrt{5} + 1)(x\sqrt{5} - 1).$
4. $x^4 - 4 = (x^2 + 2)(x^2 - 2) = (x^2 + 2)(x + \sqrt{2})(x - \sqrt{2}).$
5. $4x^4 - 1 = (2x^2 + 1)(2x^2 - 1) = (2x^2 + 1)(x\sqrt{2} + 1)(x\sqrt{2} - 1).$
6. $x^3 - 2 = (x - \sqrt[3]{2})(x^2 + x\sqrt[3]{2} + \sqrt[3]{4}).$
7. $x^3 + 6 = (x + \sqrt[3]{6})(x^2 - x\sqrt[3]{6} + \sqrt[3]{36}).$
8. $3x^3 - 1 = (x\sqrt[3]{3} - 1)(x^2\sqrt[3]{9} + x\sqrt[3]{3} + 1).$
9. $x^3 + 4 = (x + \sqrt[3]{4})(x^2 - x\sqrt[3]{4} + \sqrt[3]{16}) = (x + \sqrt[3]{4})(x^2 - x\sqrt[3]{4} + 2\sqrt[3]{2}).$

10. $2x^3+8=2(x^3+4)=2(x+\sqrt[3]{4})(x^2-x\sqrt[3]{4}+\sqrt[3]{16})$
$=2(x+\sqrt[3]{4})(x^2-x\sqrt[3]{4}+2\sqrt[3]{2})$.

11. $2x^3-8=2(x^3-4)=2(x-\sqrt[3]{4})(x^2+x\sqrt[3]{4}+\sqrt[3]{16})$
$=2(x-\sqrt[3]{4})(x^2+x\sqrt[3]{4}+2\sqrt[3]{2})$.

12. $6x^3+24=6(x^3+4)=6(x+\sqrt[3]{4})(x^2-\sqrt[3]{4}\,x+2\sqrt[3]{2})$.

13. $x^2-2=0.$
$(x+\sqrt{2})(x-\sqrt{2})=0.$
Whence $x=\pm\sqrt{2}$.

14. $x^2-6=0.$
$(x+\sqrt{6})(x-\sqrt{6})=0.$
Whence $x=\pm\sqrt{6}$.

15. $2x^2-1=0.$
$(\sqrt{2}\,x+1)(\sqrt{2}\,x-1)=0.$
Whence $x=\pm\dfrac{1}{\sqrt{2}}=\pm\dfrac{\sqrt{2}}{2}$.

16. $x^4+6=5x^2.$
$(x^2-2)(x^2-3)=0.$
Whence $x=\pm\sqrt{2}$ or $\pm\sqrt{3}$.

17. $4x^4+5=12x^2.$
$(2x^2-5)(2x^2-1)=0.$
Whence $x=\pm\sqrt{\tfrac{5}{2}}$ or $\pm\sqrt{\tfrac{1}{2}}$
$=\pm\dfrac{\sqrt{10}}{2}$ or $\pm\dfrac{\sqrt{2}}{2}$.

18. $3x^4+8=14x^2.$
$(3x^2-2)(x^2-4)=0.$
Whence $x=\pm 2$ or $\pm\tfrac{1}{3}\sqrt{6}$.

19. $5x^4-16x^2+3=0.$
$(5x^2-1)(x^2-3)=0.$
Whence $x=\pm\tfrac{1}{5}\sqrt{5}$ or $\pm\sqrt{3}$.

20. $x^4+8a=4x^2+2ax^2.$
$(x^2-4)(x^2-2a)=0.$
Whence $x=\pm 2$ or $\pm\sqrt{2a}$.

21. $ax^4-x^2+3a=3a^2x^2.$
$(ax^2-1)(x^2-3a)=0.$
Whence $x=\pm\dfrac{\sqrt{a}}{a}$ or $\pm\sqrt{3a}$.

22. $4x^4+a=x^2+4ax^2.$
$(4x^2-1)(x^2-a)=0.$
Whence $x=\pm\tfrac{1}{2}$ or $\pm\sqrt{a}$.

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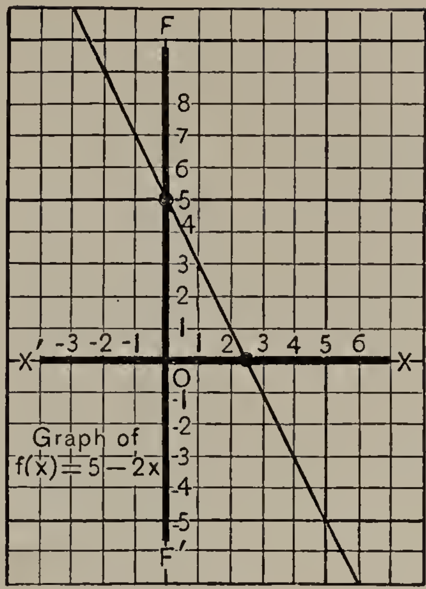
1. When $f(x)=0$, $x=-\tfrac{3}{2}$.
2. If $2x+3=0$, $x=-\tfrac{3}{2}$.
3. They are equal.
4. When $f(x)=4$, $x=\tfrac{1}{2}$.
5. If $2x+3=4$, $x=\tfrac{1}{2}$.
6. They are equal.
7. Yes. When $f(x)=6$, $x=\tfrac{3}{2}$.
8. When $f(x)$, $2x+3=-2$, $x=-\tfrac{5}{2}$.

9.

x	0	$\frac{5}{2}$
$f(x)$	5	0



10. Yes. When $f(x)$, $5 - 2x = 0$, $x = \frac{5}{2}$.

11. (*a*) When $f(x)$, $5 - 2x = 9$, $x = -2$.
    (*b*) When $f(x)$, $5 - 2x = 5$, $x = 0$.
    (*c*) When $f(x)$, $5 - 2x = -3$, $x = 4$.
    (*d*) When $f(x)$, $5 - 2x = -7$, $x = 6$.

12. The roots obtained from the graph check in the equation.

13. The graph of any linear function of x should give a straight line.

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1. When $f(x)$,
$$4x^2 - 4x - 15 = 0,$$
$$x = -\tfrac{3}{2} \text{ or } \tfrac{5}{2}.$$

2. If $4x^2 - 4x - 15 = 0$,
then $(2x + 3)(2x - 5) = 0$.
Whence $x = -\frac{3}{2}$ or $\frac{5}{2}$.

3. They are the same.

4. When $f(x)$,
$$4x^2 - 4x - 15 = 20,$$
$$x = -\tfrac{5}{2} \text{ or } \tfrac{7}{2}.$$

5. The roots are $-\frac{5}{2}$ and $\frac{7}{2}$.

6. If $4x^2 - 4x - 15 = 20$,
then $(2x + 5)(2x - 7) = 0$.
Whence $x = -\frac{5}{2}$ or $\frac{7}{2}$.

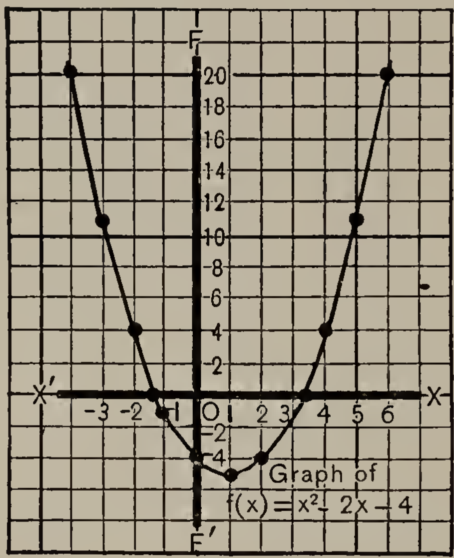
7. If $f(x)$,
$$4x^2 - 4x - 15 = 9,$$
then $x = 3$ or -2.
If $f(x)$,
$$4x^2 - 4x - 15 = -7,$$
then $x = 2$ or -1.

8. If $4x^2 - 4x - 15 = 9$,
then $(2x - 6)(2x + 4) = 0$.
Whence $x = 3$ or -2.
If $4x^2 - 4x - 15 = -7$,
then $(2x - 4)(2x + 2) = 0$.
Whence $x = 2$ or -1.

9. Values of x for $f(x) = -25$ or -20 cannot be read from the graph, for the graph does not reach these values.

10.

When $x =$	-4	-3	-2	-1	0	1	2	3	4	5	6
$fx,\ x^2 - 2x - 4 =$	20	11	4	-1	-4	-5	-4	-1	4	11	20



11. Yes.

12. When $f(x),\ x^2 - 2x - 4 = 0,\ x = -\frac{5}{4}$ or $\frac{13}{4}$, nearly.

13. The roots are $-\frac{5}{4}$ or $\frac{13}{4}$, nearly.

14. No. It can be obtained graphically.

15. Yes. Read values of x for $f(x) = 0$.

16.

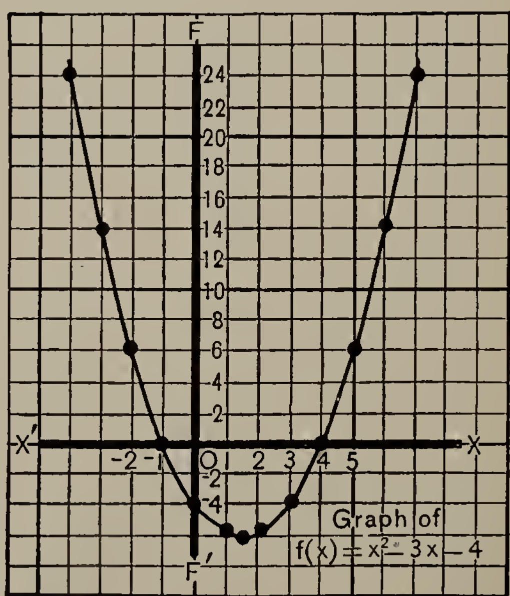
x	-4	-3	-2	-1	0	1	2	3	4	5	6	7
$f(x)$	24	14	6	0	-4	-6	-6	-4	0	6	14	24

When $f(x) = 0,\ x = 4$ or -1.

If $x^2 - 3x - 4 = 0$,

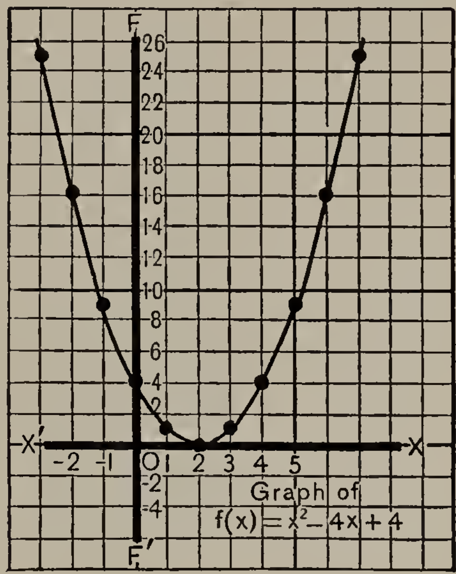
then $(x - 4)(a + 1) = 0$.

Whence $x = 4$ or -1.



17.

x	-3	-2	-1	0	1	2	3	4	5	6	7
$f(x)$	25	16	9	4	1	0	1	4	9	16	25



When $f(x)$,

$$x^2 - 4x + 4 = 0,$$
$$x = +2.$$

If $x^2 - 4x + 4 = 0,$

then $(x - 2)(x - 2) = 0.$

Whence $x = +2.$

18. It touches the x-axis. The equation $x^2 - 4x + 4 = 0$ has one root. The only value of x which satisfies the equation $x^2 - 4x + 4 = 0$ is $+2$.

If $f(x)$, $x^2 - 4x + 4 = 0$, $x = 2$.

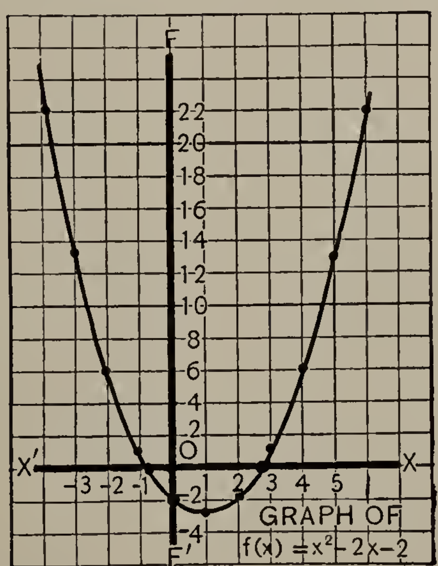
If $f(x)$, $x^2 - 4x + 4 = 4$, $x = 0$ or 4.

If $f(x)$, $x^2 - 4x + 4 = -1$, x has no real value.

If $f(x)$, $x^2 - 4x + 4 = -10$, x has no real value.

19.

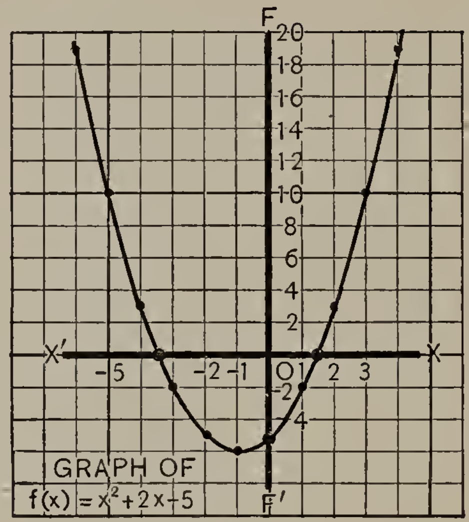
x	-4	-3	-2	-1	0	1	2	3	4	5	6
$f(x)$	22	13	6	1	-2	-3	-2	1	6	13	22



When $f(x)$, $x^2 - 2x - 2 = 0$, $x = 2.7$ or $-.7$.

20.

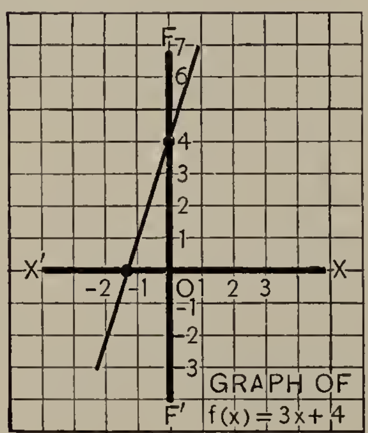
x	-6	-5	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	19	10	3	-2	-5	-6	-5	-2	3	10	19

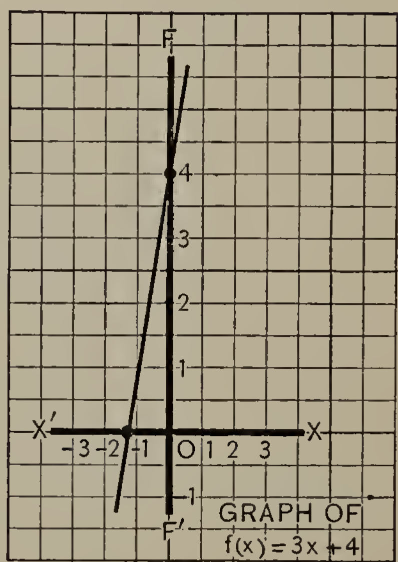


When $f(x)$, $x + 2x - 5 = 0$, $x = 1.4$ or -3.4.

21.

x	0	$-\frac{4}{3}$
$f(x)$	4	0

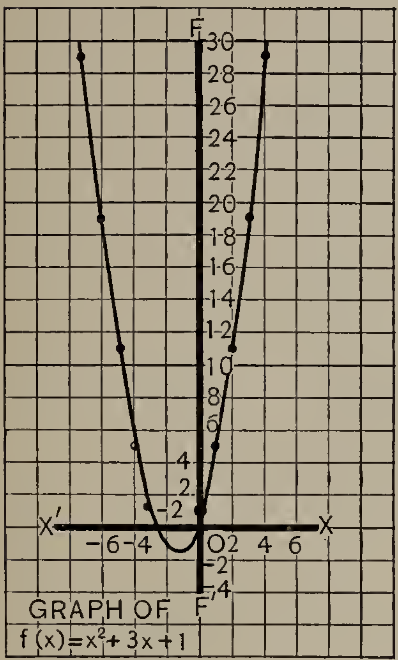


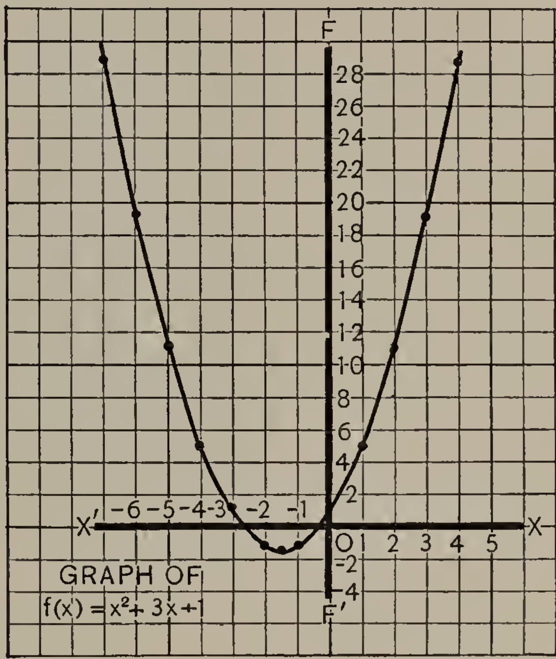


The lines cross the axes at points which have the same values respectively.

22.

x	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	29	19	11	5	1	-1	-1	1	5	11	19	29





The lines cross the axes at points which have the same values respectively.

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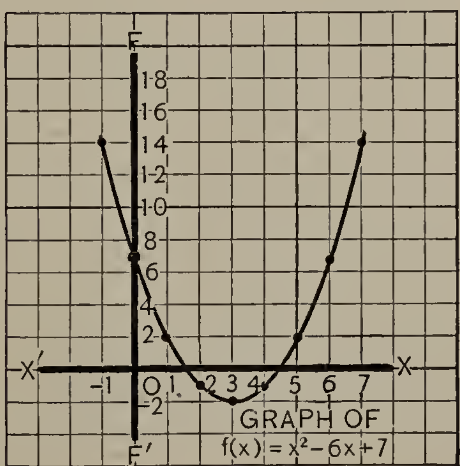
1.

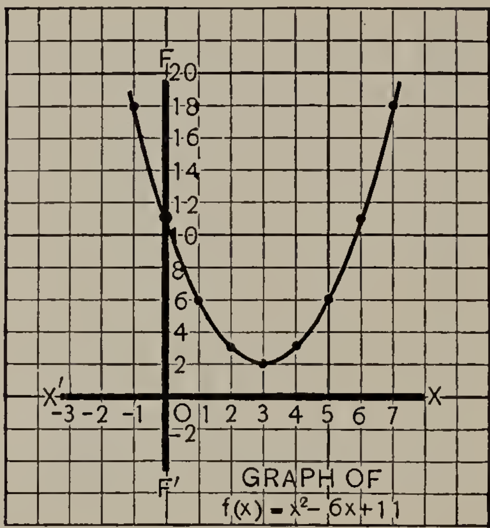
x	-1	0	1	2	3	4	5	6	7
$f(x)$	14	7	2	-1	-2	-1	2	7	14



When $f(x) = 0$, $x = 1.6$ or 4.4.

2.

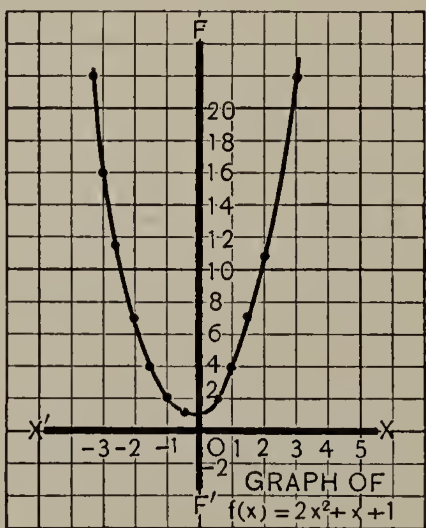
x	-1	0	1	2	3	4	5	6	7
$f(x)$	18	11	6	3	2	3	6	11	18



The equation $x^2 - 6x + 11 = 0$ has no graphical solution.

3.

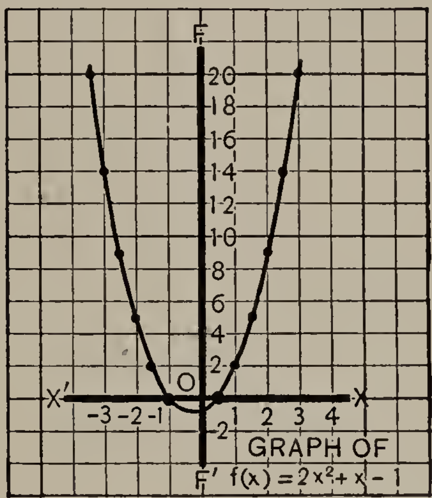
x	$-\frac{7}{2}$	-3	2	$-\frac{3}{2}$	-1	0	$\frac{1}{2}$	1	2	3
$f(x)$	22	16	7	4	2	1	2	4	11	22



The equation $2x^2 + x + 1 = 0$ has no graphical solution.

4.

x	$-\frac{7}{2}$	-3	-2	$-\frac{3}{2}$	-1	0	$\frac{1}{2}$	1	2	3
$f(x)$	20	14	5	2	0	-1	0	2	9	20



When $f(x) = 0$, $x = -1$ or $\frac{1}{2}$.

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1. $x^2 - 8x - 48 = 0,$

or $x^2 - 8x = 48.$

$(x-4)^2 = 64.$

Whence $x = +12$ or -4.

2. $x^2 - 5x - 14 = 0,$

or $x^2 - 5x = 14.$

$(x - \frac{5}{2})^2 = \frac{81}{4}.$

Whence $x = +7$ or -2.

3. $x(x+2) - 5(x+2) = 0,$

or $x^2 - 3x = 10.$

$(x - \frac{3}{2})^2 = \frac{49}{4}.$

Whence $x = -2$ or $+5$.

4. $\frac{3}{4} - y = y^2,$

or $y^2 + y = \frac{3}{4}.$

$(y + \frac{1}{2})^2 = 1.$

Whence $y = \frac{1}{2}$ or $-\frac{3}{2}$.

5. $2y^2 - 9y + 4 = 0,$

or $y^2 - \frac{9y}{2} = -2.$

$(y - \frac{9}{4})^2 = \frac{49}{16}.$

Whence $y = 4$ or $\frac{1}{2}$.

6. $2y^2 + 5y = 0,$

or $y^2 + \frac{5y}{2} = 0.$

$(y + \frac{5}{4})^2 = \frac{25}{16}.$

Whence $y = 0$ or $-\frac{5}{2}$.

7. $t^2 - 2t - 15 = 0,$

or $t^2 - 2t = 15.$

$(t-1)^2 = 16.$

Whence $t = 5$ or -3.

8. $3t^2 - 7t = 6,$

or. $t^2 - \frac{7t}{3} = 2.$

$(t - \frac{7}{6})^2 = \frac{121}{36}.$

Whence $t = 3$ or $-\frac{2}{3}$.

9. $9v = 5v^2 - 2,$

or $v^2 - \frac{9v}{5} = \frac{2}{5}.$

$(v - \frac{9}{10})^2 = \frac{121}{100}.$

Whence $v = 2$ or $-\frac{1}{5}$.

10. $x^2 - 2x - 4 = 0,$

or $x^2 - 2x = 4.$

$(x-1)^2 = 5.$

Whence $x = 1 \pm \sqrt{5}$

$= 1 \pm 2.236$

$= 3.236$ or $-1.236.$

11. $v^2 - \frac{7}{12}v - 1 = 0,$

or $v^2 - \dfrac{7v}{12} = 1.$

$(v - \frac{7}{24})^2 = \frac{625}{576},$

or $v = \frac{4}{3}$ or $-\frac{3}{4}.$

12. $s^2 - 2s - 3\frac{1}{2} = 0,$

or $s^2 - 2s = \frac{7}{2}.$

$(s-1)^2 = \frac{9}{2}.$

Whence $s = 1 \pm \frac{3}{2}\sqrt{2}.$

13. $h^2 + 10h + 13 = 0,$

or $h^2 + 10h = -13.$

$(h+5)^2 = 12.$

Whence $h = -5 \pm 2\sqrt{3}$

$= -1.535$ or $-8.464.$

14. $12t^2 - 25t + 12 = 0,$

or $t^2 - \dfrac{25t}{12} = -1.$

$(t - \frac{25}{24})^2 = \frac{49}{576}.$

Whence $t = \frac{4}{3}$ or $\frac{3}{4}.$

15. $42 + 2z^2 = -19z,$

or $z^2 + \dfrac{19z}{2} = -21.$

$(z + \frac{19}{4})^2 = \frac{25}{16}.$

Whence $z = -\frac{7}{2}$ or $-6.$

16. $15x^2 + 4x = 4,$

or $x^2 + \dfrac{4x}{15} = \dfrac{4}{15}.$

$(x + \frac{2}{15})^2 = \frac{64}{225}.$

Whence $x = \frac{2}{5}$ or $-\frac{2}{3}.$

17. $1 - 6v^2 = 2v,$

or $v^2 + \dfrac{v}{3} = \dfrac{1}{6}.$

$(v + \frac{1}{6})^2 = \frac{7}{36}.$

Whence $v = \dfrac{-1 \pm \sqrt{7}}{6}.$

18. $20s^2 + s = 1,$

or $s^2 + \dfrac{s}{20} = \dfrac{1}{20}.$

$(s + \frac{1}{40})^2 = (\frac{9}{40})^2.$

Whence $s = \frac{1}{5}$ or $-\frac{1}{4}.$

19. $9x + 4 = 9x^2,$

or $x^2 - x = \frac{4}{9}.$

$(x - \frac{1}{2})^2 = \frac{25}{36}.$

Whence $x = \frac{4}{3}$ or $-\frac{1}{3}.$

20. $25x^2 - 20x - 12 = 0,$

or $x^2 - \dfrac{4x}{5} = \dfrac{12}{25}.$

$(x - \frac{2}{5})^2 = \frac{16}{25}.$

Whence $x = \frac{6}{5}$ or $-\frac{2}{5}.$

21. $4x^2 = 1 - 4x,$

or $x^2 + x = \frac{1}{4}.$

$(x + \frac{1}{2})^2 = \frac{2}{4}.$

Whence $x = \dfrac{-1 \mp \sqrt{2}}{2}$

$= -1.207$ or $.207.$

22. $x^2 + 4\sqrt{5}x = 25,$

$(x + 2\sqrt{5})^2 = 45.$

Whence $x = \sqrt{5}$ or $-5\sqrt{5}.$

23. $x^2 - 3\sqrt{2}x + 4 = 0,$

or $x^2 - 3\sqrt{2}x = -4.$

$(x - \frac{3}{2}\sqrt{2})^2 = \frac{2}{4}.$

Whence $x = 2\sqrt{2}$ or $\sqrt{2}.$

24. $1 - \dfrac{3\sqrt{3}}{2R} + \dfrac{3}{2R^2} = 0,$

or $R^2 - \dfrac{3\sqrt{3}R}{2} = -\dfrac{3}{2}.$

$(R - \frac{3}{4}\sqrt{3})^2 = \frac{3}{16}.$

Whence $R = \sqrt{3}$ or $\frac{1}{2}\sqrt{3}.$

25. $$\frac{2x}{9}+\frac{2}{3}=\frac{7}{18x},$$

or $$x^2+3x=\tfrac{7}{4}.$$

$$(x+\tfrac{3}{2})^2=4.$$

Whence $$x=\tfrac{1}{2} \text{ or } -\tfrac{7}{2}.$$

26. $$\frac{1}{3}+\frac{x}{9}-\frac{2}{x}=0,$$

or $$x^2+3x=18.$$

$$(x+\tfrac{3}{2})^2=\tfrac{81}{4}.$$

Whence $$x=3 \text{ or } -6.$$

27. $$\frac{2}{5v^2}+\frac{1}{4}-\frac{11}{10v}=0,$$

or $$v^2-\frac{22v}{5}=-\frac{8}{5}.$$

$$(v-\tfrac{11}{5})^2=\tfrac{81}{25}.$$

Whence $$v=4 \text{ or } \tfrac{2}{5}.$$

28. $$\frac{3a}{2}+\frac{1}{2}-\frac{1}{3a}=0,$$

or $$a^2+\frac{a}{3}=\frac{2}{9}.$$

$$(a+\tfrac{1}{6})^2=\tfrac{1}{4}.$$

Whence $$a=\tfrac{1}{3} \text{ or } -\tfrac{2}{3}.$$

29. $$\frac{25c}{2}-\frac{40}{c}-40=0,$$

or $$c^2-\frac{16c}{5}=\frac{16}{5}.$$

$$(c-\tfrac{8}{5})^2=\tfrac{144}{25}.$$

Whence $$c=4 \text{ or } -\tfrac{4}{5}.$$

30. $(x-4)^2-3(x-9)=15,$

or $$x^2-11x=-28.$$

$$(x-\tfrac{11}{2})^2=\tfrac{9}{4}.$$

Whence $$x=7 \text{ or } 4.$$

31. $(x-2)(x+3)=x(5x-9)-2,$

or $$x^2-\frac{5x}{2}=-1.$$

$$(x-\tfrac{5}{4})^2=\tfrac{9}{16}.$$

Whence $$x=2 \text{ or } \tfrac{1}{2}.$$

32. $$x-\frac{3}{x+2}=0,$$

or $$x^2+2x=3.$$

$$(x+1)^2=4.$$

Whence $$x=1 \text{ or } -3.$$

33. $$\frac{v^2}{v-5}+\frac{5}{2}=0,$$

or $$v^2+\frac{5v}{2}=\frac{25}{2}.$$

$$(v+\tfrac{5}{4})^2=\tfrac{225}{16}.$$

Whence $$v=\tfrac{5}{2} \text{ or } -5.$$

34. $$\frac{3}{t-7}+\frac{t}{4}=0,$$

or $$t^2-7t=-12.$$

$$(t-\tfrac{7}{2})^2=\tfrac{1}{4}.$$

Whence $$t=4 \text{ or } 3.$$

35. $$\frac{s}{s-2}+\frac{s-2}{s}=\frac{5}{2},$$

or $$s^2-2s=8.$$

$$(s-1)^2=9.$$

Whence $$s=4 \text{ or } -2.$$

36. $$\frac{7}{2y-3}-\frac{5}{1-y}=12,$$

or $$y^2-\frac{77y}{24}=-\frac{58}{24}.$$

$$(y-\tfrac{77}{48})^2=\tfrac{361}{2304}.$$

Whence $$y=2 \text{ or } \tfrac{29}{24}.$$

37. $$\frac{3+x}{4+x}-\frac{x-5}{x-6}=\frac{1}{12},$$

or $$x^2+22x=48.$$

$$(x+11)^2=169.$$

$$x=2 \text{ or } -24.$$

38. If $y=2$,

$$x^2-xy-3y^2=-12 \text{ becomes}$$

$$x^2-2x=0.$$

Whence $(x-1)^2=1,$

and $$x=2 \text{ or } 0.$$

39. If $x = -3$, $x^2 - 4xy + x^3 + y^2 + 5 = 0$ becomes $y^2 + 12y = 13$.

Whence $(y+6)^2 = 49,$

and $y = 1 \text{ or } -13.$

41. $x^4 - 13x^2 + 36 = 0,$

or $x^4 - 13x^2 = -36.$

$(x^2 - \frac{13}{2})^2 = \frac{25}{4}.$

Whence $x^2 = 9 \text{ or } 4,$

and $x = \pm 3 \text{ or } \pm 2.$

42. $4x^4 - 5x^2 + 1 = 0,$

or $x^4 - \dfrac{5x^2}{4} = -\dfrac{1}{4}.$

$(x^2 - \frac{5}{8})^2 = \frac{9}{64}.$

Whence $x^2 = 1 \text{ or } \frac{1}{4},$

and $x = \pm 1 \text{ or } \pm \frac{1}{2}.$

43. $9x^4 - 37x^2 + 4 = 0,$

or $x^4 - \dfrac{37x^2}{9} = -\dfrac{4}{9}.$

$(x^2 - \frac{37}{18})^2 = \frac{1225}{324}.$

Whence $x^2 = \frac{1}{9} \text{ or } 4,$

and $x = \pm \frac{1}{3} \text{ or } \pm 2.$

44. $4k^4 = 9k^2 - 2,$

or $k^4 - \dfrac{9k^2}{4} = -\dfrac{1}{2}.$

$(k^2 - \frac{9}{8})^2 = \frac{49}{64}.$

Whence $k^2 = 2 \text{ or } \frac{1}{4},$

and $k = \pm\sqrt{2} \text{ or } \pm \frac{1}{2}.$

45. $9s^4 + 12 = 31s^2,$

or $s^4 - \dfrac{31s^2}{9} = -\dfrac{12}{9}.$

$(s^2 - \frac{31}{18})^2 = (\frac{23}{18})^2.$

Whence $s^2 = 3 \text{ or } \frac{4}{9},$

and $s = \pm\sqrt{3} \text{ or } \pm \frac{2}{3}.$

46. $4v^4 + 5 = 21v^2,$

or $v^4 - \dfrac{21v^2}{4} = -\dfrac{5}{4}.$

$(v^2 - \frac{21}{8})^2 = \frac{361}{64}.$

Whence $v^2 = 5 \text{ or } \frac{1}{4},$

and $v = \pm\sqrt{5} \text{ or } \pm \frac{1}{2}.$

47.
$$9 - 30 = 49 - 70. \quad (1)$$
$$9 - 30 + 25 = 49 - 70 + 25. \quad (2)$$
$$(3-5)^2 = (7-5)^2. \quad (3)$$
$$3 - 5 = 7 - 5. \quad (4)$$
$$3 = 7. \quad (5)$$

The error occurred in passing from (3) to (4). Square roots of numerically equal expressions are equal only when the signs of the roots are properly chosen. Line (4) could have read

$$+(3-5) = -(7-5) \text{ or } -(3-5) = +(7-5),$$

but not $+(3-5) = +(7-5) \text{ or } -(3-5) = -(7-5).$

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2. $x^2 - ax - 2a^2 = 0,$

or $x^2 - ax = 2a^2.$

$$\left(x - \frac{a}{2}\right)^2 = \frac{9a^2}{4}.$$

Whence $x = 2a$ or $-a$.

3. $x^2 + 2ax + a^2 - 4 = 0,$

or $x^2 + 2ax = 4 - a^2.$

$(x+a)^2 = 4.$

Whence $x = -a \pm 2.$

4. $$x^2 + 1 = a + 2x,$$
or $$x^2 - 2x = a - 1.$$
$$(x-1)^2 = a.$$
Whence $x = 1 \pm \sqrt{a}.$

5. $$3x^2 - ax = 10a^2,$$
or $$x^2 - \frac{ax}{3} = \frac{10a^2}{3}.$$
$$\left(x - \frac{a}{6}\right)^2 = \frac{121a^2}{36}.$$
Whence $x = 2a$ or $-\dfrac{5a}{3}.$

6. $$3mx + 2m^2 = 2x^2,$$
or $$x^2 - \frac{3mx}{2} = m^2.$$
$$\left(x - \frac{3m}{4}\right)^2 = \frac{25m^2}{16}.$$
Whence $x = 2m$ or $-\dfrac{m}{2}.$

7. $a^2x^2 - 7ax + 10 = 0,$
or $$x^2 - \frac{7x}{a} = -\frac{10}{a^2}.$$
$$\left(x - \frac{7}{2a}\right)^2 = \frac{9}{4a^2}.$$
Whence $x = \dfrac{5}{a}$ or $\dfrac{2}{a}.$

8. $4x^2 + 4ax - 3a^2 = 0,$
or $$x^2 + ax = \frac{3a^2}{4}.$$
$$\left(x + \frac{a}{2}\right)^2 = a^2.$$
Whence $x = \dfrac{a}{2}$ or $-\dfrac{3a}{2}.$

9. $x^2 + 4\sqrt{a}x - 5a = 0,$
or $$x^2 + 4a\sqrt{x} = 5a.$$
$$(x + 2\sqrt{a})^2 = 9a.$$
Whence $x = \sqrt{a}$ or $-5\sqrt{a}.$

10. $$2x^2 + 9x\sqrt{h} = 5h,$$
or $$x^2 + \frac{9\sqrt{h}x}{2} = \frac{5h}{2}.$$
$$\left(x + \frac{9\sqrt{h}}{4}\right)^2 = \frac{121h}{16}.$$
Whence $x = \dfrac{\sqrt{h}}{2}$ or $-5\sqrt{h}.$

11. $$a^2x^2 + 2ab = a^2 + b^2,$$
or $$x^2 = \frac{a^2 - 2ab + b^2}{a^2}.$$
$$x^2 = \frac{(a-b)^2}{a^2}.$$
Whence $x = \pm\dfrac{a-b}{a}.$

12. $$5x^2 + ax = x,$$
or $$x^2 + \frac{(a-1)x}{5} = 0.$$
$$\left(x + \frac{a-1}{10}\right)^2 = \left(\frac{a-1}{10}\right)^2.$$
Whence $x = 0$ or $\dfrac{1-a}{5}.$

13. $$x(x-b) = a(a+b),$$
or $$x^2 - bx = a^2 + ab.$$
$$\left(x - \frac{b}{2}\right)^2 = \left(a + \frac{b}{2}\right)^2.$$
Whence $x = a + b$ or $-a.$

14. $$1 - \frac{4}{x} + \frac{4}{x^2} = \frac{b}{4x^2},$$
or $$x^2 - 4x + 4 = \frac{b}{4}.$$
$$(x-2)^2 = \frac{b}{4}.$$
Whence $x = 2 \pm \dfrac{\sqrt{b}}{2}.$

15. $$x^2 + 2x = ax + 2a,$$
or $$x^2 + (2-a)x = 2a.$$
Whence $x = a$ or $-2.$

16. $x^2-(a+1)x+a=0,$

or $x^2-(a+1)x=-a.$

$$\left(x-\frac{a+1}{2}\right)^2=\left(\frac{a-1}{2}\right)^2.$$

Whence $x=a$ or 1.

17. $x^2+bx+cx+bc=0,$

or $x^2+(b+c)x=-bc.$

$$\left(x+\frac{b+c}{2}\right)^2=\left(\frac{b-c}{2}\right)^2.$$

Whence $x=-c$ or $-b$.

18. $x^2-ax+4x-4a=0,$

or $x^2-(a-4)x=4a.$

$$\left(x-\frac{a-4}{2}\right)^2=\left(\frac{a+4}{2}\right)^2.$$

Whence $x=a$ or -4.

19. $x^2+2a^2b^2=a^2x+2b^2x,$

or $x^2-(a^2+2b^2)x=-2a^2b^2.$

$$\left(x-\frac{a^2+2b^2}{2}\right)^2=\left(\frac{a^2-2b^2}{2}\right)^2.$$

Whence $x=a^2$ or $2b^2$.

20. $\frac{1}{a}-\frac{1}{5x}=1-\frac{5x}{a},$

or $x^2+\frac{(1-a)x}{5}=\frac{a}{25}.$

$$\left(x+\frac{1-a}{10}\right)^2=\left(\frac{1+a}{10}\right)^2.$$

Whence $x=\frac{a}{5}$ or $-\frac{1}{5}$.

21. $x^2+bx+c=0,$

or $x^2+bx=-c.$

$$\left(x+\frac{b}{2}\right)^2=\frac{b^2-4c}{4}.$$

Whence $x=\frac{-b\pm\sqrt{b^2-4c}}{2}.$

22. $ax^2+bx+c=0,$

or $x^2+\frac{b}{a}x=-\frac{c}{a}.$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}.$$

Whence $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$

Page 274 (First set)

1. $2x^2+5x+2=0.\quad\therefore x=\frac{-5\pm\sqrt{25-4\cdot2\cdot2}}{4}=-\frac{1}{2}$ or -2.

2. $3x^2+5x=2$, or $3x^2+5x-2=0.$

$$\therefore x=\frac{-5\pm\sqrt{25+24}}{6}=\tfrac{1}{3}\text{ or }-2.$$

3. $x^2-3x-10=0.\quad\therefore x=\frac{3\pm\sqrt{9+40}}{2}=5$ or -2.

4. $2x+2=x^2$, or $x^2-2x-2=0.\quad\therefore x=\frac{2\pm\sqrt{4+8}}{2}=1\pm\sqrt{3}.$

5. $x^2-x=1.\quad\therefore x=\frac{1\pm\sqrt{1+4}}{2}=\frac{1\pm\sqrt{5}}{2}.$

6. $2x^2-\frac{11x}{2}-\frac{15}{2}=0$, or $4x^2-11x-15=0.$

$$\therefore x=\frac{11\pm\sqrt{121+240}}{8}=\frac{15}{4}\text{ or }-1.$$

7. $2x^2 - 3x = 1. \quad \therefore x = \dfrac{3 \pm \sqrt{9+8}}{4} = \dfrac{3 \pm \sqrt{17}}{4}.$

8. $4x + 5 = x^2$, or $x^2 - 4x - 5 = 0. \quad \therefore x = \dfrac{4 \pm \sqrt{16+20}}{2} = 5$ or -1.

9. $x^2 + x\sqrt{5} = 10$, or $x^2 + x\sqrt{5} - 10 = 0$.

$\therefore x = \dfrac{-\sqrt{5} \pm \sqrt{5+40}}{2} = \sqrt{5}$ or $-2\sqrt{5}$.

10. $12x = 1 - 72x^2$, or $72x^2 + 12x - 1 = 0$.

$\therefore x = \dfrac{-12 \pm \sqrt{144+288}}{144} = \dfrac{-1 \pm \sqrt{3}}{12}.$

11. $x^2 + 2hx - 3h^2 = 0. \quad \therefore x = \dfrac{-2h \pm \sqrt{4h^2 + 12h^2}}{2} = h$ or $-3h$.

12. $2m^2 = 9mx + 5x^2$, or $5x^2 + 9mx - 2m^2 = 0$.

$\therefore x = \dfrac{-9m \pm \sqrt{81m^2 + 40m^2}}{10} = \dfrac{m}{5}$ or $-2m$.

13. $2x^2 + kx - 3k^2 = 0. \quad x = \dfrac{-k \pm \sqrt{k^2 + 24k^2}}{4} = k$ or $-\dfrac{3k}{2}.$

14. $x^2 + 2x\sqrt{a} - 3a = 0. \quad \therefore x = \dfrac{-2\sqrt{a} \pm \sqrt{4a + 12a}}{2} = \sqrt{a}$ or $-3\sqrt{a}$.

15. $mx = -m^2 + 6x^2$, or $6x^2 - mx - m^2 = 0$.

$\therefore x = \dfrac{m \pm \sqrt{m^2 + 24m^2}}{12} = \dfrac{m}{2}$ or $-\dfrac{m}{3}$.

16. $x^2 + \dfrac{kx}{2}\sqrt{2} - k^2 = 0. \quad \therefore x = \dfrac{-\dfrac{k\sqrt{2}}{2} \pm \sqrt{\dfrac{2k^2}{4} + 4k^2}}{2} = \dfrac{k}{2}\sqrt{2}$ or $-k\sqrt{2}$.

17. $n^2x^2 - 3knx - 10k^2 = 0. \quad \therefore x = \dfrac{3kn \pm \sqrt{9k^2n^2 + 40k^2n^2}}{2n^2} = \dfrac{5k}{n}$ or $-\dfrac{2k}{n}$.

18. $6m^2x^2 + 19mnx = 7n^2$, or $6m^2x^2 + 19mnx - 7n^2 = 0$.

$\therefore x = \dfrac{-19mn \pm \sqrt{361m^2n^2 + 168m^2n^2}}{12m^2} = \dfrac{n}{3m}$ or $-\dfrac{7n}{2m}$.

19. $x^2 + 2x = hx + 2h$, or $x^2 + (2-h)x - 2h = 0. \quad \therefore x = h$ or -2.

20. $x^2 + rx - sx - rs = 0$, or $x^2 + (r-s)x - rs = 0$.

$\therefore x = \dfrac{s - r \pm \sqrt{(r-s)^2 + 4rs}}{2} = s$ or $-r$.

21. $2x^2 + rs = rx + 2sx$ or $2x^2 - (r+2s)x + rs = 0$.

$\therefore x = \dfrac{r + 2s \pm \sqrt{(r+2s)^2 - 8rs}}{4} = s$ or $\dfrac{r}{2}$.

22. $mnx^2 + nx = 3mx + 3$, or $mnx^2 + (n - 3m)x - 3 = 0$.

$$\therefore x = \frac{3m - n \pm \sqrt{(n - 3m)^2 + 12mn}}{2mn} = \frac{3}{n} \text{ or } -\frac{1}{m}.$$

23. $mhx^2 + 4hx = 3mx + 12$, or $mhx^2 + (4h - 3m)x - 12 = 0$.

$$\therefore x = \frac{3m - 4h \pm \sqrt{(4h - 3m)^2 + 48hm}}{2hm} = \frac{3}{h} \text{ or } \frac{-4}{m}.$$

Page 274 (Second set)

1. Let $x =$ the number.
Then $x^2 + 2x = 15$.
Whence $x = -5$ or 3.

2. Let $x =$ the less number.
Then $x + 11 =$ the greater number.
Hence
$x(x + 11) = 42$,
or $x^2 + 11x = 42$.
Whence $x = -14$ or $+3$,
and $x + 11 = -3$ or $+14$.

3. Let $n =$ the number.
Then $2n^2 - n = 45$.
Whence $n = 5$ or $-\frac{9}{2}$.

4. Let $x =$ the less number.
Then $x + 1 =$ the greater number.
Therefore
$x(x + 1) = 462$,
or $x^2 + x = 462$.
Whence $x = 21$ or -22,
and $x + 1 = 22$ or -21.

5. Let n and $n + 2$ represent the odd numbers.
Then $n(n+2) = 255$,
or $n^2 + 2n = 255$.
Whence $n = 15$ or -17,
and $n + 2 = 17$ or -15.

6. Let n, $n + 2$, and $n + 4$ represent the even numbers.
Then
$$n + n + 2 + n + 4 = \frac{n(n + 2)}{6},$$
or $n^2 - 16n - 36 = 0$.
Whence $n = 18$ or -2,
$n + 2 = 20$ or 0,
and $n + 4 = 22$ or 2.

7. Let $n =$ the width in rods.
Then $n + 16 =$ the length in rods.
Therefore
$n(n + 16) = 32 \cdot 160$,
or $n^2 + 16n = 5120$.
Whence $n = 64$ or -80,
and $n + 16 = 80$ or -64.
The negative roots are rejected.

8. Let x and $\frac{1}{x}$ represent the number and its reciprocal.
Then $x + \frac{1}{x} = \frac{41}{20}$,
or $x^2 - \frac{41x}{20} = -1$.
Whence $x = \frac{5}{4}$ or $\frac{4}{5}$.
Then $\frac{1}{x} = \frac{4}{5}$ or $\frac{5}{4}$.

9. Let $n =$ the altitude in rods.
Then $n + 51 =$ the base in rods.
Therefore
$$\frac{n(n + 51)}{2} = \frac{45}{8} \cdot 160,$$

or $\quad n^2 + 51\,n = 1800.$

Whence $\quad n = 24$ or $-75,$

and $\quad n + 51 = 75$ or $-24.$

The negative values are rejected.

10. Let $\quad s =$ the side of one in rods.

Then $\quad s + 20 =$ the side of the other in rods.

Therefore

$$s^2 + (s + 20)^2 = 62.5 \cdot 160,$$

or $\quad s^2 + 20\,s = 4800.$

Whence $\quad s = 60$ or $-80,$

and $\quad s + 20 = 80$ or $-60.$

The negative values are rejected.

11. Let $\quad s =$ the side of the square and the breadth of the rectangle in inches.

Then $\quad s + 7 =$ the side of the rectangle.

Therefore $\quad s(s + 7) + 18 = 2\,s^2,$

or $\quad s^2 - 7\,s = 18.$

Whence $\quad s = 9$ or $-2.$

The negative value is rejected.

12. Let $\quad l =$ the shorter leg in feet.

Then $\quad l + 31 =$ the longer leg.

Therefore

$$l^2 + (l + 31)^2 = (41)^2,$$

or $\quad l^2 + 31\,l = 360.$

Whence $\quad l = 9$ or $-40,$

and $\quad l + 31 = 40$ or $-9.$

The negative roots are rejected.

13. Let $3\,x$ and $4\,x$ represent the legs.

Then

$$9\,x^2 + 16\,x^2 = 400,$$

or $\quad x^2 = 16.$

Whence $\quad x = 4$ or $-4.$

Then $\quad 3\,x = 12,$

and $\quad 4\,x = 16.$

14. Let x and $2\,x$ represent the other leg and the hypotenuse.

Then $x^2 + 81 = 4\,x^2,$

or $\quad 27 = x^2.$

Whence $\quad x = \pm 3\sqrt{3}.$

But $\quad 3\sqrt{3} = 5.19+.$

Then $\quad x = 5.19+,$

and $\quad 2\,x = 10.39+.$

15. Let $\quad l =$ the other leg.

Then

$$l^2 + (5\sqrt{3})^2 = 100,$$

or $\quad l^2 = 25.$

Whence $\quad l = \pm 5.$

Since one leg is $\frac{1}{2}$ the hypotenuse, one angle is 30°, one 60°, and the third 90°.

16. Let $\quad x =$ the side of the square in feet.

Then $\quad x^2 = 4\,x.$

Whence $\quad x = 4$ or $0.$

17. Let $x =$ the side of the square in inches.

Then $\quad \left(\dfrac{x}{12}\right)^2 = 4\,x,$

or $\quad x^2 - 576\,x = 0.$

Whence $\quad x = 576$ or $0,$

and $\quad \frac{576}{12} = 48$ feet.

18. Let $s =$ the side of the square in inches.

Then $\quad s^2 = 4\left(\dfrac{s}{12}\right),$

or $\quad 3\,s^2 = s.$

Whence $\quad s = \frac{1}{3}$ or $0.$

19. Let x, $x+2$, and $x+4$ represent the dimensions and the longest straight line respectively in inches.

Then
$$x^2+(x+2)^2=(x+4)^2,$$
or $$x^2-4x=12.$$
Whence $x=6$ or -2.
Whence $x=6$,
$x+2=8$,
and $x+4=10$.

20. Let x, $2x$, and $3x$ represent the dimensions in inches.

Then $2\cdot 2x^2+2\cdot 3x^2+2\cdot 6x^2=792,$
or $$22x^2=792.$$
Whence $x=\pm 6$.
Then $x=6$,
$2x=12$,
and $3x=18$.

21. Let x and $x+1$ represent the edges in yards.

Then
$$(x+1)^3-x^3=61,$$
or $$x^2+x-20=0.$$
Whence $x=-5$ or $+4$.
Therefore $x=4$,
and $x+1=5$.

22. Let r and $r+5$ represent the rates in miles per hour.

Then
$$\frac{280}{r}-\frac{280}{r+5}=1,$$
or $$r^2+5r=1400.$$
Whence $r=35$ or -40.
Therefore $r=35$,
and $r+5=40$.

23. Let r and $r+4$ represent the rates in miles per hour.

Then
$$\frac{80}{r}+\frac{80}{r+4}=9,$$
or $$9r^2-124r=320.$$
Whence $r=16$ or $-2\frac{2}{9}$.
Therefore $r=16$,
and $r+4=20$.

24. Let x and $x+2$ represent the width and the length in inches respectively.

Then
$$2(x-2)(x)=x(x+2),$$
or $$x^2-6x=0.$$
Whence $x=6$ or 0.
Then $x=6$,
and $x+2=8$.

25. Let x and $x-40$ represent the number of acres bought and sold respectively.

Then $\dfrac{16{,}000}{x}+20=\dfrac{16{,}000}{x-40},$
or $x^2-40x-32{,}000=0.$
Whence $x=200$ or -160,
and $x-40=160$.

26. Let $x=$ the original cost of an orange in cents.

Then $\dfrac{50}{x}-5=\dfrac{50}{x+\frac{10}{12}},$
or $x^2+\dfrac{5x}{6}-\dfrac{25}{3}=0.$
Whence $x=2\frac{1}{2}$ or $-3\frac{1}{3}$.

27. Let x and $x+2$ represent the time in hours required by each respectively.

Then
$$\frac{1}{x}+\frac{1}{x+2}=\frac{4}{3},$$
or $$2x^2+x=3.$$

Whence $x = 1$ or $-\frac{3}{2}$.
Therefore $x = 1$,
and $x + 2 = 3$.

28. Let r and $r + 7$ represent the rates in miles per hour respectively.

Then
$$\frac{200}{r} - \frac{200}{r+7} = \frac{7}{4},$$
or $r^2 + 7r = 800$.
Whence $r = 25$ or -32.
Therefore $r = 25$,
and $r + 7 = 32$.

29. Let $x =$ the number of miles B goes,
and $x + 17 =$ the number of miles A goes.

Then
$$x^2 + (x + 17)^2 = (53)^2,$$
or $x^2 + 17x = 1260$.
Whence $x = 28$ or -45.
Therefore $\frac{x}{5} = \frac{28}{5}$, B's rate in miles per hour;
and $\frac{x + 17}{5} = \frac{45}{5} = 9$, A's rate in miles per hour.

30. Let $x =$ the distance between them at the end of 5 hours.
Then $x - 9 =$ A's distance at the end of 5 hours,
and $x - 8 =$ B's distance at the end of 5 hours.
Then $(x - 8)^2 + (x - 9)^2 = x^2$,
or $x^2 - 34x + 145 = 0$.
Whence $x = 5$ or 29.
The value 5 for x is rejected.

Hence A's rate $= \frac{29 - 9}{5} = 4$ miles per hour,

and B's rate $= \frac{29 - 8}{5} = 4\frac{1}{5}$ miles per hour.

31. Let $s =$ the height in feet.

Then $\frac{32 \cdot (15)^2}{2} = s$,
or $3600 = s$.

32. Let $x =$ the time in seconds for falling,
and $6\frac{1}{2} - x =$ the time in seconds for the sound to return.
Then $16x^2 =$ the distance fallen,
and $(6\frac{1}{2} - x)\,1152 =$ the distance the sound returns.
Hence $16x^2 = (\frac{13}{2} - x)\,1152$,
or $x^2 + 72x - 468 = 0$.
Whence $x = 6$ or -78.

Then height $= \frac{32 \cdot 6^2}{2} = 576$ feet.

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1. $55 \cdot 3 \div 11 = 15$, the number of revolutions.

2. $132 \cdot 3 \div (4 \cdot \frac{22}{7}) = 31\frac{1}{2}$, the number of revolutions.

3. Let $x =$ the circumference of the fore wheel in feet.

Then $x + 2 =$ the circumference of the rear wheel in feet.

Therefore $$\frac{420}{x} - \frac{420}{x+2} = 5,$$

or $$x^2 + 2x = 168.$$

Whence $$x = 12 \text{ or } -14,$$

and $$x + 2 = 14.$$

4. Let $x =$ the circumference of the fore wheel in feet.

Then $x + 2 =$ the circumference of the rear wheel in feet.

Therefore $$\frac{300}{x} - \frac{300}{x+2} = 5,$$

or $$x^2 + 2x = 120.$$

Whence $$x = 10 \text{ or } -12,$$

and $$x + 2 = 12.$$

Again $$10 \div \frac{2 \cdot 22}{7} = 1\frac{10}{11}, \text{ radius of the fore wheel.}$$

$$12 \div \frac{2 \cdot 22}{7} = 1\frac{13}{22}, \text{ radius of the rear wheel.}$$

5. Let x and $x - 2$ represent the circumference of the rear and the fore wheels respectively in feet.

Then $$\frac{60}{x} + \frac{60}{x-2} = 11,$$

or $$11x^2 - 142x + 120 = 0.$$

Whence $$x = 12 \text{ or } \frac{10}{11},$$

and $$x - 2 = 10 \text{ or } -\frac{12}{11}.$$

Using the first values, $12 \div \frac{22}{7} = 3\frac{9}{11}$, diameter of rear wheel in feet.

and $10 \div \frac{22}{7} = 3\frac{2}{11}$, diameter of fore wheel in feet.

6. Let $r =$ the radius of the larger circle in inches.

Then $r - 7 =$ the radius of the smaller circle in inches.

Therefore $\frac{22}{7} r^2 - \frac{22}{7}(r - 7)^2 = 770,$

or $$44r = 924.$$

Whence $$r = 21,$$

and $$r - 7 = 14.$$

7.
$$AB(18 - AB) = 4 \cdot 20,$$
or
$$\overline{AB}^2 - 18\,\overline{AB} + 80 = 0.$$
Whence
$$AB = 8 \text{ or } 10.$$

8.
$$AB(5 - AB) = 4 \cdot \tfrac{3}{2},$$
or
$$\overline{AB}^2 - 5\,\overline{AB} + 6 = 0.$$
Whence
$$AB = 2 \text{ or } 3 \text{ feet},$$
and
$$BC = 3 \text{ or } 2 \text{ feet}.$$

9.
$$81 = BC(30 - BC),$$
or
$$\overline{BC}^2 - 30\,\overline{BC} + 81 = 0.$$
Whence
$$BC = 27 \text{ or } 3,$$
and
$$BD = 3 \text{ or } 27.$$

10.
$$400 = 2\,BD \cdot BD,$$
or
$$\overline{BD}^2 = 200.$$
Whence
$$BD = 10\sqrt{2}.$$

11.
$$\frac{12}{AP} = \frac{AP}{12 - AP},$$
or
$$\overline{AP}^2 - 12\,\overline{AP} - 144 = 0.$$
$$AP = -6 \pm 6\sqrt{5}.$$

The negative answer has a meaning in the dividing of a line externally by a point.

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1. $3 - 5 + 6 - 8 = -4.$
2. $6 \div 2 + 1 - 4 = 3 + 1 - 4 = 0.$
3. $24 \div 8 \cdot 4 - 4 + 6 = 3 \cdot 4 - 4 + 6 = 12 - 4 + 6 = 14.$
4. $(7 - 6)(18 - 2 \cdot 4) \div (20 \div 4) = 1 \cdot 10 \div 5 = 2.$
5. $42 - 2(18 - 2 \cdot 3) \div 4 + 3 \cdot 5 = 42 - 24 \div 4 + 15 = 42 - 6 + 15 = 51.$
6. $16 + 4 \div 8 - 10 + 51 \div 16 - 4 - 6 \cdot 3 \cdot 0 \cdot 2 + 18 \cdot 8 \div 48 - 2 \cdot 18 \div 12$
   $= 16 + \frac{1}{2} - 10 + 3\frac{3}{16} - 4 + 3 - 3 = 5\frac{11}{16}.$
7. $(16 \div 32 \times 48 \div 8 - 4 - 8 + 3)[12 \div 4 \div 3 - 1] + (42 \div 6 \cdot 7 - 42 - 6) \cdot 6$
   $= -6[0] + 1 \cdot 6 = 6.$
8. No; No; Yes.
9. In a^4, 4 is an exponent; in $4a$, 4 is a coefficient.

See pages 7–8. Also see pages 352–353.

If a number is the product of two factors, either of these factors is called the coefficient of the other in that product.

10. The power of a number is the result obtained when the operation indicated by the exponent has been performed.

An exponent indicates an operation of which the power is the result.

11. $x^2 - 5x + 6 = 5^2 - 5 \cdot 5 + 6 = 6$, when $x = 5$.

12. $x^3 - 3x^2 + 3x - 1 = 3^3 - 3 \cdot 3^2 + 3 \cdot 3 - 1 = 8$, when $x = 3$.

13. $x^3 - 3x^2y + 3xy^2 - y^3 = 4^3 - 3 \cdot 4^2 \cdot 2 + 3 \cdot 4 \cdot 2^2 - 2^3 = 8$, when $x = 4$ and $y = 2$.

14. $\dfrac{x^4 + x^2y^2 + y^4}{x^2 - xy + y^2} - \dfrac{x^3 + y^3}{x + y} = \dfrac{3^4 + 3^2 \cdot 2^2 + 2^4}{3^2 - 3 \cdot 2 + 2^2} - \dfrac{3^3 + 2^3}{3 + 2} = 19 - 7 = 12$, when $x = 3$ and $y = 2$.

15. The absolute value of a number is its value without regard to sign.

The absolute values of -4, $+7$, $-9a$ are 4, 7, and $9a$ respectively.

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3. $16 - 3 + 2 - 8 - 7 + 4 = 4$.

4. $4a - 6a - 10a + 2a + 18a = 8a$.

5 and 7.

$$\begin{array}{rcl}
7x - 4y - z & = & 7 - 8 - 3 = -4 \\
3x - 8y + z & = & 3 - 16 + 3 = -10 \\
-17x + 18y - 14z & = & -17 + 36 - 42 = -23 \\
\hline
-7x + 6y - 14z & = & -7 + 12 - 42 = -37
\end{array}$$

6.

$$\begin{array}{r}
4a^2 - 3a^2c - 4ac^2 \\
-8a^2 + 3a^2c - 8ac^2 \\
3a^2 - 6a^2c \qquad \\
\hline
-a^2 - 6a^2c - 12ac^2
\end{array}$$

The sum of the three numerical values in Exercise 7 equals the numerical value of the result.

8. Substitute for each letter in the several expressions and in their sum a convenient numerical value. The sum of the numerical values of the several expressions should equal the numerical value of their sum.

9. $ay + by + cy = (a + b + c)y$.

10. $3ax - 4bx + 6x = (3a - 4b + 6)x$.

11. $4x - abx - x = (3 - ab)x$.

12. $3(a + b) - c(a + b) = (3 - c)(a + b)$.

13. $4b(3x - 2) - 8c(3x - 2) = (4b - 8c)(3x - 2)$.

14. $4m(5a - 3c) - 6n(-3c + 5a) = (4m - 6n)(5a - 3c)$.

15. $6a - 4a = 2a$.

16. $5a^3 - 8a^3 = -3a^3$.

17. $(8x + 6) - (4x + 3) = 4x + 3$.

18. $(5x^2 + 20) - (7x^2 - 10) = -2x^2 + 30$.

19. $(4x - y^2 + 8 - 5ax + 9ac) - (x - 3y^2 + z - 4ac + 7ax)$
$= 3x + 2y^2 - z + 8 - 12ax + 13ac$.

20. $(4a^3 + m - 8x - 10ac + 4a^2m) - (a^3 - c + 3x - a^2m - 8ac)$
$= 3a^3 + m - 11x - 2ac + 5a^2m + c$.

21. $(3x^2 - 5x + 2) - (x^2 - 5x + 6) = 2x^2 - 4$.

22. $(8c^2 + 7cx - 10x^2 + 8) - (4x^2 - 3cx + c^2)$
$= -14x^2 + 10cx + 7c^2 + 8.$

23. $(4a^2 - 2ab + b^2) - (7a^2 - 10ab + 6b^2) = -3a^2 + 8ab - 5b^2.$

24. $(c^2 - 10cx + 8x^2) - (9x^2 - 10cx + 4 + c^2) = -x^2 - 4.$

25. The sum of the remainder and the subtrahend should equal the minuend.

26.

$$\begin{array}{lr}
 & ax - ac - 3c^2 \quad (1) \\
 & -3ac + 4c^2 \quad (2) \\
\hline
(1)+(2), & ax - 4ac + c^2 \quad (3) \\
 & -5ax + 4ac - c^2 + a^2 \quad (6) \\
\hline
(3)-(6), & 6ax - 8ac + 2c^2 - a^2
\end{array}
\qquad
\begin{array}{r}
4c^2 - 8ax + a^2 \quad (4) \\
-5c^2 + 3ax + 4ac \quad (5) \\
\hline
-c^2 - 5ax + a^2 + 4ac \quad (6)
\end{array}$$

27. $4x - 3 - (a - 2x) + (3x - a)$
$= 4x - 3 - a + 2x + 3x - a$
$= 9x - 3 - 2a.$

28. $6x + (3c - 8x + 2) - (c - x - 2)$
$= 6x + 3c - 8x + 2 - c + x + 2$
$= -x + 2c + 4.$

29. $6x - [-(a - c) + (3c - 4a)]$
$= 6x - [-a + c + 3c - 4a]$
$= 6x + a - c - 3c + 4a$
$= 6x + 5a - 4c.$

30. $7c - [(3c - 4) - 6 - (4x - 3a - c)]$
$= 7c - [3c - 4 - 6 - 4x + 3a + c]$
$= 7c - 3c + 10 + 4x - 3a - c$
$= 3c + 10 + 4x - 3a.$

31. $6x - 4(3 - 5x) - 4[2(x - 4) + 3(2x - 1) - (x - 7)]$
$= 6x - 12 + 20x - 4[2x - 8 + 6x - 3 - x + 7]$
$= 6x - 12 + 20x - 28x + 16$
$= -2x + 4.$

32. $3x - 2[1 - 3(2x - 3 - a) - 5\{a - (3x - 2a) - 4\}]$
$= 3x - 2[1 - 6x + 9 + 3a - 5\{a - 3x + 2a - 4\}]$
$= 3x - 2[10 - 6x + 3a - 15a + 15x + 20]$
$= 3x - 60 - 18x + 24a$
$= -15x - 60 + 24a.$

33. See pages 55–56.

34. $+(x^2 + 2xy + y^2) - (a^2).$

35. $+(x^2) - (49a^2 - 14ab + b^2).$

36. $+(y^2 + 6xy + 9x^2) - (m^2 + 10m + 25).$

37. $+(x^4 + 10x^2y^3 + 25y^6) - (c^8 - 12c^4d + 36d^2).$

38. See page 58.

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2. $(4x^2 - 3x)(2x) = 8x^3 - 6x^2.$

3. $(2x + 3)(5x - 6) = 10x^2 + 3x - 18.$

4. Substituting in Exercise 2, $(4 \cdot 2^2 - 3 \cdot 2)(2 \cdot 2) = 8 \cdot 2^3 - 6 \cdot 2^2 = 40.$ See page 63.

5. $(3x^2 - 1)^2 = 9x^4 - 6x^2 + 1.$

6. $(7x^{2a} - 8x^a + 3)^2 = 49x^{4a} + 106x^{2a} + 9 - 112x^{3a} - 48x^a.$

7. $(x^{\frac{1}{2}} + x^{\frac{1}{3}})^2 = x + 2x^{\frac{5}{6}} + x^{\frac{2}{3}}.$

8. $(e^x + 2e^{-x})^2 = e^{2x} + 4 + 4e^{-2x}.$

9. $(e^x - e^{-x})^3 = (e^x - e^{-x})^2(e^x - e^{-x}) = e^{3x} - 3e^x + 3e^{-x} - e^{-3x}.$

10. $(e^{2x} - 3e^{-x})^4 = [(e^{2x} - 3e^{-x})^2]^2 = e^{8x} - 12e^{5x} + 54e^{2x} - 108e^{-x} + 81e^{-4x}.$

11. $(x^{\frac{1}{2}} + x^{\frac{1}{4}} + 1)(x^{\frac{1}{2}} - x^{\frac{1}{4}} + 1) = x + x^{\frac{1}{2}} + 1.$

12.
$$\begin{array}{rrrrrrr}
 & & 4x^{3e} & -6x^e & +3 & & \\
 & & 7x^{3e} & -x^{2e} & +4 & & \\
\hline
28x^{6e} & & -42x^{4e} & +21x^{3e} & & & \\
 & -4x^{5e} & & +6x^{3e} & -3x^{2e} & & \\
 & & & +16x^{3e} & & -24x^e & +12 \\
\hline
28x^{6e} & -4x^{5e} & -42x^{4e} & +43x^{3e} & -3x^{2e} & -24x^e & +12
\end{array}$$

13. $(x^2 - 2xy^2 + y^4)(x^2 + 2xy^2 + y^4) = x^4 - 2x^2y^4 + y^8.$

14. $(x^{-1} - 3x - 2x^{-2})^2 = x^{-2} + 9x^2 + 4x^{-4} - 6 - 4x^{-3} + 12x^{-1}.$

15. $(x^{-\frac{1}{2}} + 2x^{\frac{1}{2}} - 3x^{\frac{3}{2}})^2 = x^{-1} - 2x + 9x^3 + 4 - 12x^2.$

16.
$$\begin{array}{rrrrrr}
\frac{2a^2}{3} & -\frac{a}{5} & +\frac{2}{7} & & & \\
\frac{2a^3}{3} & +\frac{a^2}{5} & -\frac{2a}{7} & & & \\
\hline
\frac{4a^5}{9} & -\frac{2a^4}{15} & +\frac{4a^3}{21} & & & \\
 & +\frac{2a^4}{15} & -\frac{a^3}{25} & +\frac{2a^2}{35} & & \\
 & & -\frac{4a^3}{21} & +\frac{2a^2}{35} & -\frac{4a}{49} & \\
\hline
\frac{4a^5}{9} & & -\frac{a^3}{25} & +\frac{4a^2}{35} & -\frac{4a}{49} &
\end{array}$$

17. $(5x^{2a} - 3x^{-2a} - 6x^{-a} + 3x^a)^2 = 25x^{4a} + 9x^{-4a} + 36x^{-2a} + 9x^{2a}$
$- 60x^a + 30x^{3a} + 36x^{-3a} - 18x^{-a} - 66.$

18. $x^2 - x - 90 = 81 + 9 - 90 = 0$, if $x = -9$.

19. $2^3 - 3 \cdot 2^2(-3) + 3 \cdot 2(-3)^2 - (-3)^3 = 125$, if $x = 2$ and $y = -3$.

20. $(-4)^3 + 3(-4)^2(-2) + 3(-4)(-2)^2 + (-2)^3 = -216$, if $x = -4$ and $y = -2$.

21. See page 59.

22. See page 61.

24. $(8x^4 - 6x^2 - 4x) \div (-2x) = -4x^3 + 3x + 2$.

25.
$$\begin{array}{l|l} x^2 - 7x + 12 & x - 3 \\ \underline{x^2 - 3x} & \overline{x - 4} \\ \quad -4x + 12 & \\ \quad \underline{-4x + 12} & \end{array}$$

26. $(e^1 - e^{-1})^2 = (2 - \frac{1}{2})^2 = (\frac{3}{2})^2 = \frac{9}{4}$, if $e = 2$.
$(e^1 - e^{-1})^2 = (-3 + \frac{1}{3})^2 = (-\frac{8}{3})^2 = \frac{64}{9}$, if $e = -3$.

27. $e^{2x} - 2e^0 + e^{-2x} = 2^4 - 2 \cdot 2^0 + 2^{-4} = 16 - 2 + \frac{1}{16} = 14\frac{1}{16}$, if $e = 2$ and $x = 2$.

28.
$$\begin{array}{r|lr} x^3 - 64 & x - 4 & \\ \underline{x^3 - 4x^2} & \overline{x^2 + 4x + 16} & (1) \\ 4x^2 & x^2 - 4x + 16 & (2) \\ \underline{4x^2 - 16x} & \overline{x^4 + 4x^3 + 16x^2} & \\ 16x - 64 & \quad -4x^3 - 16x^2 - 64x & \\ \underline{16x - 64} & \quad \underline{+16x^2 + 64x + 256} & \end{array}$$
$$(1) \cdot (2) = x^4 + 16x^2 + 256$$

29.
$$\begin{array}{r|l} x^4 \qquad - 8x^2 + 33x - 30 & x^2 + 3x - 5 \\ \underline{x^4 + 3x^3 - 5x^2} \qquad\qquad & \overline{x^2 - 3x + 6} \\ -3x^3 - 3x^2 \qquad\qquad & \\ \underline{-3x^3 - 9x^2 + 15x} \qquad & \\ 6x^2 + 18x - 30 & \\ \underline{6x^2 + 18x - 30} & \end{array}$$

30. See pages 79–80.

31.
$$\begin{array}{r|l} 8x^3 - \quad x^2 - 5 & 2x - 3 \\ \underline{8x^3 - 12x^2} \qquad & \overline{4x^2 + \frac{11}{2}x + \frac{33}{4}} \\ 11x^2 \qquad & \\ \underline{11x^2 - \frac{33}{2}x} \quad & \\ \frac{33}{2}x - 5 & \\ \underline{\frac{33}{2}x - \frac{99}{4}} & \\ \frac{79}{4}, \text{ remainder} & \end{array}$$

32.
$$\begin{array}{l|l}
4x^4 - x^2 - 3 & 2x^2 - x - 1 \\ \cline{2-2}
4x^4 - 2x^3 - 2x^2 & 2x^2 + x + 1 \\ \cline{1-1}
\quad 2x^3 + x^2 & \\
\quad 2x^3 - x^2 - x & \\ \cline{1-1}
\qquad 2x^2 + x - 3 & \\
\qquad 2x^2 - x - 1 & \\ \cline{1-1}
\qquad\qquad 2x - 2, \text{ remainder} &
\end{array}$$

33.
$$\begin{array}{l|l}
x^3 - 30xy + 125 + 8y^3 & x + 2y + 5 \\ \cline{2-2}
x^3 + 2x^2y + 5x^2 & x^2 - 5x - 2xy + 25 - 10y + 4y^2 \\ \cline{1-1}
\quad -5x^2 - 2x^2y - 30xy & \\
\quad -5x^2 - 10xy - 25x & \\ \cline{1-1}
\qquad -2x^2y + 25x - 20xy & \\
\qquad -2x^2y - 10xy - 4xy^2 & \\ \cline{1-1}
\qquad\quad 25x - 10xy + 4xy^2 + 125 & \\
\qquad\quad 25x + 50y + 125 & \\ \cline{1-1}
\qquad\qquad -10xy + 4xy^2 - 50y & \\
\qquad\qquad -10xy - 50y - 20y^2 & \\ \cline{1-1}
\qquad\qquad\quad 4xy^2 + 20y^2 + 8y^3 & \\
\qquad\qquad\quad 4xy^2 + 20y^2 + 8y^3 & \\ \cline{1-1}
\end{array}$$

34.
$$\begin{array}{l|l}
x^3 - 3xyz + y^3 + z^3 & x + y + z \\ \cline{2-2}
x^3 + x^2y + x^2z & x^2 - xy - xz + y^2 - yz + z^2 \\ \cline{1-1}
\quad -x^2y - x^2z & \\
\quad -x^2y - xy^2 - xyz & \\ \cline{1-1}
\qquad -x^2z + xy^2 - 2xyz & \\
\qquad -x^2z - xyz - xz^2 & \\ \cline{1-1}
\qquad\quad xy^2 - xyz + xz^2 & \\
\qquad\quad xy^2 + y^3 + y^2z & \\ \cline{1-1}
\qquad\qquad -xyz + xz^2 - y^2z & \\
\qquad\qquad -xyz - y^2z - yz^2 & \\ \cline{1-1}
\qquad\qquad\quad xz^2 + yz^2 + z^3 & \\
\qquad\qquad\quad xz^2 + yz^2 + z^3 & \\ \cline{1-1}
\end{array}$$

35.
$$\begin{array}{l|l}
x^6 - 4x^{-6} + 3x^{-10} & x^2 + 2x^{-2} + 3x^{-6} \\ \cline{2-2}
x^6 + 2x^2 + 3x^{-2} & x^4 - 2 + x^{-4} \\ \cline{1-1}
\quad -2x^2 - 3x^{-2} - 4x^{-6} & \\
\quad -2x^2 - 4x^{-2} - 6x^{-6} & \\ \cline{1-1}
\qquad x^{-2} + 2x^{-6} + 3x^{-10} & \\
\qquad x^{-2} + 2x^{-6} + 3x^{-10} & \\ \cline{1-1}
\end{array}$$

36. $(x^{\frac{3}{5}} - y^{\frac{3}{5}}) \div (x^{\frac{1}{10}} - y^{\frac{1}{10}})$

$$= x^{\frac{5}{10}} + x^{\frac{4}{10}}y^{\frac{1}{10}} + x^{\frac{3}{10}}y^{\frac{2}{10}} + x^{\frac{2}{10}}y^{\frac{3}{10}} + x^{\frac{1}{10}}y^{\frac{4}{10}} + y^{\frac{5}{10}}$$

$$= x^{\frac{1}{2}} + x^{\frac{2}{5}}y^{\frac{1}{10}} + x^{\frac{3}{10}}y^{\frac{1}{5}} + x^{\frac{1}{5}}y^{\frac{3}{10}} + x^{\frac{1}{10}}y^{\frac{2}{5}} + y^{\frac{1}{2}}.$$

37.

$$\begin{array}{l|l}
9m - 13 + 4m^{-1} & 3m^{\frac{1}{2}} - 5 + 2m^{-\frac{1}{2}} \\ \cline{2-2}
9m - 15m^{\frac{1}{2}} + 6 & 3m^{\frac{1}{2}} + 5 + 2m^{-\frac{1}{2}} \\ \cline{1-1}
\quad 15m^{\frac{1}{2}} - 19 & \\
\quad 15m^{\frac{1}{2}} - 25 + 10m^{-\frac{1}{2}} & \\ \cline{1-1}
\qquad 6 - 10m^{-\frac{1}{2}} + 4m^{-1} & \\
\qquad 6 - 10m^{-\frac{1}{2}} + 4m^{-1} & \\ \cline{1-1}
\end{array}$$

38.

$$\begin{array}{l|l}
x^{2a} - 29 + 4x^{-2a} & x^{a} - 5 - 2x^{-a} \\ \cline{2-2}
x^{2a} - 5x^{a} - 2 & x^{a} + 5 - 2x^{-a} \\ \cline{1-1}
\quad 5x^{a} - 27 & \\
\quad 5x^{a} - 25 - 10x^{-a} & \\ \cline{1-1}
\qquad - 2 + 10x^{-a} + 4x^{-2a} & \\
\qquad - 2 + 10x^{-a} + 4x^{-2a} & \\ \cline{1-1}
\end{array}$$

39.

$$\begin{array}{l|l}
9x^{2a} - 19x^{-a} + 25x^{-4a} & 3x^{a} - 7x^{-\frac{a}{2}} + 5x^{-2a} \\ \cline{2-2}
9x^{2a} - 21x^{\frac{a}{2}} + 15x^{-a} & 3x^{a} + 7x^{-\frac{a}{2}} + 5x^{-2a} \\ \cline{1-1}
\quad 21x^{\frac{a}{2}} - 34x^{-a} & \\
\quad 21x^{\frac{a}{2}} - 49x^{-a} + 35x^{-\frac{5a}{2}} & \\ \cline{1-1}
\qquad 15x^{-a} - 35x^{-\frac{5a}{2}} + 25x^{-4a} & \\
\qquad 15x^{-a} - 35x^{-\frac{5a}{2}} + 25x^{-4a} & \\ \cline{1-1}
\end{array}$$

40.

$$\begin{array}{l|l}
6a^{3} + \frac{35}{3}a^{2}x + \frac{35ax^{2}}{2} + 6x^{3} & \frac{3a}{2} + \frac{2x}{3} \\ \cline{2-2}
6a^{3} + \frac{8a^{2}x}{3} & 4a^{2} + 6ax + 9x^{2} \\ \cline{1-1}
\quad 9a^{2}x & \\
\quad 9a^{2}x + 4ax^{2} & \\ \cline{1-1}
\qquad \frac{27}{2}ax^{2} + 6x^{3} & \\
\qquad \frac{27}{2}ax^{2} + 6x^{3} & \\ \cline{1-1}
\end{array}$$

41.
$$\begin{array}{r|l}
\frac{9a^5}{5} - \frac{43a^4}{8} + \frac{243a^3}{20} - \frac{443a^2}{30} + \frac{59a}{4} - 12 & \frac{3a^3}{4} - \frac{5a^2}{6} + a - \frac{3}{2} \\ \cline{2-2}
\frac{9a^5}{5} - 2a^4 + \frac{12a^3}{5} - \frac{18a^2}{5} \qquad\qquad\qquad & \frac{12a^2}{5} - \frac{9a}{2} + 8 \\ \cline{1-1}
- \frac{27a^4}{8} + \frac{195a^3}{20} - \frac{67a^2}{6} \qquad\qquad\qquad & \\
- \frac{27a^4}{8} + \frac{15a^3}{4} - \frac{9a^2}{2} + \frac{27a}{4} \qquad & \\ \cline{1-1}
6a^3 - \frac{20a^2}{3} + 8a - 12 & \\
6a^3 - \frac{20a^2}{3} + 8a - 12 & \\ \cline{1-1}
\end{array}$$

Page 287 (First set)

1.
$$\begin{array}{r}
1 - 8 + 16 \qquad\quad \\
2 - 3 \qquad\qquad\quad \\ \hline
2 - 16 + 32 \qquad\quad \\
- 3 + 24 - 48 \\ \hline
2 - 19 + 56 - 48
\end{array}$$

$\therefore$ the product is $2x^3 - 19x^2 + 56x - 48$.

2.
$$\begin{array}{r}
1 - 4 + 4 \qquad\qquad\quad \\
1 + 4 + 4 \qquad\qquad\quad \\ \hline
1 - 4 + 4 \qquad\qquad\quad \\
+ 4 - 16 + 16 \qquad\quad \\
+ 4 - 16 + 16 \\ \hline
1 + 0 - 8 + 0 + 16
\end{array}$$

$\therefore$ the product is $x^4 - 8x^2 + 16$.

3.
$$\begin{array}{r}
1 - 1 + 1 \qquad\qquad \\
1 + 1 + 1 \qquad\qquad \\ \hline
1 - 1 + 1 \qquad\qquad \\
+ 1 - 1 + 1 \qquad \\
+ 1 - 1 + 1 \\ \hline
1 + 0 + 1 + 0 + 1
\end{array}$$

$\therefore$ the product is $a^4 + a^2b^2 + b^4$.

4.
$$\begin{array}{r|l}
2 + 5 + 2 & 2 + 1 \\ \cline{2-2}
2 + 1 \qquad & 1 + 2 \\ \cline{1-1}
4 + 2 & \\
4 + 2 & \\ \cline{1-1}
\end{array}$$

$x + 2$, the quotient.

5.
$$\begin{array}{r|l}
1 + 0 + 4 - 16 & 1 - 2 \\ \cline{2-2}
1 - 2 \qquad\qquad\quad & 1 + 2 + 8 \\ \cline{1-1}
2 + 4 \qquad\quad & \\
2 - 4 \qquad\quad & \\ \cline{1-1}
8 - 16 & \\
8 - 16 & \\ \cline{1-1}
\end{array}$$

$x^2 + 2x + 8$, the quotient.

6.
$$\begin{array}{r}
- 2 + 3 - 6 \qquad\qquad\qquad\quad \\
8 - 5 - 6 \qquad\qquad\qquad\quad \\ \hline
- 16 + 24 - 48 \qquad\qquad\qquad\quad \\
+ 10 - 15 + 30 \qquad\quad \\
+ 12 - 18 + 36 \\ \hline
- 16 + 34 - 51 + 12 + 36
\end{array}$$

$\therefore$ the product is $-16x^4 + 34x^3y - 51x^2y^2 + 12xy^3 + 36y^4$.

7.
$$\begin{array}{r|l}
9 - 6 + 13 - 4 + 4 & 3 - 1 + 2 \\ \cline{2-2}
9 - 3 + 6 \qquad\qquad\quad & 3 - 1 + 2 \\ \cline{1-1}
- 3 + 7 \qquad\qquad\quad & \\
- 3 + 1 - 2 \qquad\quad & \\ \cline{1-1}
6 - 2 + 4 & \\
6 - 2 + 4 & \\ \cline{1-1}
\end{array}$$

$3x^2 - x + 2$, the quotient.

8.
$$
\begin{array}{l|l}
1+0+0+0+4 & 1-2+2 \\ \cline{2-2}
\underline{1-2+2} \qquad & 1+2+2 \\
\quad 2-2 & \\
\quad \underline{2-4+4} & \\
\qquad 2-4+4 & \\
\qquad \underline{2-4+4} &
\end{array}
$$

$x^2 + 2xy + 2y^2$, the quotient.

10.
$$
\begin{array}{l|l}
4-2+2+0-5-3 & 2+0-2-1 \\ \cline{2-2}
\underline{4+0-4-2} & 2-1+3 \\
\quad -2+6+2 & \\
\quad \underline{-2+0+2+1} & \\
\qquad 6+0-6-3 & \\
\qquad \underline{6+0-6-3} &
\end{array}
$$

$2a - 1 + 3a^{-1}$, the quotient.

9.
$$
\begin{array}{l|l}
81+\ \ 0-171+\ \ 0+25 & 9+9-5 \\ \cline{2-2}
\underline{81+81-\ \ 45} & 9-9-5 \\
\quad -81-126 & \\
\quad \underline{-81-\ \ 81+45} & \\
\qquad -\ \ 45-45+25 & \\
\qquad \underline{-\ \ 45-45+25} &
\end{array}
$$

$9a^2 - 9ab - 5b^2$, the quotient.

11.
$$
\begin{array}{l|l}
8-12+6-1 & 2-1 \\ \cline{2-2}
\underline{8-\ \ 4} & 4-4+1 \\
\quad -\ \ 8+6 & \\
\quad \underline{-\ \ 8+4} & \\
\qquad 2-1 & \\
\qquad \underline{2-1} &
\end{array}
$$

$4x^{\frac{2}{3}} - 4x^{\frac{1}{3}}y^{-1} + y^{-2}$, the quotient.

12. (*a*) The expressions in Exercises 10 and 11 are not integral.
(*b*) Neither expression in Exercise 11 is rational.

Page 287 (Second set)

1. See pages 93–100.

2. $(x+3)^2 = x^2 + 6x + 9$.

3. $(x-5)^2 = x^2 - 10x + 25$.

4. $(2x+4)^2 = 4x^2 + 16x + 16$.

5. $(4x-3)^2 = 16x^2 - 24x + 9$.

6. $(x^2 - x)^2 = x^4 - 2x^3 + x^2$.

7. $(x - 3c^2)^2 = x^2 - 6c^2x + 9c^4$.

8. $(3x^2 - 4xc)^2 = 9x^4 - 24x^3c + 16x^2c^2$.

9. $(x^2 - x^{-2})^2 = x^4 - 2 + x^{-4}$.

10. $(x^4 - 3x^{-4})^2 = x^8 - 6 + 9x^{-8}$.

11. $(16x^2 + 8x^2c + x^2c^2) \div (4x + xc) = 4x + xc$. Formula I.

12. $(2a^x - a^{-x})(2a^x - a^{-x})(2a^x - a^{-x}) \div (2a^x - a^{-x}) = 4a^{2x} - 4 + a^{-2x}$.

13. $(x-c)(x+c) = x^2 - c^2$.

14. $(x+6)(x-6) = x^2 - 36$.

15. $(a-3c)(a+3c) = a^2 - 9c^2$.

16. $(m-x)(x+m) = m^2 - x^2$.

17. $(4x-3c)(3c+4x) = 16x^2 - 9c^2$.

18. $(x^3 - cx)(x^3 + cx) = x^6 - c^2x^2$.

19. $(4c^3 - a^5)(a^5 + 4c^3) = 16c^6 - a^{10}$.

20. $(x+3)(x+4) = x^2 + 7x + 12$.

21. $(b-3)(b-4) = b^2 - 7b + 12$.

22. $(c-1)(c+2) = c^2 + c - 2$.

23. $(x-3)(x+5) = x^2 + 2x - 15$.

24. $(a^2 - 4a)(a^2 + 6a) = a^4 + 2a^3 - 24a^2$.

25. $(a^x - 2a^{-x})(a^x + 5a^{-x}) = a^{2x} + 3 - 10a^{-2x}$.

26. $(cx - 4c^2)(cx + 8c^2) = c^2x^2 + 4c^3x - 32c^4$.

27. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$.

28. $(a+c+x)^2 = a^2 + c^2 + x^2 + 2ac + 2ax + 2cx$.

29. $(a-c+x)^2 = a^2 + c^2 + x^2 - 2ac + 2ax - 2cx$.

30. $(a-c-x)^2 = a^2 + c^2 + x^2 - 2ac - 2ax + 2cx$.

31. $(a-c+2)^2=a^2+c^2+4-2\,ac+4\,a-4\,c.$

32. $(x-c-3\,a)^2=x^2+c^2+9\,a^2-2\,cx-6\,ax+6\,ac.$

33. $\dfrac{a^2+9\,c^2+x^2-6\,ac+2\,ax-6\,cx}{a-3\,c+x}=a-3\,c+x.$ Formula V.

34. $(3\,c-5\,a+2\,x)^2=9\,c^2+25\,a^2+4\,x^2-30\,ac+12\,cx-20\,ax.$

35. $(x^{2a}+x^a-5)^2=x^{4a}+25+2\,x^{3a}-9\,x^{2a}-10\,x^a.$

36. $(a+a^{-1}-3)^2=a^2+a^{-2}+11-6\,a-6\,a^{-1}.$

37. $(a^x-a^{-x}+4)^2=a^{2x}+a^{-2x}+14+8\,a^x-8\,a^{-x}.$

38. Yes; regard $a+b$ as one term and c as the other. Then use Formula I.

39. $(x+c)^3=x^3+3\,cx^2+3\,c^2x+c^3.$

40. $(x-c)^3=x^3-3\,cx^2+3\,c^2x-c^3.$

41. $(x-1)^3=x^3-3\,x^2+3\,x-1.$

42. $(x+2)^3=x^3+6\,x^2+12\,x+8.$

43. $(x+3)^3=x^3+9\,x^2+27\,x+27.$

44. $(x-4)^3=x^3-12\,x^2+48\,x-64.$

45. $(x^2-2)^3=x^6-6\,x^4+12\,x^2-8.$

46. $(5\,a-4\,c)^3=125\,a^3-300\,a^2c+240\,ac^2-64\,c^3.$

47. $(2\,x^5-7\,x^2)^3=8\,x^{15}-84\,x^{12}+294\,x^9-343\,x^6.$

48. $(x^3-6\,x^2+12\,x-8)\div(x-2)=(x-2)^2.$ By § 46.

49. $(8-12\,x+6\,x^2-x^3)\div(4-4\,x+x^2)=2-x.$ By § 46.

50. $25-70+49=(-2)^2=4.$

51. $x^2+18\,x+81.$ Results are the same. Even powers of both positive and negative numbers are positive.

52. $81-9-12=60$, which equals $5\cdot 12$.

53. (*a*) $(42)^2=(40+2)^2=1600+160+4=1764.$

(*b*) $(59)^2=(60-1)^2=3600-120+1=3481.$

(*c*) $(73)^2=(70+3)^2=4900+420+9=5329.$

(*d*) $(105)^2=(100+5)^2=10{,}000+1000+25=11{,}025.$

(*e*) $(97)^2=(100-3)^2=10{,}000-600+9=9409.$

(*f*) $(1005)^2=(1000+5)^2=1{,}000{,}000+10{,}000+25=1{,}010{,}025.$

54. $16+81+25+72-40-90=64$, which equals $8\cdot 8$.

55. $27-54+36-8=1$, which equals 1^3.

56. $(a-2\,b)^3=a^3-6\,a^2b+12\,ab^2-8\,b^3.$ $(2\,b-a)^3=8\,b^3-12\,ab^2+6\,a^2b-a^3.$ Results have opposite signs. Odd powers of any number have the same sign as the number. Here one expression is the negative of the other.

57. $+1.$

58. $24\,a.$ When arranged the middle term of a trinomial square is twice the product of the square roots of the other two.

59. $x^2-6\,x+9.$

60. $4\,x^2-12\,ax+9\,a^2.$

61. $x^2+4\,x+4.$

62. $9\,x^2\pm 24\,x+16.$

63. $4\,x^2\pm 12\,x+9.$

64. $9\,x^2-4\,ax+\dfrac{4\,a^2}{9}.$

65. $a^2+2+a^{-2}.$

66. $a^{2x} - 2 + a^{-2x}$.
67. $a^{2x} - 8 + 16a^{-2x}$.
68. $a^{4x} + 10 + 25a^{-4x}$.
69. $a^{10} - 14a^2 + 49a^{-6}$.
70. $a^6 - 6a^2 + 9a^{-2}$.
71. $a^{4x} - 12a^x + 36a^{-2x}$.
72. $4a^{6x} + 20a^{2x} + 25a^{-2x}$.

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1. $4x + 8 = 4(x + 2)$.
2. $ax - 7ay = a(x - 7y)$.
3. $a^2c - ac^2 - 4ac = ac(a - c - 4)$.
4. $3xy + 21y^5 - 15y^3 = 3y(x + 7y^4 - 5y^2)$.
5. $x^{2a} - 3x^a + 12x = x(x^{2a-1} - 3x^{a-1} + 12)$.
6. $y^{2a} - 6y^a + 2y^{a-1} = y^a(y^a - 6 + 2y^{-1})$.
7. $5xy + 30y(x^2 + xy) = 5xy(1 + 6x + 6y)$.
8. $(7a^2 - 21ab + 7a) - 14ax = 7a(a - 3b + 1 - 2x)$.
9. $(3c^2 - 3cd) - a(45c^2 - 15c^3x) = 3c(c - d - 15ac + 5ac^2x)$.
10. $2r^{2x+3} + 12r^{x-7} - 16sr^{x+2} + 8sr^{x+4} = 2r^x(r^{x+3} + 6r^{-7} - 8sr^2 + 4sr^4)$.
11. $3(x + y) + a(x + y) = (x + y)(3 + a)$.
12. $a(x - 3) - b(x - 3) = (x - 3)(a - b)$.
13. $4x - 4y + bx - by = (x - y)(4 + b)$.
14. $3cx + 6ac + 8ax + 4x^2 = 3c(x + 2a) + 4x(2a + x) = (x + 2a)(3c + 4x)$.
15. $-6x^2 + 10x + 21xm - 35m = 2x(-3x + 5) - 7m(-3x + 5)$
$= (5 - 3x)(2x - 7m)$.
16. $rs - 2s + 3r - 6 - 5rx + 10x = s(r - 2) + 3(r - 2) - 5x(r - 2)$
$= (r - 2)(s + 3 - 5x)$.
17. $x^{3a} - 3x^{2a} - x^a + 3 - 6x^{4a} + 2x^{5a}$
$= x^{2a}(x^a - 3) - 1(x^a - 3) + 2x^{4a}(x^a - 3)$
$= (x^a - 3)(x^{2a} - 1 + 2x^{4a})$.
18. $x^{3a-2} + 2x^{a+1} - 15x^{2a-3} - 10 + 10x^{2a-3}$
$= x^{a+1}(x^{2a-3} + 2) - 5(x^{2a-3} + 2) = (x^{2a-3} + 2)(x^{a+1} - 5)$.
19. $x^2 + 6x + 9 = (x + 3)^2$.
20. $x^2 + x + \frac{1}{4} = (x + \frac{1}{2})^2$.
21. $9b^2 - 12b + 4 = (3b - 2)^2$.
22. $9r^2 + 49 - 42r = (3r - 7)^2$.
23. $a^2 - 2 + a^{-2} = (a - a^{-1})^2$.
24. $a^4 - 6 + 9a^{-4} = (a^2 - 3a^{-2})^2$.
25. $\frac{x^2}{9a^2} - \frac{8x}{a} + 144 = \left(\frac{x}{3a} - 12\right)^2$.
26. $x^{4a} - 10ax^{2a} + 25a^2 = (x^{2a} - 5a)^2$.
27. $r^{4x+2} + 4 - 4r^{2x+1} = (r^{2x+1} - 2)^2$.
28. $a^{2x} + 4a^{-2x} - 4 = (a^x - 2a^{-x})^2$.
29. $a^{4x} - 2a^x + a^{-2x} = (a^{2x} - a^{-x})^2$.
30. $4(a + 5)^2 - 12b(a + 5) + 9b^2 = [2(a + 5) - 3b]^2$.
31. $(a - b)^{2x} - 18x(a - b)^x + 81x^2 = [(a - b)^x - 9x]^2$.
32. $x^2 + 2x - 24 = (x + 6)(x - 4)$.
33. $x^2 + x - \frac{3}{4} = (x + \frac{3}{2})(x - \frac{1}{2})$.
34. $a^2 + .3a - .1 = (a + \frac{1}{2})(a - \frac{1}{5})$.
35. $c^2 - ac - 90a^2 = (c - 10a)(c + 9a)$.
36. $r^2s^2 + 6rs - 40 = (rs + 10)(rs - 4)$.

37. $a^2 + 5 + 6a^{-2} = (a + 3a^{-1})(a + 2a^{-1})$.
38. $15m^2 - 14mx - x^2 = (m - x)(x + 15m)$.
39. $90 + x - x^2 = (10 - x)(9 + x)$.
40. $12a^4 - a^2x - x^2 = (3a^2 - x)(x + 4a^2)$.
41. $a^{2x} - 20 + 19a^x = (a^x + 20)(a^x - 1)$.
42. $a^2 + 12a^{-2} + 7 = (a + 4a^{-1})(a + 3a^{-1})$.
43. $a^{2x} - 8a^{-2x} - 2 = (a^x - 4a^{-x})(a^x + 2a^{-x})$.
44. $120 + 7m^n - m^{2n} = (8 + m^n)(15 - m^n)$.
45. $a^{4x} - a^x - 6a^{-2x} = (a^{2x} - 3a^{-x})(a^{2x} + 2a^{-x})$.
46. $(m + n)^{4e} - 9(m + n)^{2e} - 22 = [(m + n)^{2e} - 11][(m + n)^{2e} + 2]$.

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2. $$\begin{aligned}2a^2 - 3a - 2 &= 2a^2 - 4a + a - 2\\ &= 2a(a - 2) + 1(a - 2)\\ &= (a - 2)(2a + 1).\end{aligned}$$
3. $$\begin{aligned}3a^2 + 8a - 3 &= 3a^2 + 9a - a - 3\\ &= 3a(a + 3) - 1(a + 3)\\ &= (a + 3)(3a - 1).\end{aligned}$$
4. $$\begin{aligned}4a^2 + a - 5 &= 4a^2 + 5a - 4a - 5\\ &= a(4a + 5) - 1(4a + 5)\\ &= (4a + 5)(a - 1).\end{aligned}$$
5. $$\begin{aligned}9c^2 - 71c - 8 &= 9c^2 - 72c + c - 8\\ &= 9c(c - 8) + 1(c - 8)\\ &= (c - 8)(9c + 1).\end{aligned}$$
6. $$\begin{aligned}5r^2 - 22r + 8 &= 5r^2 - 20r - 2r + 8\\ &= 5r(r - 4) - 2(r - 4)\\ &= (r - 4)(5r - 2).\end{aligned}$$
7. $$\begin{aligned}7x^2 + 62x - 9 &= 7x^2 + 63x - x - 9\\ &= 7x(x + 9) - 1(x + 9)\\ &= (x + 9)(7x - 1).\end{aligned}$$
8. $$\begin{aligned}6x^2 + 19x - 7 &= 6x^2 + 21x - 2x - 7\\ &= 3x(2x + 7) - 1(2x + 7)\\ &= (2x + 7)(3x - 1).\end{aligned}$$
9. $$\begin{aligned}6x^2 + 13x - 5 &= 6x^2 + 15x - 2x - 5\\ &= 3x(2x + 5) - 1(2x + 5)\\ &= (2x + 5)(3x - 1).\end{aligned}$$
10. $$\begin{aligned}2x^2 + 7x - 15 &= 2x^2 + 10x - 3x - 15\\ &= 2x(x + 5) - 3(x + 5)\\ &= (x + 5)(2x - 3).\end{aligned}$$
11. $$\begin{aligned}3x^2 - ax - 2a^2 &= 3x^2 - 3ax + 2ax - 2a^2\\ &= 3x(x - a) + 2a(x - a)\\ &= (x - a)(3x + 2a).\end{aligned}$$

12. $4a^4 - 12a^2 + 9 = (2a^2 - 3)^2.$

13. $25 + 4c^2d^2 - 20cd = (2cd - 5)^2.$

14. $-8n^4 + 3n^8 - 3 = 3n^8 - 9n^4 + n^4 - 3$
$$= 3n^4(n^4 - 3) + 1(n^4 - 3)$$
$$= (n^4 - 3)(3n^4 + 1).$$

15. $6x^{2y} - 13x^y + 6 = 6x^{2y} - 9x^y - 4x^y + 6$
$$= 3x^y(2x^y - 3) - 2(2x^y - 3)$$
$$= (2x^y - 3)(3x^y - 2).$$

16. $20x^2 - 9xy^3 - 20y^6 = 20x^2 + 16xy^3 - 25xy^3 - 20y^6$
$$= 4x(5x + 4y^3) - 5y^3(5x + 4y^3)$$
$$= (5x + 4y^3)(4x - 5y^3).$$

17. $c^2x^a + x^{2a} - 12c^4 = x^{2a} - 3c^2x^a + 4c^2x^a - 12c^4$
$$= x^a(x^a - 3c^2) + 4c^2(x^a - 3c^2)$$
$$= (x^a + 4c^2)(x^a - 3c^2).$$

18. $2x^2 - (a + 2b)x + ab = 2x^2 - ax - 2bx + ab$
$$= x(2x - a) - b(2x - a)$$
$$= (2x - a)(x - b).$$

19. $5m^{2n-4} + 9am^{n-2} - 2a^2 = 5m^{2n-4} + 10am^{n-2} - am^{n-2} - 2a^2$
$$= 5m^{n-2}(m^{n-2} + 2a) - a(m^{n-2} + 2a)$$
$$= (m^{n-2} + 2a)(5m^{n-2} - a).$$

20. $6x^{4a} + (3 - 2y^b)x^{2a} - y^b = 6x^{4a} - 2x^{2a}y^b + 3x^{2a} - y^b$
$$= 2x^{2a}(3x^{2a} - y^b) + 1(3x^{2a} - y^b)$$
$$= (3x^{2a} - y^b)(2x^{2a} + 1).$$

21. $20a^2b^{4-2x} - 9a - 20b^{2x-4} = 20a^2b^{4-2x} + 16a - 25a - 20b^{2x-4}$
$$= 4ab^{2-x}(5ab^{2-x} + 4b^{x-2}) - 5b^{x-2}(5ab^{2-x} + 4b^{x-2})$$
$$= (5ab^{2-x} + 4b^{x-2})(4ab^{2-x} - 5b^{x-2}).$$

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1. $m^2 - n^2 = (m + n)(m - n).$

2. $x^2 - 4 = (x + 2)(x - 2).$

3. $x^2 - \frac{1}{9} = (x + \frac{1}{3})(x - \frac{1}{3}).$

4. $25x^2 - 49b^2 = (5x + 7b)(5x - 7b).$

5. $a^2 - a^{-2} = (a + a^{-1})(a - a^{-1}).$

6. $a^4 - \dfrac{4c^2}{25} = \left(a^2 + \dfrac{2c}{5}\right)\left(a^2 - \dfrac{2c}{5}\right).$

7. $16a^6 - 25c^4d^{10} = (4a^3 + 5c^2d^5)(4a^3 - 5c^2d^5).$

8. $a^{2x} - a^{-2x} = (a^x + a^{-x})(a^x - a^{-x}).$

9. $a^4 - a^{-4} = (a^2 + a^{-2})(a + a^{-1})(a - a^{-1}).$

10. $(a + c)^2 - 1 = (a + c + 1)(a + c - 1).$

11. $(a - x)^4 - 4 = (a^2 - 2ax + x^2 + 2)(a^2 - 2ax + x^2 - 2).$

12. $9 - (2 + x)^2 = (5 + x)(1 - x).$

13. $16 - (x - a)^{6a} = [4 + (x - a)^{3a}][4 - (x - a)^{3a}].$

14. $5^{2a} - (m - 1)^{10a} = [5^a + (m - 1)^{5a}][5^a - (m - 1)^{5a}].$

15. $(a+c)^2-(m-n)^2=(a+c+m-n)(a+c-m+n).$

16. $(a-x)^{2r}-(c-5)^2=[(a-x)^r+c-5][(a-x)^r-c+5].$

17. $a^2+2ax+x^2-9=(a+x+3)(a+x-3).$

18. $25-10x+x^2-16m^2=(5-x)^2-16m^2$
$$=(5-x+4m)(5-x-4m).$$

19. $9-12a+4a^2-b^{2m}=(3-2a)^2-b^{2m}$
$$=(3-2a+b^m)(3-2a-b^m).$$

20. $m^2-a^2-6a-9=(m+a+3)(m-a-3).$

21. $x^2-4y^2+20y-25=x^2-(2y-5)^2$
$$=(x+2y-5)(x-2y+5).$$

22. $-28c^3d^2-49c^6+1-4d^4=1-(7c^3+2d^2)^2$
$$=(1+7c^3+2d^2)(1-7c^3-2d^2).$$

23. $12r^s-36+5^{2a}-r^{2s}=5^{2a}-(r^s-6)^2$
$$=(5^a+r^s-6)(5^a-r^s+6).$$

24. $(m-2)^2-4n^2+28n-49=(m-2)^2-(2n-7)^2$
$$=(m-2+2n-7)(m-2-2n+7)$$
$$=(m+2n-9)(m-2n+5).$$

25. $x^2-6x+9-y^2+8ay-16a^2=(x-3)^2-(y-4a)^2$
$$=(x-3+y-4a)(x-3-y+4a).$$

26. $4bd+4c^2-4d^2-4c-b^2+1=(2c-1)^2-(2d-b)^2$
$$=(2c-1+2d-b)(2c-1-2d+b).$$

27. $6c+h^2-1-9c^2-4hk+4k^2=(h-2k)^2-(1-3c)^2$
$$=(h-2k+1-3c)(h-2k-1+3c).$$

28. $h^2-4y^2-10h+8xy+25-4x^2=(h-5)^2-(2y-2x)^2$
$$=(h-5+2y-2x)(h-5-2y+2x).$$

29. $25x^2-20x+4-4y^2-9a^2-12ay=(5x-2)^2-(2y+3a)^2$
$$=(5x-2+2y+3a)(5x-2-2y-3a).$$

Page 294 (First set)

1. $x^4+x^2+1=x^4+2x^2+1-x^2=(x^2+1)^2-x^2=(x^2+1+x)(x^2+1-x).$

2. $x^4+x^2y^2+y^4=x^4+2x^2y^2+y^4-x^2y^2$
$$=(x^2+y^2)^2-x^2y^2$$
$$=(x^2+xy+y^2)(x^2-xy+y^2).$$

3. $x^4+4x^2+16=x^4+8x^2+16-4x^2$
$$=(x^2+4)^2-4x^2$$
$$=(x^2+2x+4)(x^2-2x+4).$$

4. $16y^4+4y^2+1=16y^4+8y^2+1-4y^2$
$$=(4y^2+1)^2-4y^2$$
$$=(4y^2+2y+1)(4y^2-2y+1).$$

5. $c^4+c^2d^2+25d^4=c^4+10c^2d^2+25d^4-9c^2d^2$
$$=(c^2+5d^2)^2-9c^2d^2$$
$$=(c^2+5d^2+3cd)(c^2+5d^2-3cd).$$

6. $1 - 19y^6 + 25y^{12} = 1 - 10y^6 + 25y^{12} - 9y^6$
$= (1 - 5y^6)^2 - 9y^6$
$= (1 - 5y^6 + 3y^3)(1 - 5y^6 - 3y^3).$

7. $4x^4 + 3x^2y^2 + 9y^4 = 4x^4 + 12x^2y^2 + 9y^4 - 9x^2y^2$
$= (2x^2 + 3y^2)^2 - 9x^2y^2$
$= (2x^2 + 3y^2 + 3xy)(2x^2 + 3y^2 - 3xy).$

8. $4x^4 - 28x^2y^6 + 9y^{12} = 4x^4 - 12x^2y^6 + 9y^{12} - 16x^2y^6$
$= (2x^2 - 3y^6)^2 - 16x^2y^6$
$= (2x^2 - 3y^6 + 4xy^3)(2x^2 - 3y^6 - 4xy^3).$

9. $9a^8 - 19a^4b^2 + 25b^4 = 9a^8 + 30a^4b^2 + 25b^4 - 49a^4b^2$
$= (3a^4 + 5b^2)^2 - 49a^4b^2$
$= (3a^4 + 5b^2 + 7a^2b)(3a^4 + 5b^2 - 7a^2b).$

10. $49h^4 - 44h^2k^4 + 4k^8 = 49h^4 - 28h^2k^4 + 4k^8 - 16h^2k^4$
$= (7h^2 - 2k^4)^2 - 16h^2k^4$
$= (7h^2 - 2k^4 + 4hk^2)(7h^2 - 2k^4 - 4hk^2).$

11. $x^8 + x^4 + 1 = x^8 + 2x^4 + 1 - x^4$
$= (x^4 + x^2 + 1)(x^4 - x^2 + 1)$
$= (x^4 - x^2 + 1)(x^2 + x + 1)(x^2 - x + 1).$

12. $c^8 - 6c^4 + 1 = c^8 - 2c^4 + 1 - 4c^4$
$= (c^4 - 1)^2 - 4c^4$
$= (c^4 + 2c^2 - 1)(c^4 - 2c^2 - 1).$

13. $16 + 4x^4 + x^8 = 16 + 8x^4 + x^8 - 4x^4$
$= (4 + x^4)^2 - 4x^4$
$= (4 + x^4 + 2x^2)(4 + x^4 - 2x^2).$

14. $y^8 + 16y^4 + 256 = y^8 + 32y^4 + 256 - 16y^4$
$= (y^4 + 16 + 4y^2)(y^4 + 16 - 4y^2)$
$= (y^2 + 4 + 2y)(y^2 + 4 - 2y)(y^4 + 16 - 4y^2).$

15. $3x^9y + 3x^5y^5 + 3xy^9 = 3xy(x^8 + x^4y^4 + y^8)$
$= 3xy(x^4 + x^2y^2 + y^4)(x^4 - x^2y^2 + y^4)$
$= 3xy(x^2 + xy + y^2)(x^2 - xy + y^2)(x^4 - x^2y^2 + y^4).$

16. $16h^4 - 33h^2k^2 + 36k^4 = 16h^4 + 48h^2k^2 + 36k^4 - 81h^2k^2$
$= (4h^2 + 6k^2 + 9hk)(4h^2 + 6k^2 - 9hk).$

17. $25c^4 - 51c^2d^2 + 49d^4 = 25c^4 + 70c^2d^2 + 49d^4 - 121c^2d^2$
$= (5c^2 + 7d^2 + 11cd)(5c^2 + 7d^2 - 11cd).$

18. $49a^4 - 32a^2b^2 + 64b^4 = 49a^4 + 112a^2b^2 + 64b^4 - 144a^2b^2$
$= (7a^2 + 8b^2 + 12ab)(7a^2 + 8b^2 - 12ab).$

19. $64x^4 + 119x^2y^2 + 81y^4 = 64x^4 + 144x^2y^2 + 81y^4 - 25x^2y^2$
$= (8x^2 + 9y^2 + 5xy)(8x^2 + 9y^2 - 5xy).$

20. $81a^4 - 171a^2b^2 + 25b^4 = 81a^4 - 90a^2b^2 + 25b^4 - 81a^2b^2$
$= (9a^2 - 5b^2 + 9ab)(9a^2 - 5b^2 - 9ab).$

21. $1 + 4x^4 = 1 + 4x^2 + 4x^4 - 4x^2$
$= (1 + 2x^2 + 2x)(1 + 2x^2 - 2x).$

22. $64c^4 + 1 = 64c^4 + 16c^2 + 1 - 16c^2$
$= (8c^2 + 1 + 4c)(8c^2 + 1 - 4c).$

23. $x^4 + 4y^4 = x^4 + 4x^2y^2 + 4y^4 - 4x^2y^2$
$= (x^2 + 2y^2 + 2xy)(x^2 + 2y^2 - 2xy).$

24. $x^8 + 4y^8 = x^8 + 4x^4y^4 + 4y^8 - 4x^4y^4$
$= (x^4 + 2y^4 + 2x^2y^2)(x^4 + 2y^4 - 2x^2y^2).$

25. $x^8 + 64 = x^8 + 16x^4 + 64 - 16x^4$
$= (x^4 + 8 + 4x^2)(x^4 + 8 - 4x^2).$

26. $x^{4a} + 4y^{8a} = x^{4a} + 4x^{2a}y^{4a} + 4y^{8a} - 4x^{2a}y^{4a}$
$= (x^{2a} + 2y^{4a} + 2x^ay^{2a})(x^{2a} + 2y^{4a} - 2x^ay^{2a}).$

27. $a^{4b}c^{12d} + 64e^{4x+4}$
$= a^{4b}c^{12d} + 16a^{2b}c^{6d}e^{2x+2} + 64e^{4x+4} - 16a^{2b}c^{6d}e^{2x+2}$
$= (a^{2b}c^{6d} + 8e^{2x+2} + 4a^bc^{3d}e^{x+1})(a^{2b}c^{6d} + 8e^{2x+2} - 4a^bc^{3d}e^{x+1}).$

Page 294 (Second set)

1. $x^3 + 64 = x^3 + 4^3 = (x + 4)(x^2 - 4x + 16).$

2. $x^3 + 27 = x^3 + 3^3 = (x + 3)(x^2 - 3x + 9).$

3. $a^3 - 64 = a^3 - 4^3 = (a - 4)(a^2 + 4a + 16).$

4. $8 + m^3 = 2^3 + m^3 = (2 + m)(4 - 2m + m^2).$

5. $27 - m^6 = 3^3 - (m^2)^3 = (3 - m^2)(9 + 3m^2 + m^4).$

6. $x^3 - \frac{a^3}{8} = x^3 - \left(\frac{a}{2}\right)^3 = \left(x - \frac{a}{2}\right)\left(x^2 + \frac{ax}{2} + \frac{a^2}{4}\right).$

7. $x^3 - y^{-3} = x^3 - (y^{-1})^3 = (x - y^{-1})(x^2 + xy^{-1} + y^{-2}).$

8. $8a^3 + 27b^3 = (2a)^3 + (3b)^3 = (2a + 3b)(4a^2 - 6ab + 9b^2).$

9. $1 - 125x^6 = 1^3 - (5x^2)^3 = (1 - 5x^2)(1 + 5x^2 + 25x^4).$

10. $\frac{x^3}{27} - y^6 = \left(\frac{x}{3}\right)^3 - (y^2)^3 = \left(\frac{x}{3} - y^2\right)\left(\frac{x^2}{9} + \frac{xy^2}{3} + y^4\right).$

11. $x^6 - y^6 = (x^3 + y^3)(x^3 - y^3) = (x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2).$

12. $x^6 + y^6 = (x^2)^3 + (y^2)^3 = (x^2 + y^2)(x^4 - x^2y^2 + y^4).$

13. $x^9 - a^3 = (x^3)^3 - a^3 = (x^3 - a)(x^6 + x^3a + a^2).$

14. $x^6 + a^9 = (x^2)^3 + (a^3)^3 = (x^2 + a^3)(x^4 - x^2a^3 + a^6).$

15. $(x + y)^3 - 8 = (x + y)^3 - 2^3 = (x + y - 2)(x^2 + 2xy + y^2 + 2x + 2y + 4).$

16. $x^3 - 9x^2 + 27x - 28 = (x^3 - 9x^2 + 27x - 27 - 1)$
$= (x - 3)^3 - 1^3$
$= (x - 3 - 1)(x^2 - 6x + 9 + x - 3 + 1)$
$= (x - 4)(x^2 - 5x + 7).$

17. $a^3b^6c^9 - 8d^{12} = (ab^2c^3)^3 - (2d^4)^3 = (ab^2c^3 - 2d^4)(a^2b^4c^6 + 2ab^2c^3d^4 + 4d^8).$

18. $x^{3e} - y^{-3e} = (x^e)^3 - (y^{-e})^3 = (x^e - y^{-e})(x^{2e} + x^ey^{-e} + y^{-2e}).$

19. $c^{6e} + 27d^{9x} = (c^{2e})^3 + (3d^{3x})^3 = (c^{2e} + 3d^{3x})(c^{4e} - 3c^{2e}d^{3x} + 9d^{6x}).$

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1.
$$\begin{array}{l|l} x^2 + bx + c & x - n \\ \cline{2-2} \underline{x^2 - nx} & x + (n + b) \\ (n + b)x + c & \\ \underline{(n + b)x - n^2 - nb} & \\ \qquad n^2 + nb + c, \text{ remainder} & \end{array}$$

2.
$$\begin{array}{l|l} x^3 + ax^2 + bx + c & x - n \\ \cline{2-2} \underline{x^3 - nx^2} & x^2 + (n + a)x + (n^2 + na + b) \\ (n + a)x^2 & \\ \underline{(n + a)x^2 - (n^2 + na)x} & \\ \quad + (n^2 + na + b)x & \\ \quad \underline{+ (n^2 + na + b)x - n^3 - n^2a - nb} & \\ \qquad n^3 + n^2a + nb + c, \text{ remainder} & \end{array}$$

3. $x^3 - x - 8 = 27 - 3 - 8 = 16$, if 3 is put for x.
16, remainder.

4.
$$\begin{array}{l|l} 1 + 0 + 0 - 1 + 6 & -2 \\ \underline{\quad - 2 + 4 - 8 + 18} & \\ 1 - 2 + 4 - 9 + 24 & \end{array}$$
By synthetic division

24, remainder.

5.
$$\begin{array}{l|l} 1 + 0 - 3 + 2 & +1 \\ \underline{\quad + 1 + 1 - 2} & \\ 1 + 1 - 2 \quad 0 & \end{array}$$
$\therefore\ x^3 - 3x + 2 = (x - 1)(x^2 + x - 2) = (x - 1)^2(x + 2).$

6.
$$\begin{array}{l|l} 1 + 0 - 4 + 3 & +1 \\ \underline{\quad + 1 + 1 - 3} & \\ 1 + 1 - 3 \quad 0 & \end{array}$$
$\therefore\ x^3 - 4x + 3 = (x - 1)(x^2 + x - 3).$

7.
$$\begin{array}{l|l} 1 + 0 + 2 + 3 & -1 \\ \underline{\quad - 1 + 1 - 3} & \\ 1 - 1 + 3 \quad 0 & \end{array}$$
$\therefore\ x^3 + 2x + 3 = (x + 1)(x^2 - x + 3).$

8.
$$\begin{array}{l|l} 1 + 0 - 1 - 6 & +2 \\ \underline{\quad + 2 + 4 + 6} & \\ 1 + 2 + 3 \quad 0 & \end{array}$$
$\therefore\ x^3 - x - 6 = (x - 2)(x^2 + 2x + 3).$

9.
$$\begin{array}{l|l} 1 + 0 - 1 + 6 & -2 \\ \underline{\quad - 2 + 4 - 6} & \\ 1 - 2 + 3 \quad 0 & \end{array}$$
$\therefore\ x^3 - x + 6 = (x + 2)(x^2 - 2x + 3).$

10.
$$\begin{array}{l|l} 1 + 0 - 11 + 6 & +3 \\ \underline{\quad + 3 + 9 - 6} & \\ 1 + 3 - 2 \quad 0 & \end{array}$$
$\therefore\ x^3 - 11x + 6 = (x - 3)(x^2 + 3x - 2).$

11.
$$\begin{array}{l|l} 1 + 0 - 11 - 6 & -3 \\ \underline{\quad - 3 + 9 + 6} & \\ 1 - 3 - 2 \quad 0 & \end{array}$$
$\therefore\ x^3 - 11x - 6 = (x + 3)(x^2 - 3x - 2).$

12.
$$\begin{array}{l|l} 1 + 0 - 14 - 8 & +4 \\ \underline{\quad + 4 + 16 + 8} & \\ 1 + 4 + 2 \quad 0 & \end{array}$$
$\therefore\ x^3 - 14x - 8 = (x - 4)(x^2 + 4x + 2).$

13.
$$\begin{array}{rrrr|l} 1 & +0 & -27 & -10 & -5 \\ & -5 & +25 & +10 & \\ \hline 1 & -5 & -2 & 0 & \end{array}$$
$\therefore x^3 - 27x - 10$
$= (x + 5)(x^2 - 5x - 2)$.

14.
$$\begin{array}{rrrr|l} 1 & +3 & +3 & +2 & -2 \\ & -2 & -2 & -2 & \\ \hline 1 & +1 & +1 & 0 & \end{array}$$
$\therefore x^3 + 3x^2 + 3x + 2$
$= (x + 2)(x^2 + x + 1)$.

15.
$$\begin{array}{rrrr|l} 1 & +4 & +5 & +2 & -1 \\ & -1 & -3 & -2 & \\ \hline 1 & +3 & +2 & 0 & \end{array}$$
$\therefore x^3 + 4x^2 + 5x + 2$
$= (x + 1)(x^2 + 3x + 2)$
$= (x + 1)(x + 1)(x + 2)$.

16.
$$\begin{array}{rrrr|l} 1 & -6 & +11 & -6 & +1 \\ & +1 & -5 & +6 & \\ \hline 1 & -5 & +6 & 0 & \end{array}$$
$\therefore x^3 - 6x^2 + 11x - 6$
$= (x - 1)(x^2 - 5x + 6)$
$= (x - 1)(x - 2)(x - 3)$.

17.
$$\begin{array}{rrrrr|l} 1 & +0 & -11 & +2 & +12 & -1 \\ & -1 & +1 & +10 & -12 & \\ \hline 1 & -1 & -10 & +12 & 0 & \end{array}$$
$$\begin{array}{rrrr|l} 1 & -1 & -10 & +12 & +3 \\ & +3 & +6 & -12 & \\ \hline 1 & +2 & -4 & 0 & \end{array}$$
$\therefore x^4 - 11x^2 + 2x + 12$
$= (x + 1)(x^3 - x^2 - 10x + 12)$
$= (x + 1)(x - 3)(x^2 + 2x - 4)$.

18.
$$\begin{array}{rrrrr|l} 4 & +0 & +0 & -3 & -1 & +1 \\ & +4 & +4 & +4 & +1 & \\ \hline 4 & +4 & +4 & +1 & 0 & \end{array}$$
$\therefore 4x^4 - 3x - 1$
$= (x - 1)(4x^3 + 4x^2 + 4x + 1)$.

19.
$$\begin{array}{rrrr|l} 1 & +0 & -5a^2 & +2a^3 & +2a \\ & +2a & +4a^2 & -2a^3 & \\ \hline 1 & +2a & -a^2 & 0 & \end{array}$$
$x^3 - 5a^2x + 2a^3$
$= (x - 2a)(x^2 + 2ax - a^2)$.

20.
$$\begin{array}{rrrr|l} 1 & +0 & -7m^2 & -6m^3 & -m \\ & -m & +m^2 & +6m^3 & \\ \hline 1 & -m & -6m^2 & 0 & \end{array}$$
$\therefore x^3 - 7m^2x - 6m^3$
$= (x + m)(x^2 - mx - 6m^2)$
$= (x + m)(x - 3m)(x + 2m)$.

21.
$$\begin{array}{rrrr|l} 1 & -2n & -5n^2 & +6n^3 & +n \\ & +n & -n^2 & -6n^3 & \\ \hline 1 & -n & -6n^2 & 0 & \end{array}$$
$\therefore x^3 - 2nx^2 - 5n^2x + 6n^3$
$= (x - n)(x^2 - nx - 6n^2)$
$= (x - n)(x - 3n)(x + 2n)$.

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1. $x^5 + 1 = (x + 1)(x^4 - x^3 + x^2 - x + 1)$.
2. $x^5 + y^5 = (x + y)(x^4 - x^3y + x^2y^2 - xy^3 + y^4)$.
3. $x^7 - y^7 = (x - y)(x^6 + x^5y + x^4y^2 + x^3y^3 + x^2y^4 + xy^5 + y^6)$.
4. $x^7 + 1 = (x + 1)(x^6 - x^5 + x^4 - x^3 + x^2 - x + 1)$.
5. $x^5 - 32 = (x - 2)(x^4 + 2x^3 + 4x^2 + 8x + 16)$.
6. $3125 - c^5 = (5 - c)(625 + 125c + 25c^2 + 5c^3 + c^4)$.
7. $c^7 - 128 = (c - 2)(c^6 + 2c^5 + 4c^4 + 8c^3 + 16c^2 + 32c + 64)$.
8. $c^7 - x^{14} = (c - x^2)(c^6 + c^5x^2 + c^4x^4 + c^3x^6 + c^2x^8 + cx^{10} + x^{12})$.
9. $x^{10} + y^5 = (x^2 + y)(x^8 - x^6y + x^4y^2 - x^2y^3 + y^4)$.
10. $1 + c^7d^{14} = (1 + cd^2)(1 - cd^2 + c^2d^4 - c^3d^6 + c^4d^8 - c^5d^{10} + c^6d^{12})$.
11. $c^{14} + 128 = (c^2 + 2)(c^{12} - 2c^{10} + 4c^8 - 8c^6 + 16c^4 - 32c^2 + 64)$.
12. $x^{10} + a^{15} = (x^2 + a^3)(x^8 - x^6a^3 + x^4a^6 - x^2a^9 + a^{12})$.

13. $x^5 - 32y^{10} = (x - 2y^2)(x^4 + 2x^3y^2 + 4x^2y^4 + 8xy^6 + 16y^8)$.
14. $128x^7 - 1 = (2x - 1)(64x^6 + 32x^5 + 16x^4 + 8x^3 + 4x^2 + 2x + 1)$.
15. $1024 - 243x^5 = (4 - 3x)(256 + 192x + 144x^2 + 108x^3 + 81x^4)$.

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1. $x^3 - x = x(x^2 - 1) = x(x + 1)(x - 1)$.
2. $x^{10} - x^2 = x^2(x^8 - 1) = x^2(x^4 + 1)(x^2 + 1)(x + 1)(x - 1)$.
3. $x^8 - 2x^4 + 1 = (x^4 - 1)^2 = (x^2 + 1)^2(x + 1)^2(x - 1)^2$.
4. $x^5 - 8x^3 + 16x = x(x^2 - 4)^2 = x(x + 2)^2(x - 2)^2$.
5. $x^4 - 10x^2 + 9 = (x^2 - 9)(x^2 - 1) = (x + 3)(x - 3)(x + 1)(x - 1)$.
6. $x^4 - 13x^2 + 36 = (x^2 - 9)(x^2 - 4) = (x + 3)(x - 3)(x + 2)(x - 2)$.
7. $18a^2x^2 - 24a^2x - 10a^2 = 2a^2(9x^2 - 12x - 5) = 2a^2(3x - 5)(3x + 1)$.
8. $x^2 - \frac{x}{6} - \frac{1}{6} = \left(x - \frac{1}{2}\right)\left(x + \frac{1}{3}\right)$.
9. $16x^4 + 8x^2 - 3 = (4x^2 + 3)(4x^2 - 1) = (4x^2 + 3)(2x + 1)(2x - 1)$.
10. $a^3 - a + a^2b - b = a(a^2 - 1) + b(a^2 - 1) = (a + b)(a + 1)(a - 1)$.
11. $3x^4 - 15x^2 + 12 = 3(x^2 - 4)(x^2 - 1)$
$= 3(x + 2)(x - 2)(x + 1)(x - 1)$.
12. $12a - 39ay - 51ay^2 = 3a(4 - 13y - 17y^2) = 3a(4 - 17y)(1 + y)$.
13. $x^4 - 3x^3 + 4x^2 - 12x = x^3(x - 3) + 4x(x - 3) = x(x^2 + 4)(x - 3)$.
14. $2a^3b + 3a^2b - 8ab - 12b = a^2b(2a + 3) - 4b(2a + 3)$
$= b(a + 2)(a - 2)(2a + 3)$.
15. $4a^2 - a^4 + 81 + 10a^2x - 36a - 25x^2 = (2a - 9)^2 - (a^2 - 5x)^2$
$= (2a - 9 + a^2 - 5x)(2a - 9 - a^2 + 5x)$.
16. $12cd^3 - 6a^3x - a^6 + 4c^2 + 9d^6 - 9x^2 = (2c + 3d^3)^2 - (a^3 + 3x)^2$
$= (2c + 3d^3 + a^3 + 3x)(2c + 3d^3 - a^3 - 3x)$.
17. $x^6 + 1 = (x^2 + 1)(x^4 - x^2 + 1)$.
18. $x^5 - \frac{a^4x}{16} = x\left(x^2 + \frac{a^2}{4}\right)\left(x + \frac{a}{2}\right)\left(x - \frac{a}{2}\right)$.
19. $x^{10} - y^{10} = (x^5 + y^5)(x^5 - y^5)$
$= (x+y)(x-y)(x^4 + x^3y + x^2y^2 + xy^3 + y^4)(x^4 - x^3y + x^2y^2 - xy^3 + y^4)$.
20. $x^8 - x^2 = x^2(x + 1)(x - 1)(x^2 + x + 1)(x^2 - x + 1)$.
21. $x^{12} + y^{12} = (x^4 + y^4)(x^8 - x^4y^4 + y^8)$.
22. $x^{12} - y^{12} = (x^6 + y^6)(x^6 - y^6)$
$= (x^6 + y^6)(x^3 + y^3)(x^3 - y^3)$
$= (x^2 + y^2)(x^4 - x^2y^2 + y^4)(x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2)$.
23. $x^{12} - 64 = (x^6 - 8)(x^6 + 8)$
$= (x^2 - 2)(x^4 + 2x^2 + 4)(x^2 + 2)(x^4 - 2x^2 + 4)$.
24. $x^{10} + xy^9 = x(x + y)(x^2 - xy + y^2)(x^6 - x^3y^3 + y^6)$.
25. $x^{12} - 8 = (x^4 - 2)(x^8 + 2x^4 + 4)$.
26. $x^4 - y^{16} = (x^2 + y^8)(x + y^4)(x - y^4)$.

27. $64x^{12} - 4x^4 = 4x^4(16x^8 - 1)$
$= 4x^4(2x^2+1)(2x^2-1)(2x^2+2x+1)(2x^2-2x+1).$

28. $x^{12} + 64y^{12} = (x^4+4y^4)(x^8 - 4x^4y^4 + 16y^8)$
$= (x^2+2y^2+2xy)(x^2+2y^2-2xy)(x^4+2x^3y+2x^2y^2+4xy^3+4y^4)$
$(x^4 - 2x^3y + 2x^2y^2 - 4xy^3 + 4y^4).$

29. $32x^{10} + y^{10} = (2x^2+y^2)(16x^8 - 8x^6y^2 + 4x^4y^4 - 2x^2y^6 + y^8).$

30. $x^{16} - y^{16} = (x^8+y^8)(x^4+y^4)(x^2+y^2)(x+y)(x-y).$

31. $16x^{16} - 1 = (4x^8+1)(2x^4+1)(2x^4-1)$
$= (2x^4+2x^2+1)(2x^4-2x^2+1)(2x^4+1)(2x^4-1).$

32. $5^{-4x} - 1 = (5^{-2x}+1)(5^{-2x}-1) = (5^{-2x}+1)(5^{-x}+1)(5^{-x}-1).$

33. $10 - 10c^{14}d^4 = 10(1 - c^{14}d^4) = 10(1+c^7d^2)(1-c^7d^2).$

34. $2cd - c^2 - d^2 = (c-d)(d-c).$

35. $x^4 - 9x^2 - x + 3 = (x-3)(x^3+3x^2-1).$

36. $x^4 - 7x^2y^2 + 81y^4 = (x^2+9y^2+5xy)(x^2+9y^2-5xy).$

37. $4c^4 + 20c^3d - 11c^2d^2 = c^2(2c+11d)(2c-d).$

38. $a^5 - a^3 + a - 1 = a^3(a^2-1) + (a-1) = (a^4+a^3+1)(a-1).$

39. $a^5 - a^4 - a^3 + a = a^4(a-1) - a(a+1)(a-1) = a(a^3-a-1)(a-1).$

40. $5d^2 - 5cd - 10c^2 = 5(d-2c)(d+c).$

41.
$$\begin{array}{rrrr|l} 1 & -3 & +8 & -12 & \underline{2} \\ & +2 & -2 & +12 & \\ \hline 1 & -1 & +6 & 0 & \end{array}$$

$$\therefore x^3 - 3x^2 + 8x - 12 = (x-2)(x^2-x+6).$$

42. $121x^4 - 476x^2y^2 + 100y^4 = (11x^2 - 10y^2 + 16xy)(11x^2 - 10y^2 - 16xy).$

43. $x^4 - x^2 + 12xy - 36y^2 = (x^2+x-6y)(x^2-x+6y).$

44. $y^4 - 18y^2 + 81 - 16x^4 - 24x^2y^3 - 9y^6 = (y^2-9)^2 - (4x^2+3y^3)^2$
$= (y^2 - 9 + 4x^2 + 3y^3)(y^2 - 9 - 4x^2 - 3y^3).$

45. $h^5k^5 - 1024k^5 = k^5(h-4)(h^4+4h^3+16h^2+64h+256).$

46. $x^3 - 83x^5 + 289x^7 = x^3(1 - 17x^2 + 7x)(1 - 17x^2 - 7x).$

47. $x^2 - y^2 - x - y = (x+y)(x-y-1).$

48. $289 - 100a^2 - b^2 - 20ab = (17 + 10a + b)(17 - 10a - b).$

49. $625a^8 - 169d^4 + 78cd^2 - 9c^2 = (25a^4 + 13d^2 - 3c)(25a^4 - 13d^2 + 3c).$

50. $x^{3n} - 125y^{6n} = (x^n - 5y^{2n})(x^{2n} + 5x^ny^{2n} + 25y^{4n}).$

51. $4x^4 - 37x^2 + 9 = (x+3)(x-3)(2x+1)(2x-1).$

52. $256 - 16k^4 + 8h^2k^2 - h^4 = (16 + 4k^2 - h^2)(16 - 4k^2 + h^2).$

53. $x^4 + 4 = x^4 + 4x^2 + 4 - 4x^2 = (x^2+2+2x)(x^2+2-2x).$

54. $x^{12} - 729 = (x^6-27)(x^6+27)$
$= (x^2-3)(x^4+3x^2+9)(x^2+3)(x^4-3x^2+9).$

55. $a^{4x+8} + 64 = a^{4x+8} + 16a^{2x+4} + 64 - 16a^{2x+4}$
$= (a^{2x+4} + 8 + 4a^{x+2})(a^{2x+4} + 8 - 4a^{x+2}).$

56. $a^4 + 225a^{-4} - 39 = a^4 - 30 + 225a^{-4} - 9$
$= (a^2 - 15a^{-2} + 3)(a^2 - 15a^{-2} - 3).$

57. $x^3 - 6x^2 + 12x - 8 = (x-2)^3.$

58. $a^2 - 9d^2 - 8ab + 6cd - c^2 + 16b^2 = (a - 4b)^2 - (3d - c)^2$
$= (a - 4b + 3d - c)(a - 4b - 3d + c).$

59. $4h^{-6} - 20h^{-3}k + 25k^2 - 6ab^{-2} - 9a^2 - b^{-4}$
$= (2h^{-3} - 5k)^2 - (3a + b^{-2})^2$
$= (2h^{-3} - 5k + 3a + b^{-2})(2h^{-3} - 5k - 3a - b^{-2}).$

60. $x^{2a} - 2x^a - 15 = (x^a - 5)(x^a + 3).$

61. $a^3 + a + b^3 + b = a^3 + b^3 + a + b = (a+b)(a^2 - ab + b^2 + 1).$

62. $x^{2a} - 12x^a + 36 = (x^a - 6)^2.$

63. $25x^{2c} + 50x^c - 39 = (5x^c + 13)(5x^c - 3).$

64. $128 - x^{28} = (2 - x^4)(64 + 32x^4 + 16x^8 + 8x^{12} + 4x^{16} + 2x^{20} + x^{24}).$

65. $3x^2 + 10x - 8 = (3x - 2)(x + 4).$

66. $a^5 - a^3 - a^2 + 1 = a^3(a+1)(a-1) - (a+1)(a-1)$
$= (a+1)(a-1)(a^3 - 1)$
$= (a+1)(a-1)^2(a^2 + a + 1).$

67. $x^3 + 3x^2 + 9x + 27 = (x+3)(x^2+9).$

68. $x^3 - 6x^2 + 12x - 7 = x^3 - 6x^2 + 12x - 8 + 1 = [(x-2)^3 + 1]$
$= (x - 2 + 1)[(x-2)^2 - (x-2) + 1]$
$= (x-1)(x^2 - 5x + 7).$

69. $4x^4 - 25y^6 + 10y^3 - 12x^2 + 8$
$= (4x^4 - 12x^2 + 9) - (25y^6 - 10y^3 + 1)$
$= (2x^2 + 5y^3 - 4)(2x^2 - 5y^3 - 2).$

70. $e^{2x} - 2 + e^{-2x} = (e^x - e^{-x})^2.$

71. $e^{2x} - 5 + 6e^{-2x} = (e^x - 3e^{-x})(e^x - 2e^{-x}).$

72. $3x^{2a} + 5x^a - 28 = (3x^a - 7)(x^a + 4).$

73. $e^{3x} - 2e^x - 24e^{-x} = e^{-x}(e^{2x} + 4)(e^{2x} - 6)$ or $(e^x + 4e^{-x})(e^{2x} - 6).$

74. $6e^{2x} - 5e^{-2x} - 13 = (2e^x - 5e^{-x})(3e^x + e^{-x}).$

75.
$$\begin{array}{llll} 1 + 0 & - 7y^2 + 6y^3 & \underline{|-3y} \\ \underline{\quad - 3y + 9y^2} & \underline{- 6y^3} & \\ 1 - 3y + 2y^2 & \quad 0 & \end{array}$$

$\therefore\ x^3 - 7xy^2 + 6y^3 = (x + 3y)(x^2 - 3xy + 2y^2) = (x + 3y)(x - 2y)(x - y).$

76. $e^{3x} + e^{2x} + e^x + 1 = (e^x + 1)(e^{2x} + 1).$

77. $a^{4x-2} - 10 + 25a^{2-4x} = (a^{2x-1} - 5a^{1-2x})^2.$

78. $e^{3x} - e^{-3x} + 3e^{-x} - 3e^x = (e^x - e^{-x})^3.$

79. $e^{x+3} + e^{x+2} - e^{3-x} - e^{2-x} = (e+1)(e^{x+2} - e^{2-x})$ or
$e^{2-x}(e+1)(e^x+1)(e^x-1).$

80. Let $x = -y$ and the expression becomes zero.
Let $x = -z$ and the expression becomes zero.
Let $y = -z$ and the expression becomes zero.
$\therefore\ (x+y)(x+z)(y+z) = xy^2 + xz^2 + x^2y + x^2z + yz^2 + y^2z + 2xyz.$

81. Let $a = -b$ and the expression becomes zero.
Let $a = c$ and the expression becomes zero.
Let $b = -c$ and the expression becomes zero.
Let $a = b - c$ and the expression becomes zero.
$\therefore (a+b)(c-a)(b+c)(a-b+c) = ab^3 - a^3b + ac^3 - a^3c + bc^3 - b^3c.$

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1. $x^2 - 25 = 0.$
$(x+5)(x-5) = 0.$
$\therefore x = 5$ or $-5.$

2. $x^2 + 10 = 7x.$
$(x-2)(x-5) = 0.$
$\therefore x = 2$ or $5.$

3. $y^2 - 9y = 0.$
$y(y-9) = 0.$
$\therefore y = 0$ or $9.$

4. $r^2 - ra - 30a^2 = 0.$
$(r-6a)(r+5a) = 0.$
$\therefore r = 6a$ or $-5a.$

5. $4z^3 - 36z = 0.$
$z(z+3)(z-3) = 0.$
$\therefore z = 0, 3,$ or $-3.$

6. $x^4 - 5x^2 + 4 = 0.$
$(x+1)(x-1)(x+2)(x-2) = 0.$
$\therefore x = -1, 1, -2,$ or $2.$

7. $t^6 - 13t^4 + 36t^2 = 0.$
$t^2(t+3)(t-3)(t+2)(t-2) = 0.$
$\therefore t = 0, -3, 3, 2,$ or $-2.$

8. $3x^2 - xb - 2b^2 = 0.$
$(3x+2b)(x-b) = 0.$
$\therefore x = -\frac{2b}{3}$ or $b.$

9. $x^3 - 2x^2 - x + 2 = 0.$
$(x-2)(x+1)(x-1) = 0.$
$\therefore x = 2, -1,$ or $1.$

10. $x^{2x} - 2x^x + 1 = 0.$
$(x^x - 1)^2 = 0.$
Then $x^x = 1.$
$\therefore x = 1.$

11. $x^{2x} - 8x^x + 16 = 0.$
$(x^x - 4)^2 = 0.$
Then $x^x = 4.$
$\therefore x = 2.$

12. $x^3 + 5x^2 - 18x - 72 = 0.$

$$\begin{array}{l}
1 + 5 - 18 - 72 \,\underline{|+4} \\
\underline{\quad\;\; + 4 + 36 + 72} \\
1 + 9 + 18 \qquad 0
\end{array}$$

$(x-4)(x+3)(x+6) = 0.$
$\therefore x = 4, -3,$ or $-6.$

13. $x^4 + 3x^3 - 8x^2 - 12x + 16 = 0.$

$$\begin{array}{l}
1 + 3 - 8 - 12 + 16 \,\underline{|-4} \\
\underline{\quad - 4 + 4 + 16 - 16} \\
1 - 1 - 4 + \;4 \qquad 0 \\
1 - 1 - 4 + 4 \,\underline{|2} \\
\underline{\quad + 2 + 2 - 4} \\
1 + 1 - 2 \quad\; 0
\end{array}$$

$\therefore (x+4)(x-2)(x+2)(x-1) = 0.$
$\therefore x = -4, +2, -2,$ or $1.$

14. $x^3 - 2ax^2 - 5a^2x + 6a^3 = 0.$

$$\begin{array}{l}
1 - 2a - 5a^2 + 6a^3 \,\underline{|3a} \\
\underline{\quad + 3a + 3a^2 - 6a^3} \\
1 + \;\; a - 2a^2 \qquad 0
\end{array}$$

$\therefore (x-3a)(x+2a)(x-a) = 0.$
$\therefore x = 3a, -2a,$ or $a.$

15. $x^5 - 9x^4a - x^3 + 9ax^2 = 0.$
$x^2(x-9a)(x+1)(x-1) = 0.$
$\therefore x = 0, 9a, -1,$ or $1.$

16. $x^3 + 5x^2c - 16xc^2 - 80c^3 = 0.$
$(x-4c)(x+4c)(x+5c) = 0.$
$\therefore x = 4c, -4c,$ or $-5c.$

17. Let $x =$ the less part.

Then $272 - x =$ the greater part.

Therefore $272 - x = x^2$.

Whence $(x - 16)(x + 17) = 0$,

or $x = 16, -17$. -17 is rejected.

Therefore 16 and 256 are the required parts.

18. Let $x =$ the least number.

Then the next greater even number $= x + 2$, and the greatest number $= x + 4$.

Therefore $2x^2 = x^2 + 6x + 8 + 104$.

Whence $x^2 - 6x - 112 = 0$,

or $(x - 14)(x + 8) = 0$.

Whence $x = 14, -8$. -8 is rejected.

Therefore the numbers are 14, 16, and 18.

19. Let $x =$ the number.

Therefore $x + 4\left(\frac{1}{x}\right) = 20.2$,

or $x^2 - 20.2x + 4 = 0$.

Factoring, $(x - 20)(x - .2) = 0$.

Whence $x = 20, .2$.

Therefore the number is 20 or .2.

20. Let $b =$ the number of days B requires.

Then $b + 2 =$ the number of days A requires.

Therefore $\frac{1}{b} =$ the portion of the work B does in one day,

and $\frac{1}{b + 2} =$ the portion of the work A does in one day.

Therefore $\frac{1}{b} + \frac{1}{b + 2} = \frac{8}{15}$.

Whence $4b^2 - 7b - 15 = 0$.

Factoring, $(b-3)(4b+5) = 0$.

Then $b = 3, -\frac{5}{4}$. $-\frac{5}{4}$ is impossible.

Therefore A and B require 5 days and 3 days respectively.

21. Let $x =$ the shorter leg.

Then $x + 8 =$ the other leg.

Therefore $x^2 + (x + 8)^2 = 1600$,

or $x^2 + 8x - 768 = 0$.

Factoring, $(x + 32)(x - 24) = 0$.

Whence $x = -32, 24$. -32 is impossible.

Therefore the sides are 40 ft., 24 ft., and 32 ft.

22. Let $x =$ the rate of the slower train.

Then $x + 10 =$ the rate of the other train.

Then $\dfrac{240}{x} = \dfrac{240}{x+10} + 2.$

Whence $x^2 + 10x - 1200 = 0.$

Factoring, $(x + 40)(x - 30) = 0.$

Whence $x = -40, 30.$ -40 is impossible.

Therefore the two rates are 30 mi. and 40 mi. per hour.

23. Let $a =$ the altitude in feet.

Then the bases are $a + 2$ and $a + 8$ respectively.

Therefore $a\dfrac{(a+2)+(a+8)}{2} = 126.$

Whence $a^2 + 5a - 126 = 0.$

Factoring, $(a - 9)(a + 14) = 0.$

Whence $a = 9, -14.$ -14 is impossible.

Therefore the altitude is 9 ft. and the bases are 11 ft. and 17 ft.

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1.
$$28 = 2^2 \cdot 7.$$
$$56 = 2^3 \cdot 7.$$
$$84 = 2^2 \cdot 3 \cdot 7.$$
$$35 = 5 \cdot 7.$$
$$\therefore \text{H.C.F.} = 7.$$

2.
$$225 = 3^2 \cdot 5^2.$$
$$120 = 2^3 \cdot 3 \cdot 5.$$
$$210 = 2 \cdot 3 \cdot 5 \cdot 7.$$
$$135 = 3^3 \cdot 5.$$
$$\therefore \text{H.C.F.} = 3 \cdot 5 = 15.$$

3.
$$198 = 2 \cdot 3^2 \cdot 11.$$
$$495 = 11 \cdot 3^2 \cdot 5.$$
$$693 = 11 \cdot 7 \cdot 3^2.$$
$$1155 = 11 \cdot 3 \cdot 7 \cdot 5.$$
$$\therefore \text{H.C.F.} = 11 \cdot 3 = 33.$$

4.
$$816 = 2^4 \cdot 3 \cdot 17.$$
$$1224 = 2^3 \cdot 3^2 \cdot 17.$$
$$1360 = 2^4 \cdot 5 \cdot 17.$$
$$4080 = 2^4 \cdot 5 \cdot 17 \cdot 3.$$
$$\therefore \text{H.C.F.} = 2^3 \cdot 17 = 136.$$

5.
$$91x^4y^3 = 13 \cdot 7x^4y^3.$$
$$133x^2y^6 = 19 \cdot 7x^2y^6.$$
$$343x^5y^2 = 7^3 \cdot x^5y^2.$$
$$\therefore \text{H.C.F.} = 7x^2y^2.$$

6.
$$a^2 - 9a + 14 = (a-2)(a-7).$$
$$a^2 - 4 = (a-2)(a+2).$$
$$5a^2 - 10a = 5a(a-2).$$
$$\therefore \text{H.C.F.} = a - 2.$$

7.
$$x^3 + 27 = (x+3)(x^2 - 3x + 9).$$
$$2x^2 + 3x - 9 = (x+3)(2x-3).$$
$$5x^3 + 15x^2 = 5x^2(x+3).$$
$$\therefore \text{H.C.F.} = x + 3.$$

8.
$$2x^5 + 8x = 2x(x^2 + 2 + 2x)(x^2 + 2 - 2x).$$
$$3ax^3 + 6ax + 6ax^2 = 3ax(x^2 + 2 + 2x).$$
$$3ax^7 + 12ax^3 = 3ax^3(x^2 + 2 + 2x)(x^2 + 2 - 2x).$$
$$\therefore \text{H.C.F.} = x(x^2 + 2x + 2).$$

9. $[(x+y)(x-y)]^3 = (x+y)^3(x-y)^3.$

$x^4 - 2x^2y^2 + y^4 = (x+y)^2(x-y)^2.$

$(x^6-y^6)^2 = (x+y)^2(x-y)^2(x^2+xy+y^2)^2(x^2-xy+y^2)^2.$

$\therefore$ H.C.F. $= (x+y)^2(x-y)^2 = x^4 - 2x^2y^2 + y^4.$

10. $x^7 + a^{7e} = (x+a^e)(x^6 - x^5a^e + x^4a^{2e} - x^3a^{3e} + x^2a^{4e} - xa^{5e} + a^{6e}).$

$x^2 - a^{2e} = (x+a^e)(x-a^e).$

$x^2 - 3xa^e - 4a^{2e} = (x+a^e)(x-4a^e).$

$x^3 + a^{3e} = (x+a^e)(x^2 - xa^e + a^{2e}).$

$\therefore$ H.C.F. $= x + a^e.$

11. $a^{5e} + 4a^eb^8 = a^e(a^{2e} + 2b^4 + 2a^eb^2)(a^{2e} + 2b^4 - 2a^eb^2).$

$a^{2e+1}b^2 + 2ab^6 - 2a^{e+1}b^4 = ab^2(a^{2e} + 2b^4 - 2a^eb^2).$

$a^{8e} - 16b^{16} = (a^{2e}+2b^4+2a^eb^2)(a^{2e}+2b^4-2a^eb^2)(a^{2e}+2b^4)(a^{2e}-2b^4).$

$\therefore$ H.C.F. $= a^{2e} + 2b^4 - 2a^eb^2.$

12. $24 = 2^3 \cdot 3.$

$30 = 2 \cdot 3 \cdot 5.$

$54 = 2 \cdot 3^3.$

$\therefore$ L.C.M. $= 2^3 \cdot 3^3 \cdot 5 = 1080.$

13. $105 = 3 \cdot 5 \cdot 7.$

$140 = 2^2 \cdot 5 \cdot 7.$

$245 = 5 \cdot 7^2.$

$\therefore$ L.C.M. $= 2^2 \cdot 3 \cdot 5 \cdot 7^2 = 2940.$

14. $174 = 2 \cdot 3 \cdot 29.$

$485 = 5 \cdot 97.$

$4611 = 3 \cdot 29 \cdot 53.$

$5141 = 97 \cdot 53.$

$\therefore$ L.C.M. $= 2 \cdot 3 \cdot 29 \cdot 5 \cdot 97 \cdot 53 = 4{,}472{,}670.$

15. $30ax^2 = 2 \cdot 3 \cdot 5ax^2.$

$225a^5xy^2 = 3^2 \cdot 5^2 \cdot a^5xy^2.$

$75a^4x^3y = 3 \cdot 5^2 \cdot a^4x^3y.$

$\therefore$ L.C.M. $= 2 \cdot 3^2 \cdot 5^2a^5x^3y^2 = 450a^5x^3y^2.$

16. $12x^2 + 6x = 6x(2x+1).$

$12x^3 - 3x = 3x(2x+1)(2x-1).$

$16x^4 + 2x = 2x(2x+1)(4x^2 - 2x + 1).$

$\therefore$ L.C.M. $= 6x(2x+1)(2x-1)(4x^2-2x+1).$

17. $a^3 - 8b^3 = (a-2b)(a^2 + 2ab + 4b^2).$

$4b^2 - a^2 = (2b+a)(2b-a).$

$a^3b + 4ab^3 + 2a^2b^2 = ab(a^2 + 4b^2 + 2ab).$

$\therefore$ L.C.M. $= ab(2b+a)(2b-a)(a^2+4b^2+2ab).$

18. $x^3 - 2x^2 - 5x + 6 = (x+2)(x-1)(x-3).$

$4 - x^2 = (2+x)(2-x).$

$a - ax^2 = a(1+x)(1-x).$

$\therefore$ L.C.M. $= a(x+1)(x+2)(x-3)(2-x)(x-1).$

19. $a^4 + 4a^2 + 16 = (a^2 + 4 + 2a)(a^2 + 4 - 2a).$
$a^2 - 4 = (a+2)(a-2).$
$a^3 + 8 = (a+2)(a^2 - 2a + 4).$
$a^3 - 8 = (a-2)(a^2 + 2a + 4).$
$\therefore$ L.C.M. $= (a^2 + 2a + 4)(a^2 - 2a + 4)(a+2)(a-2).$

20. $x^3 - 2a^2x + ax^2 = x(x + 2a)(x - a).$
$2a^3 + 3a^2x + ax^2 = a(x+2a)(x+a).$
$4a^4x^2 - a^2x^4 = a^2x^2(2a + x)(2a - x).$
$\therefore$ L.C.M. $= a^2x^2(x-a)(x+2a)(2a-x)(x+a).$

21. $m^4 - 3m^2n^2 + 9n^4 = (m^2 + 3n^2 + 3mn)(m^2 + 3n^2 - 3mn).$
$m^3 + 3mn^2 + 3m^2n = m(m^2 + 3n^2 + 3mn).$
$3n^3 + m^2n - 3mn^2 = n(3n^2 + m^2 - 3mn).$
$\therefore$ L.C.M. $= mn(m^2 + 3n^2 + 3mn)(m^2 + 3n^2 - 3mn).$

22. $e^{2x} + e^{-2x} - 2 = (e^x - e^{-x})^2.$
$e^{2x} - e^{-2x} = (e^x + e^{-x})(e^x - e^{-x}).$
$e^{2x} - 3 + 2e^{-2x} = (e^x - e^{-x})(e^x - 2e^{-x}).$
$\therefore$ L.C.M. $= (e^x - e^{-x})^2(e^x + e^{-x})(e^x - 2e^{-x}).$

23. $x^3 - 2x^2 - 2x - 3 = (x-3)(x^2 + x + 1).$
$x^3 - 27 = (x-3)(x^2 + 3x + 9).$
$x^2 + x - 12 = (x-3)(x+4).$
$\therefore$ L.C.M. $= (x-3)(x+4)(x^2 + x + 1)(x^2 + 3x + 9).$

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1. $\dfrac{18a^3c^2}{24a^2c^3} = \dfrac{3a}{4c}.$

2. $\dfrac{3x-6}{x^2-4} = \dfrac{3(x-2)}{(x+2)(x-2)} = \dfrac{3}{x+2}.$

3. $\dfrac{7a^2 - 14b^2}{a^4 + a^2b^2 - 6b^4} = \dfrac{7(a^2 - 2b^2)}{(a^2 - 2b^2)(a^2 + 3b^2)} = \dfrac{7}{a^2 + 3b^2}.$

4. $\dfrac{x^3 - 8}{(x-2)^3} = \dfrac{(x-2)(x^2 + 2x + 4)}{(x-2)^3} = \dfrac{x^2 + 2x + 4}{(x-2)^2}.$

5. $\dfrac{x^{2a} - c^{2b}}{(x^a + c^b)^2} = \dfrac{(x^a + c^b)(x^a - c^b)}{(x^a + c^b)^2} = \dfrac{x^a - c^b}{x^a + c^b}.$

6. $\dfrac{(x^2 - c^2)^2}{x^4 - c^4} = \dfrac{(x+c)^2(x-c)^2}{(x^2 + c^2)(x+c)(x-c)} = \dfrac{x^2 - c^2}{x^2 + c^2}.$

7. $\dfrac{2x^{6e} - 128}{x^{2e} - 4} = \dfrac{2(x^e + 2)(x^{2e} - 2x^e + 4)(x^e - 2)(x^{2e} + 2x^e + 4)}{(x^e + 2)(x^e - 2)}$
$= 2x^{4e} + 8x^{2e} + 32.$

8. $\dfrac{x^6 - 1}{x^4 + x^2 + 1} = \dfrac{(x^2 - 1)(x^4 + x^2 + 1)}{x^4 + x^2 + 1} = x^2 - 1.$

9. $\dfrac{3x^2 + 2x - 21}{27x^4 - 147x^2} = \dfrac{(3x-7)(x+3)}{3x^2(3x-7)(3x+7)} = \dfrac{x+3}{9x^3 + 21x^2}.$

10. $\dfrac{x^4 - x^2 + x - 1}{x^4 - 1} = \dfrac{(x-1)(x^3 + x^2 + 1)}{(x^2+1)(x+1)(x-1)} = \dfrac{x^3 + x^2 + 1}{(x^2+1)(x+1)}.$

11. $-\dfrac{c}{-2a} = \dfrac{c}{2a} = \dfrac{-c}{-2a} = -\dfrac{-c}{2a}.$

12. $\dfrac{c}{a-2c} = \dfrac{-c}{2c-a} = -\dfrac{-c}{a-2c} = -\dfrac{c}{2c-a}.$

13. $\dfrac{2x - 3y}{3x^2 - 6y^2 - xy} = \dfrac{-2x + 3y}{-3x^2 + 6y^2 + xy}$

$$= -\dfrac{-2x + 3y}{3x^2 - 6y^2 - xy}$$

$$= -\dfrac{2x - 3y}{-3x^2 + 6y^2 + xy}.$$

14. $\dfrac{-3}{(c-a)(b-a)} = \dfrac{-3}{(a-c)(a-b)}.$

15. $\dfrac{x-y}{(y-x)(z-x)(z-y)} = \dfrac{y-x}{(x-y)(x-z)(y-z)}.$

16. $\dfrac{x-3}{5-x-x^2} \neq \dfrac{x+3}{x^2+x-5}$ for $x + 3 \neq -(x-3)$.

17. $\frac{7}{24}$, $\frac{9}{56}$ equal $\frac{49}{168}$, $\frac{27}{168}$ respectively.

18. $\dfrac{4}{ab}, \dfrac{3}{a^2c}$ equal $\dfrac{4ac}{a^2bc}, \dfrac{3b}{a^2bc}$ respectively.

19. $\dfrac{3a+b}{6a}, \dfrac{a-2b}{4ab}$ equal $\dfrac{6ab + 2b^2}{12ab}, \dfrac{3a - 6b}{12ab}$ respectively.

20. $\dfrac{2}{6x-12}, \dfrac{3}{x^2-2x}$ equal $\dfrac{x}{3x^2-6x}, \dfrac{9}{3x^2-6x}$ respectively.

21. $\dfrac{x-1}{x^2-5x+6}, \dfrac{x}{x^2-9}$ equal $\dfrac{x^2+2x-3}{(x-3)(x+3)(x-2)},$

$\dfrac{x^2-2x}{(x-3)(x-2)(x+3)}$ respectively.

22. No. The subtraction of equal numbers from both numerator and denominator of a fraction changes the value of the fraction. See pages 135–136.

23. No. The term may be *subtracted* from each member of the equation, but not *canceled*. Cancellation means division.

24. $\dfrac{3a}{6} - \dfrac{a-x}{9} - \dfrac{3x}{4} = \dfrac{18a - (4a - 4x) - 27x}{36} = \dfrac{14a - 23x}{36}.$

25. $\dfrac{x-3}{4x^2} - \dfrac{3x^2-2}{14x^3} - \dfrac{x^3-8}{3x^4} = \dfrac{21x^3 - 63x^2 - (18x^3 - 12x) - (28x^3 - 224)}{84x^4}$

$$= \dfrac{-25x^3 - 63x^2 + 12x + 224}{84x^4}.$$

26. $\dfrac{5-x}{x-4}-\dfrac{5}{7}=\dfrac{35-7x-(5x-20)}{7(x-4)}=\dfrac{55-12x}{7x-28}.$

27. $\dfrac{4m^2}{x^2-9m^2}-\dfrac{3m-x}{x+3m}=\dfrac{4m^2-(3m-x)(x-3m)}{x^2-9m^2}=\dfrac{x^2-6mx+13m^2}{x^2-9m^2}.$

28. $\dfrac{x-3}{x^2-9x+14}-\dfrac{3}{x^2-4}=\dfrac{(x-3)(x+2)-3(x-7)}{(x-2)(x-7)(x+2)}$

$=\dfrac{x^2-4x+15}{(x-2)(x-7)(x+2)}.$

29. $\dfrac{x}{x^2-2x}-\dfrac{1}{x}-\dfrac{2+x}{x^2-4x+4}=\dfrac{x(x-2)-(x-2)^2-x(2+x)}{x(x-2)^2}$

$=\dfrac{-x^2-4}{x(x-2)^2}.$

30. $2-\dfrac{1}{v-1}+\dfrac{1}{1-v}=2+\dfrac{1}{1-v}+\dfrac{1}{1-v}=2+\dfrac{2}{1-v}=\dfrac{4-2v}{1-v}.$

31. $x^2+x+1-\dfrac{x^3}{x-1}=\dfrac{(x^2+x+1)(x-1)-x^3}{x-1}=\dfrac{1}{1-x}.$

32. $x^2-\dfrac{x^4-2x^2}{x^2-x+1}+x+1=\dfrac{(x^2+x+1)(x^2-x+1)-(x^4-2x^2)}{x^2-x+1}$

$=\dfrac{3x^2+1}{x^2-x+1}.$

33. $\dfrac{2x^2-3a^2}{27x^3-64a^3}-\dfrac{2x+a}{9x^2-16a^2}$

$=\dfrac{(2x^2-3a^2)(3x+4a)-(2x+a)(9x^2+12ax+16a^2)}{(3x-4a)(9x^2+12ax+16a^2)(3x+4a)}$

$=\dfrac{-12x^3-53a^2x-25ax^2-28a^3}{(27x^3-64a^3)(3x+4a)}.$

34. $\dfrac{2}{(a-c)(x-c)}-\dfrac{3}{(c-a)(c-x)}=\dfrac{-2+3}{(a-c)(c-x)}=\dfrac{1}{(a-c)(c-x)}.$

35. $\dfrac{2x-4}{9-x^2}-\dfrac{3x+1}{x-3}-\dfrac{x}{9-6x+x^2}=\dfrac{4-2x}{x^2-9}-\dfrac{3x+1}{x-3}-\dfrac{x}{x^2-6x+9}$

$=\dfrac{(4-2x)(x-3)-(3x+1)(x+3)(x-3)-x(x+3)}{(x+3)(x-3)^2}$

$=\dfrac{-3x^3-4x^2+34x-3}{(x+3)(x-3)^2}.$

36. $\dfrac{2x+12c}{6x^2-13cx-5c^2}-\dfrac{3x-7c}{4x^2+4cx-35c^2}$

$=\dfrac{(2x+12c)(2x+7c)-(3x-7c)(3x+c)}{(2x-5c)(3x+c)(2x+7c)}$

$=\dfrac{-5x^2+56cx+91c^2}{(2x-5c)(3x+c)(2x+7c)}.$

37. $\dfrac{(a-s)(c-s)}{s(s-m)}+\dfrac{ac}{ms}+\dfrac{(a-m)(c-m)}{m(m-s)}$
$=\dfrac{(s-a)(c-s)m+ac(m-s)+s(a-m)(c-m)}{ms(m-s)}$
$=1.$

38. $\dfrac{2x^3-x^2}{x^2-x+1}-\dfrac{x^2-3}{x^4+x^2+1}+\dfrac{3x-5}{x^3+1}$
$=\dfrac{(2x^3-x^2)(x+1)(x^2+x+1)-(x^2-3)(x+1)+(3x-5)(x^2+x+1)}{(x^2-x+1)(x^2+x+1)(x+1)}$
$=\dfrac{2x^6+3x^5+2x^4+2x^3-4x^2+x-2}{(x^4+x^2+1)(x+1)}.$

39. $\dfrac{e^{3x}-e^{-3x}}{e^x-e^{-x}}=\dfrac{e^{3x}-\dfrac{1}{e^{3x}}}{e^x-\dfrac{1}{e^x}}=\dfrac{e^{6x}-1}{e^{2x}(e^{2x}-1)}=\dfrac{4^3-1}{4(4-1)}=5\frac{1}{4}$, when $e^{2x}=4$.

40. $c(c-y)\div d-(y+d)d\div c=\dfrac{c(c-c+d)}{d}-\dfrac{(c-d+d)d}{c}$
$=c-d$, when $y=c-d$.

41. $\dfrac{a}{b}-\dfrac{c}{d}=\dfrac{x+2}{x^2+2x+4}-\dfrac{x-2}{x^2-2x+4}$
$=\dfrac{(x+2)(x^2-2x+4)-(x-2)(x^2+2x+4)}{(x^2-2x+4)(x^2+2x+4)}=\dfrac{16}{x^4+4x^2+16},$
when $a=x+2$, $b=x^2+2x+4$, $c=x-2$, and $d=x^2-2x+4$.

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1. $\dfrac{4x^2c^2}{9a^3}\cdot\dfrac{27a^2}{10x^3c}=\dfrac{6c}{5ax}.$

2. $\left(\dfrac{2a}{cx}\right)^2\cdot\left(\dfrac{3c^2x}{4a^2}\right)^3=\dfrac{4a^2}{c^2x^2}\cdot\dfrac{27c^6x^3}{64a^6}=\dfrac{27c^4x}{16a^4}.$

3. $\left(\dfrac{a}{6c^2}\right)\cdot\left(\dfrac{3a}{c}\right)^2\cdot\left(\dfrac{2c^2}{3a^4}\right)^2=\dfrac{a}{6c^2}\cdot\dfrac{9a^2}{c^2}\cdot\dfrac{4c^4}{9a^8}=\dfrac{2}{3a^5}.$

4. $\left(\dfrac{-a}{x}\right)^3\cdot\left(\dfrac{-2x}{a}\right)^2\cdot\left(\dfrac{-c}{2a}\right)^4=\dfrac{-a^3}{x^3}\cdot\dfrac{4x^2}{a^2}\cdot\dfrac{c^4}{16a^4}=\dfrac{-c^4}{4a^3x}.$

5. $\left(\dfrac{3c^2}{2a}\right)^2\div\left(\dfrac{9c^2x^3}{4a^2}\right)^2\div\left(\dfrac{-2a^3c^2}{5x^3}\right)^3\dfrac{(3a^2c)^4}{(10x^3)^2}$
$=\dfrac{9c^4}{4a^2}\cdot\dfrac{16a^4}{81c^4x^6}\cdot\dfrac{125x^9}{(-8a^9c^6)}\cdot\dfrac{81a^8c^4}{100x^6}=-\dfrac{45a}{8c^2x^3}.$

6. $\dfrac{4x^e-2y^e}{(2x^e-y^e)^2}\cdot\dfrac{4x^{2e}-y^{2e}}{(2x^e+y^e)^2}\cdot\dfrac{2x^e+y^e}{3}$
$=\dfrac{2(2x^e-y^e)}{(2x^e-y^e)^2}\cdot\dfrac{(2x^e+y^e)(2x^e-y^e)}{(2x^e+y^e)^2}\cdot\dfrac{2x^e+y^e}{3}=\dfrac{2}{3}.$

7. $\dfrac{x^2-1}{x^4-1}\left(1-\dfrac{2}{x^6+1}\right)\dfrac{x^4-x^2+1}{x^2+x+1}$

$$=\frac{x^2-1}{(x^2+1)(x^2-1)}\cdot\frac{(x-1)(x^2+x+1)(x+1)(x^2-x+1)}{(x^2+1)(x^4-x^2+1)}\cdot\frac{x^4-x^2+1}{x^2+x+1}$$

$$=\frac{x^4-x^3+x-1}{(x^2+1)^2}.$$

8. $\left(x^2-2x+4-\dfrac{16}{x+2}\right)\dfrac{x^2-4}{3}=\dfrac{x^3-8}{x+2}\cdot\dfrac{x^2-4}{3}=\dfrac{x^4-2x^3-8x+16}{3}.$

9. $\dfrac{a^2-a-90}{a^2-100}\div\dfrac{a^3+9a^2}{a^2+10a}\div\dfrac{4a+6}{2a^2x+3ax}$

$$=\frac{(a-10)(a+9)}{(a+10)(a-10)}\cdot\frac{a(a+10)}{a^2(a+9)}\cdot\frac{ax(2a+3)}{2(2a+3)}=\frac{x}{2}.$$

10. $\left(a-\dfrac{13a}{a+6}\right)\div\dfrac{a^2-a-42}{a^2-36}\div\dfrac{2a^3-12a}{ax+6x}$

$$=\frac{a(a-7)}{a+6}\cdot\frac{(a-6)(a+6)}{(a-7)(a+6)}\cdot\frac{x(a+6)}{2a(a^2-6)}=\frac{ax-6x}{2a^2-12}.$$

11. $\left(\dfrac{3a^4-75a^2}{3a-7}\right)\left(6a-\dfrac{7}{a}-11\right)\div\left(2+\dfrac{5}{a^2}+\dfrac{11}{a}\right)$

$$=\frac{3a^2(a+5)(a-5)}{3a-7}\cdot\frac{(3a-7)(2a+1)}{a}\cdot\frac{a^2}{(2a+1)(a+5)}$$

$$=3a^4-15a^3.$$

12. $\left(\dfrac{4c}{a}-\dfrac{15c^2}{a^2}+4\right)\left(3-\dfrac{4a+20c}{2a+5c}\right)\div\left(4-\dfrac{16c}{a}+\dfrac{15c^2}{a^2}\right)$

$$=\frac{(2a+5c)(2a-3c)}{a^2}\cdot\frac{2a-5c}{2a+5c}\cdot\frac{a^2}{(2a-5c)(2a-3c)}=1.$$

13. $\dfrac{1}{mx}\left(\dfrac{m}{x}+\dfrac{x}{m}\right)\div\left(\dfrac{m^6+x^6}{m^3x^3}\right)\left(m^2-x^2+\dfrac{x^4}{m^2}\right)\left(\dfrac{am+mx}{cx-ax}\right)$

$$=\frac{1}{mx}\cdot\frac{m^2+x^2}{mx}\cdot\frac{m^3x^3}{(m^2+x^2)(m^4-m^2x^2+x^4)}\cdot\frac{m^4-m^2x^2+x^4}{m^2}\cdot\frac{m(a+x)}{x(c-a)}$$

$$=\frac{a+x}{c-a}.$$

14. $\dfrac{x^3-8}{x^4+4x^2+16}\cdot\dfrac{x^2-4}{(x-2)^2}\cdot\left(1\div\dfrac{(x+2)^2}{x^3+8}\right)$

$$=\frac{x^3-8}{(x^2+4+2x)(x^2+4-2x)}\cdot\frac{x^2-4}{(x-2)(x-2)}\cdot\frac{x^3+8}{(x+2)^2}=1.$$

15. $\dfrac{2x^2+5x+3}{6x^2+x-1}\cdot\dfrac{3x^2+3ax-x-a}{2x^2-2cx+3x-3c}\cdot\left(\dfrac{c+2a+(2a+1+2c)x}{x^2+x+ax+a}-2\right)$

$$=\frac{(2x+3)(x+1)}{(3x-1)(2x+1)}\cdot\frac{(x+a)(3x-1)}{(x-c)(2x+3)}\cdot\frac{(c-x)(2x+1)}{(x+a)(x+1)}=-1.$$

16. $\left(\dfrac{2x^3}{c^2}\right)^5 - 5\left(\dfrac{2x^3}{c^2}\right)^4\left(\dfrac{c^3}{15x^4}\right) + 10\left(\dfrac{2x^3}{c^2}\right)^3\left(\dfrac{c^3}{15x^4}\right)^2 - 10\left(\dfrac{2x^3}{c^2}\right)^2\left(\dfrac{c^3}{15x^4}\right)^3$

$$=\frac{32x^{15}}{c^{10}} - \frac{16x^8}{3c^5} + \frac{16x}{45} - \frac{8c^5}{675x^6}.$$

17. $\left(\dfrac{a^2}{b^3}\right)^{10} - 10\left(\dfrac{a^2}{b^3}\right)^9\left(\dfrac{b^2}{6a^4}\right) + \dfrac{10\cdot 9}{2}\left(\dfrac{a^2}{b^3}\right)^8\left(\dfrac{b^2}{6a^4}\right)^2 - \dfrac{10\cdot 9\cdot 8}{2\cdot 3}\left(\dfrac{a^2}{b^3}\right)^7\left(\dfrac{b^2}{6a^4}\right)^3$

$$=\frac{a^{20}}{b^{30}} - \frac{5a^{14}}{3b^{25}} + \frac{5a^8}{4b^{20}} - \frac{5a^2}{9b^{15}}.$$

18. For $1 \div \frac{2}{5}$ we may write $\frac{5}{5} \div \frac{2}{5}$, which equals $\frac{5}{2}$. Similarly, $1 \div \frac{c}{a} = \frac{a}{a} \div \frac{c}{a} = \frac{a}{c}$. Hence the number of times a fraction is contained in the number 1 is the reciprocal of the fraction. In the number n a number will be contained n times as often as in the number 1. Therefore $n \div \frac{c}{a} = n \times \frac{a}{c} = \frac{na}{c}$.

19. For $\frac{a}{b} \div \frac{c}{d}$ we may (using the principle of § 28) write $\frac{ad}{bd} \div \frac{bc}{bd} = \frac{ad}{bc}$; that is, the result of dividing one fraction by another is a fraction whose numerator is the product of the numerator of the dividend and the denominator of the divisor, and whose denominator is the product of the denominator of the dividend and the numerator of the divisor. Hence $\frac{a}{c} \div \frac{b}{d} = \frac{a}{c} \times \frac{d}{b} = \frac{ad}{bc}$ gives in abbreviated form the essentials, the reduction of divisor and dividend to similar fractions being unnecessary except for the purpose of explanation.

20. $\dfrac{8-\frac{1}{27}}{2-\frac{1}{3}} = \dfrac{215}{27}\cdot\dfrac{3}{5} = 4\frac{7}{9}$.

21. $\dfrac{7\frac{3}{7}}{40\frac{5}{9}}\cdot\dfrac{146}{17\frac{1}{3}} = \dfrac{52}{7}\cdot\dfrac{9}{365}\cdot 146\cdot\dfrac{3}{52} = \dfrac{54}{35}$.

22. $\dfrac{3\frac{1}{2}\cdot 3\frac{1}{2}\cdot 3\frac{1}{2}-1}{3\frac{1}{2}\cdot 3\frac{1}{2}-1} = \dfrac{335}{8}\cdot\dfrac{4}{45} = \dfrac{67}{18}$.

23. $\dfrac{\frac{a}{x}+3}{\frac{a^2}{x^2}-9} = \dfrac{a+3x^2}{x}\cdot\dfrac{x^2}{a^2-9x^2} = \dfrac{x}{a-3x}$.

24. $\dfrac{1-\frac{m}{x}}{\frac{4m^4}{x^4}-4} = \dfrac{x-m}{x}\cdot\dfrac{x^4}{4m^4-4x^4} = \dfrac{-x^3}{4(m^3+m^2x+mx^2+x^3)}$.

25. $\dfrac{2a-\frac{a^2+c^2}{c}}{\frac{a}{c}-1} = \dfrac{2ac-a^2-c^2}{c}\cdot\dfrac{c}{a-c} = c-a$.

26. $$\frac{3-\frac{a}{c}}{\frac{(a+3c)^2}{2ac}-6}=\frac{3c-a}{c}\cdot\frac{2ac}{a^2-6ac+9c^2}=\frac{2a}{3c-a}.$$

27. $$\frac{c-\frac{4}{c}}{\frac{1}{c^2}+\frac{2}{c^3}-\frac{8}{c^4}}=\frac{c^2-4}{c}\cdot\frac{c^4}{c^2+2c-8}$$
$$=\frac{(c+2)(c-2)}{c}\cdot\frac{c^4}{(c+4)(c-2)}=\frac{c^4+2c^3}{c+4}.$$

28. $$\frac{\left(\frac{4c+3a}{3a}\right)^2-\frac{6c}{a}}{\frac{(3a+2c)^2}{3a}-8c}=\frac{16c^2-30ac+9a^2}{9a^2}\cdot\frac{3a}{9a^2-12ac+4c^2}$$
$$=\frac{(2c-3a)(8c-3a)}{9a^2}\cdot\frac{3a}{(3a-2c)^2}=\frac{8c-3a}{6ac-9a^2}.$$

29. $$1+\frac{1}{1+\frac{1}{1+2}}=1+\frac{1}{\frac{4}{3}}=1+\frac{3}{4}=\frac{7}{4}.$$

30. $$1-\frac{2}{3+\frac{4}{5-\frac{6}{7}}}=1-\frac{2}{3+\frac{28}{29}}=1-\frac{2}{\frac{115}{29}}=1-\frac{58}{115}=\frac{57}{115}.$$

31. $$\frac{\frac{m^2}{n^2}+\frac{mn}{mn}+\frac{n^2}{m^2}}{\frac{m^2+n^2}{mn}-1}=\frac{m^4+m^2n^2+n^4}{m^2n^2}\cdot\frac{mn}{m^2+n^2-mn}=\frac{m^2+mn+n^2}{mn}.$$

32. $$\frac{1}{1-\frac{1}{1-\frac{1}{x}}}=\frac{1}{1-\frac{1}{\frac{x-1}{x}}}=\frac{1}{1-\frac{x}{x-1}}=\frac{1}{\frac{-1}{x-1}}=1-x.$$

33. $$3-\frac{1}{1-\frac{4}{6+\frac{3}{7-\frac{2}{5}}}}=3-\frac{1}{1-\frac{4}{\frac{71}{11}}}=3-\frac{1}{\frac{27}{71}}=\frac{10}{27}.$$

34. $$\frac{6x-11-\frac{7}{x}}{2+\frac{11}{x}+\frac{5}{x^2}}\div\frac{\frac{5-x}{3x}}{x^2-25}=\frac{6x^2-11x-7}{x}\cdot\frac{x^2}{2x^2+11x+5}\div\frac{5-x}{3x(x^2-25)}$$
$$=\frac{(3x-7)(2x+1)}{x}\cdot\frac{x^2}{(2x+1)(x+5)}\cdot\frac{3x(x-5)(x+5)}{5-x}$$
$$=21x^2-9x^3.$$

35. $$\frac{1-\dfrac{a^2b^2}{(a^2+b^2)^2}}{\dfrac{ab}{a^2+b^2}+1}=\frac{a^4+a^2b^2+b^4}{(a^2+b^2)^2}\cdot\frac{a^2+b^2}{a^2+ab+b^2}=\frac{a^2-ab+b^2}{a^2+b^2}.$$

36. $$\frac{1+\dfrac{y}{x}+\dfrac{x}{y}}{\dfrac{x^2}{y^2}+1+\dfrac{y^2}{x^2}}=\frac{xy+y^2+x^2}{xy}\cdot\frac{x^2y^2}{x^4+x^2y^2+y^4}=\frac{xy}{x^2-xy+y^2}.$$

37. $$\frac{\dfrac{1}{x}-\dfrac{x-a}{x^2-a^2}}{\dfrac{1}{a}-\dfrac{a-x}{a^2+x^2}}=\frac{x^2-a^2-x^2+ax}{(x+a)(x-a)x}\div\frac{a^2+x^2-a^2+ax}{a(a^2+x^2)}$$

$$=\frac{a(x-a)}{(x+a)(x-a)x}\cdot\frac{a(a^2+x^2)}{x(a+x)}$$

$$=\frac{a^2(a^2+x^2)}{x^2(a+x)^2}.$$

38. $$\frac{\dfrac{xy+1}{y}}{x+\dfrac{1}{\dfrac{yz+1}{z}}}-\frac{1}{y(x+xyz+z)}=\frac{xy+1}{y}\cdot\frac{yz+1}{xyz+x+z}-\frac{1}{y(x+xyz+z)}$$

$$=\frac{xy^2z+xy+yz+1-1}{y(xyz+x+z)}=1.$$

39. $$\frac{1}{1+\dfrac{a}{a+1+\dfrac{2a^2}{1-a}}}=\frac{1}{1+\dfrac{a-a^2}{1+a^2}}=\frac{1}{\dfrac{1+a}{1+a^2}}=\frac{a^2+1}{a+1}.$$

40. $$\frac{a^3-b^3}{a^2+b^2}=\frac{(e^x-e^{-x})^3-(e^x+e^{-x})^3}{(e^x-e^{-x})^2+(e^x+e^{-x})^2}$$

$$=\frac{e^{3x}-3e^x+3e^{-x}-e^{-3x}-e^{3x}-3e^x-3e^{-x}-e^{-3x}}{e^{2x}-2+e^{-2x}+e^{2x}+2+e^{-2x}}$$

$$=\frac{-6e^x-2e^{-3x}}{2e^{2x}+2e^{-2x}}=-\frac{3e^x+e^{-3x}}{e^{2x}+e^{-2x}}$$

$$=\frac{-3e^{4x}-1}{e^{5x}+e^x},\text{ if } a=e^x-\frac{1}{e^x},\text{ and } b=e^x+\frac{1}{e^x}.$$

41. The fraction continues thus:

$$(a)\quad 2+\cfrac{81}{2+\cfrac{121}{2+\cfrac{169}{2+\cfrac{225}{2+}}}}.$$

$$(b)\ \frac{1}{1+\cfrac{1}{2+\cfrac{9}{2+\cfrac{25}{\left[2+\cfrac{49}{\frac{85}{2}}\right]}}}}=\frac{1}{1+\cfrac{1}{2+\cfrac{9}{\left[2+\cfrac{25}{\frac{268}{85}}\right]}}}$$

$$=\frac{1}{1+\left[\cfrac{1}{2+\cfrac{9}{\frac{2661}{268}}}\right]}=\frac{1}{1+\frac{2661}{7734}}=\frac{7734}{10395}.$$

$\frac{7734}{10395}\cdot 4 = 2.9760+.$ $\qquad 3.1416 - 2.9760 + = .1655.$

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1. See pages 33, 43, 34, 44, 309, 310, 30.

2. (*a*) $3 \cdot 2 + 16 = 24 - 2$.
(*b*) $(a - 2b)^2 = a^2 - 4ab + 4b^2$.
(*c*) $4x - 3 = 18$.
(*d*) $\frac{2}{x} - \frac{3}{x+2} = 1,\ x^2 + 3x = 4$.
(*e*) $x - 5 = x + 5$. See also pages 161 and 377.

3. See pages 44–46.
(*a*) The subtraction axiom.
(*b*) Yes. See page 309, § 123, Principle I.

4. (*a*) $x = -4$. (*d*) $x = 0$.
(*b*) None. (*e*) None.
(*c*) $x = c$. (*f*) $x = -2, -1$.

5. (*a*) $x = 2$. (*d*) $x = 0$.
(*b*) $x = 3$. (*e*) $x = +2$ and -2.
(*c*) $x = -4$. (*f*) None.

6. $$\frac{5x}{21} - \frac{a}{6} = \frac{x}{14}. \qquad (1)$$

$(1) \cdot 42,\ 10x - 7a = 3x.$
$x = a.$

7. $$\frac{3}{2x} - \frac{3}{20} = \frac{7}{5x}. \qquad (1)$$

$(1) \cdot 20x,\ 30 - 3x = 28.$
$x = \frac{2}{3}.$

8. $$\frac{m-2}{m-3} = \frac{17}{18}. \qquad (1)$$

$(1) \cdot 18(m-3), \qquad 18m - 36 = 17m - 51.$
$m = -15.$

9. $$\frac{2x+5}{10x} - \frac{3(2x+1)}{2x} = 3\tfrac{1}{5}. \qquad (1)$$

$(1) \cdot 10x, \qquad 2x + 5 - 30x - 15 = 32x.$
$x = -\frac{1}{6}.$

10. $$\frac{5x-7}{6} - \frac{5}{2}\left(\frac{4-x}{10} = \right)\frac{15x-22}{6}. \qquad (1)$$

$(1) \cdot 60, \qquad 50x - 70 - 60 + 15x = 150x - 220.$
$-85x = -90,\ x = \frac{18}{17}.$

11. $$\frac{1-8x}{5}-\frac{2(1-6x)}{24x-3}=\frac{-24x+1}{15}. \qquad (1)$$

$(1)\cdot 15(8x-1)$, $-3+48x-192x^2-10+60x$
$$=-192x^2+32x-1.$$
$$76x=12,\ x=\tfrac{3}{19}.$$

12. $$\frac{3x-9}{3x-5}-2=\frac{3x-5}{8-3x}.$$

$(1)\cdot(3x-5)(8-3x)$, $-9x^2+51x-72+18x^2-78x+80$
$$=9x^2-30x+25.$$
$$3x=17,\ x=\tfrac{17}{3}.$$

13. $$\frac{1}{a}+\frac{1}{b}-\frac{1}{c}=\frac{1}{x}. \qquad (1)$$

$(1)\cdot abcx$, $\quad bcx+acx-abx=abc.$
$$(bc+ac-ab)x=abc.$$
$$x=\frac{abc}{bc+ac-ab}.$$

14. $$\frac{x}{a}+\frac{x+m}{b}=1. \qquad (1)$$

$(1)\cdot ab$, $\quad bx+ax+am=ab.$
$$(b+a)x=ab-am.$$
$$x=\frac{ab-am}{a+b}.$$

15. $$\frac{cy}{2d}-c^2=\frac{2dy-c^3}{c}. \qquad (1)$$

$(1)\cdot 2cd$, $\quad c^2y-2c^3d=4d^2y-2c^3d.$
$$(c^2-4d^2)y=0.$$
$$y=0.$$

16. $$\frac{2x-3b}{a}-\frac{2a-3x}{b}+\frac{9b}{a}=\frac{4a}{b}-5. \qquad (1)$$

$(1)\cdot ab$, $\quad 2bx-3b^2-2a^2+3ax+9b^2=4a^2-5ab.$
$$(2b+3a)x=6a^2-5ab-6b^2.$$
$$x=2a-3b.$$

17. $$\frac{3}{b(a-x)}+\frac{1}{a-x}=\frac{-3(b+3)}{2ab}. \qquad (1)$$

$(1)\cdot 2ab(a-x)$, $\quad 6a+2ab=-3ab+3bx-9a+9x.$
$$15a+5ab=x(3b+9).$$
$$\frac{5a}{3}=x.$$

18. $$\frac{3}{2y+1}+\frac{1+2y}{-2y+1}=\frac{4y^2}{1-4y^2}. \qquad (1)$$

$(1)\cdot(1-4y^2),\quad 3-6y+1+4y+4y^2=4y^2.$

$$-2y=-4,\ y=2.$$

19. $$\left(1-\frac{1}{c}\right)\div\left(1+\frac{1}{z}\right)+\frac{c+z}{z+1}=1. \qquad (1)$$

$$\frac{cz-z}{c(z+1)}+\frac{c+z}{z+1}=1. \qquad (2)$$

$(2)\cdot c(z+1),\quad cz-z+c^2+cz=cz+c.$

$$(c-1)z=c-c^2.$$

$$z=-c.$$

20. $$\frac{3b+9x}{9a+6x}-\frac{a-2x}{2x+3a}=2.$$

$$\frac{b+3x-a+2x}{2x+3a}=2. \qquad (1)$$

$(1)\cdot(2x+3a),\quad 5x-a+b=4x+6a.$

$$x=7a-b.$$

21. $$\frac{8}{x-7}-\frac{2-6x}{x^2-6x-7}=\frac{27}{x+1}. \qquad (1)$$

$(1)\cdot(x-7)(x+1),\quad 8x+8-2+6x=27x-189.$

$$-13x=-195,\ x=15.$$

22. $$\frac{x}{2(a+b)}-\frac{5}{b-a}=\frac{bx}{b^2-a^2}. \qquad (1)$$

$(1)\cdot 2(b+a)(b-a),\quad bx-ax-10b-10a=2bx.$

$$(-a-b)x=10b+10a.$$

$$x=-10.$$

23. $$\frac{2x}{3}-\frac{d}{5}\left(\frac{6x}{c}-10d\right)=2cd\left(\frac{2}{3}-\frac{d}{5c}\right). \qquad (1)$$

$(1)\cdot 15c,\quad 10cx-18dx+30cd^2=20c^2d-6cd^2.$

$$(10c-18d)x=20c^2d-36cd^2.$$

$$x=2cd.$$

24. $$82.4-13x=32x-52.6.$$

$$-45x=-135.0.$$

$$x=3.$$

25. $$.01(2x+.205)-.0125(1.5x-.5)=.01955. \qquad (1)$$

$(1)\cdot 10{,}000,\quad 100(2x+.205)-125(1.5x-.5)=195.5.$

$$200x+20.5-187.5x+62.5=195.5.$$

$$12.5x=112.5.$$

$$x=9.$$

26.
$$\frac{1}{x-.33\,a}+\frac{1}{\dfrac{2\,x}{3}-.22\,a}=\frac{5}{4\,a}.$$

$$\frac{100}{100\,x-33\,a}+\frac{300}{200\,x-66\,a}=\frac{5}{4\,a}.$$

$$\frac{500}{200\,x-66\,a}=\frac{5}{4\,a}.$$

$$400\,a=200\,x-66\,a.$$

$$2.33\,a=x.$$

27.
$$\frac{c^2}{ax}+\frac{1}{c}=\frac{3\,c-3\,a}{x}+\frac{1}{a}+\frac{a^2}{cx}. \quad (1)$$

(1) $\cdot\, acx$,
$$c^3+ax=3\,ac^2-3\,a^2c+cx+a^3.$$

$$(a-c)\,x=a^3-3\,a^2c+3\,ac^2-c^3.$$

$$x=(a-c)^2.$$

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2. Let x equal the number of minute spaces that the minute hand travels from its position at 7 o'clock until it overtakes the hour hand. Obviously the minute hand travels 35 minute spaces more than the hour hand.

Then $\quad x=(x-35)\,12.$

Whence $\quad x=38\tfrac{2}{11}.$

3. Let x equal the number of minute spaces that the minute hand travels from its position at 2 o'clock until it is in the required position. For the first of the required positions the minute hand must gain 10 minute spaces over the hour hand; for the second position 40 spaces.

Then $\quad x=(x-10)\,12.$

Whence $\quad x=10\tfrac{10}{11},$

and $\quad x=(x-40)\,12.$

Whence $\quad x=43\tfrac{7}{11}.$

4. Let x equal the number of minute spaces passed over by the minute hand in going from its position at 6 o'clock to the required position. For the first it must gain 40 spaces and for the second 20 spaces.

Then $\quad x=(x-40)\,12.$

Whence $\quad x=43\tfrac{7}{11},$

and $\quad x=(x-20)\,12.$

Whence $\quad x=21\tfrac{9}{11}.$

5. Let x equal the number of minute spaces passed over by the minute hand in going from its position at 5 o'clock to the required position. Now if the hands are at right angles, one is 15 minute spaces from the other. For the first position the minute hand must gain 10 spaces and for the second 40 spaces.

Then $x = (x - 10)\,12.$

Whence $x = 10\frac{10}{11},$

and $x = (x - 40)\,12.$

Whence $x = 43\frac{7}{11}.$

7. Let x equal the number of days from April 28, 1909, until the next conjunction.

Then, as in Exercise 6, $\frac{x}{225} = \frac{x}{365} + 1.$

Whence $x = 586 + .$

Therefore the required date is December 5, 1910.

8. Let x equal the number of days from opposition to opposition.

Then $\frac{x}{365} = \frac{x}{4332} + 1.$

Whence $x = 398 + .$

Therefore the required date is April 2, 1910.

9. Let x equal the number of hours from the start until they are again together.

Then $\frac{x}{2\frac{1}{2}} = \frac{x}{3} + 1.$

Whence $x = 15.$

10. Reducing $2\frac{3}{4}$, $3\frac{1}{3}$, and $4\frac{2}{5}$ to a common denominator gives $\frac{165}{60}$, $\frac{200}{60}$, and $\frac{264}{60}$.

The L.C.M. of the numerators is 6600.

The L.C.M. of the fractions is $\frac{6600}{60} = 110.$

Hence the three are together 110 hours after the start.

11. Yes, $15 \div 2\frac{1}{2} = 6$, and $15 \div 3 = 5$. Yes.

12. Yes; Yes.

13. $\frac{11}{4}, \frac{10}{3}, \frac{22}{5}, \frac{\text{L.C.M. of numerators}}{\text{G.C.D. of denominators}} = \frac{110}{1}.$

14. Rule. Divide the L.C.M. of the numerators by the G.C.D. of the denominators.

L.C.M. of $\frac{7}{6}$, $\frac{7}{3}$, and $\frac{28}{9}$ is $\frac{28}{3}$, or $9\frac{1}{3}$.

15. Assuming that a and c have no common integral factor and that b and d have none, the L.C.M. of $\frac{a}{b}$ and $\frac{c}{d} = \frac{ac}{1} = ac$.

With an assumption similar to the preceding one, the L.C.M. of

$$\frac{a}{b}, \frac{c}{d}, \text{ and } \frac{e}{f} = \frac{ace}{1} = ace.$$

16. Let x equal the number of ounces needed.

Then $$\frac{56}{56 + x} = \frac{70}{100}.$$

Whence $$x = 24.$$

17. $$\frac{\frac{16 \cdot 410}{100} + x}{410 + x} = \frac{18}{100}.$$

$$6560 + 100\,x = 7380 + 18\,x.$$

Whence $$x = 10.$$

18. Let m equal the number of gallons of milk and w the number of gallons of water.

Then $$m + w = 30, \quad (1)$$

and $$\frac{\frac{20\,m}{100}}{30} = \frac{16}{100}. \quad (2)$$

Solving (1) and (2), $$m = 24,$$

and $$w = 6.$$

19. Let x equal the number of gallons of 90% alcohol.

Then $$\frac{\frac{90}{100}x + \frac{95}{100} \cdot 10}{x + 10} = \frac{92}{100}.$$

Whence $$x = 15.$$

20. Let e equal the diameter of the earth in miles, and m the diameter of the moon in miles.

Then $$e = \frac{11\,m}{3},$$

and $$e - m = 5760.$$

Whence $$e = 7920,$$

and $$m = 2160.$$

21. Let s equal the diameter of the sun in miles, and e the diameter of the earth in miles.

Then $$s = 3220 + 109\,e,$$

and $$s + e = 874{,}420.$$

Whence $$s = 866{,}500,$$

and $$e = 7920.$$

22. Let s equal the sun's distance from the earth in miles, and m the moon's distance from the earth in miles.

Then $$s = 387\tfrac{1}{2}\,m,$$

and $$\frac{s}{186000} = \frac{m}{186000} + 498\tfrac{22}{31}.$$

Whence $$s = 93{,}000{,}000,$$

and $$m = 240{,}000.$$

23. Let j equal the diameter of Jupiter in miles, and e equal the diameter of the earth in miles.

Then $$j = \frac{120\,e}{11},$$

and $$j + e = 94{,}320.$$

Whence $$j = 86{,}400,$$

and $$e = 7920.$$

24. Let x be the number of days B requires to finish the work.

Then $$3 \cdot \tfrac{1}{15} + 3 \cdot \tfrac{1}{25} + x \cdot \tfrac{1}{25} = 1.$$

Whence $$x = 17.$$

25. Let x be the number of days required to do the job, when all work together.

Then $$\frac{1}{a} + \frac{1}{b} + \frac{1}{a+b} = \frac{1}{x}.$$

Whence $$x = \frac{a^2b + ab^2}{a^2 + 3\,ab + b^2}.$$

26. Let h be the number of hours required to fill the cistern when both pipes are open.

Then $$\frac{1}{2\,m} - \frac{1}{\dfrac{n+1}{3}} = \frac{1}{h}.$$

Whence $$h = \frac{2\,mn + 2m}{n + 1 - 6\,m}.$$

27. By the conditions of the problem the numerator $2\,mn + 2\,m$ is a positive number. Therefore, if $6\,m$ is less than $n + 1$, h in Problem 26 is positive. This would mean that the water comes in faster than it goes out.

If $6\,m$ equals $n + 1$, h equals a fraction whose denominator is zero. This would mean that the water runs out as fast as it runs in.

If $6\,m$ is greater than $n + 1$, h equals a fraction whose denominator is negative. This would mean that more water could run out than runs in.

28. Here
$$\frac{1}{2m} + \frac{1}{\frac{n+1}{3}} = \frac{1}{h} \cdot \frac{1}{x}.$$

Whence
$$h = \frac{2mn + 2m}{nx + x + 6mx}.$$

29. Let r equal the length of the radius in inches.

Then
$$\tfrac{22}{7} r^2 + 440 = \tfrac{22}{7}(r+7)^2.$$

Whence
$$r = 6\tfrac{1}{2}.$$

30. Let R be the radius of the earth and x be the radius of the hoop, both in feet.

Then
$$2\pi R + 1 = 2\pi x.$$

Whence
$$x = \frac{2\pi R + 1}{2\pi} = R + \frac{1}{2\pi}.$$

The distance from the earth to the hoop $= R + \dfrac{1}{2\pi} - R = \dfrac{1}{\frac{44}{7}}$ feet, or $1\frac{10}{11}$ inches.

31. Here if x equals radius of the hoop in inches, $2\pi \cdot 3 + 12 = 2\pi x$.

Whence
$$x = 3 + \frac{6}{\pi}.$$

The distance of the hoop from the pole is $3 + \dfrac{6}{\pi} - 3 = 1\frac{10}{11}$ inches.

32. Let x equal the rate of the freight in miles per hour.

Then
$$42\left(b + \frac{a}{60}\right) = x\left(a + \frac{b}{60} + b + \frac{a}{60}\right).$$

Whence
$$x = \frac{2520\,b + 42\,a}{61\,a + 61\,b}.$$

33. Let x be the required weight in pounds. Then by the law of the lever (see page 168),
$$3x = 4 \cdot 100.$$

Whence
$$x = 133\tfrac{1}{3}.$$

Therefore the weight is $133\frac{1}{3}$ pounds.

34. Let AB be the beam and C the point on which the load rests, AC being 7 feet. Then A may be regarded as the fulcrum, and the force downward at C equal to the upward force at B. Then, if x is the pressure in tons on B,
$$AC \cdot 3 = AB \cdot x,$$

or
$$7 \cdot 3 = 12 \cdot x.$$

Whence $x = \frac{7}{4}$, the pressure on B.

Since the pressure on A and B together is 3 tons, the pressure on A alone equals 3 tons minus $\frac{7}{4}$ tons, or $\frac{5}{4}$ tons.

35. Let the load on the shorter and on the longer arms be $3\,l$ and $2\,l$ respectively, and let x be the required distance.

Then $$x \cdot 8 + (2\,l - 4)\,12 = 8 \cdot 3\,l.$$

Solving, $$x = 6.$$

36. Let x equal the pressure in pounds on the support. Then, as in the solution of Problem 35, we have

$$8 \cdot x = 12 \cdot 130.$$

Whence $$x = 195.$$

37. (*a*) 100 pounds on each support.

(*b*) The support 3 feet from the end does exactly what it did at the end, — support the beam. In one case it does it at the end of an arm 14 feet long, in the other at the end of an arm 11 feet long.

Let x equal the pressure in pounds on the support.

Then $$11\,x = 14 \cdot 100.$$

$$\therefore x = 127\tfrac{3}{11}.$$

38. Solution like (*b*) in Problem 37. Let x equal the weight in pounds on the support which is 4 feet from one end.

Then $$12 \cdot x = 16 \cdot 150.$$

$$x = 200, \text{ weight on one support.}$$

$$300 - 200 = 100, \text{ weight on the other support.}$$

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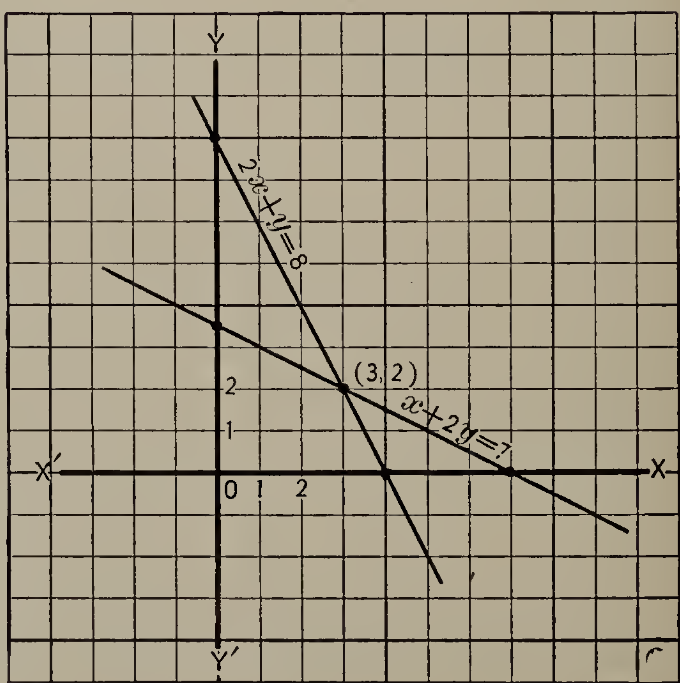
1. $2\,x + y = 8$

x	0	4	2
y	8	0	4

$x + 2\,y = 7$

x	0	7	1
y	$\frac{7}{2}$	0	3



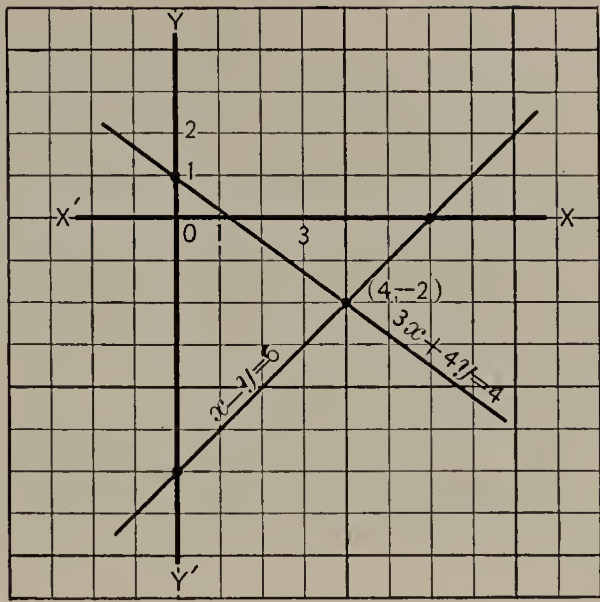
$x = 3,\ y = 2.$

2. $x - y = 6$

x	0	6	2
y	-6	0	-4

$3x + 4y = 4$

x	0	$\frac{4}{3}$	4
y	1	0	-2



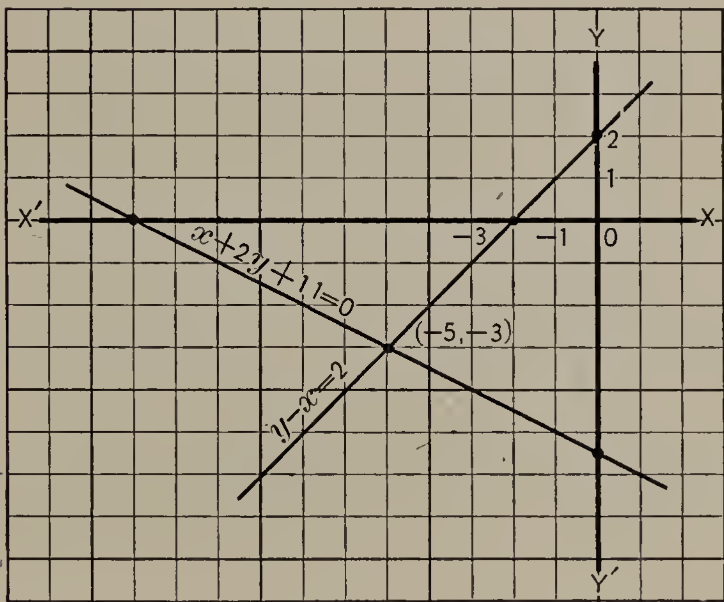
$x = 4, y = -2.$

3. $x + 2y + 11 = 0$

x	0	-11	1
y	$-\frac{11}{2}$	0	-6

$y - x = 2$

x	0	-2	-4
y	2	0	-2



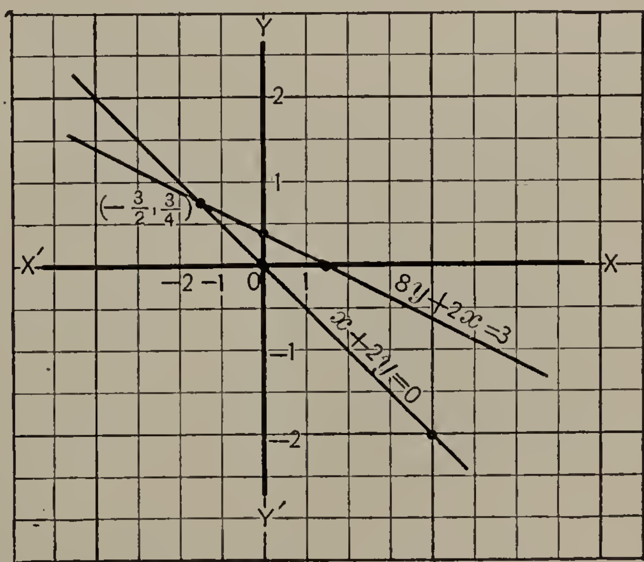
$x = -5, y = -3.$

4. $x + 2y = 0$

x	0	4	-2
y	0	-2	1

$8y + 2x = 3$

x	0	$\frac{3}{2}$	8
y	$\frac{3}{8}$	0	$-\frac{13}{8}$



$x = -\frac{3}{2}, y = \frac{3}{4}.$

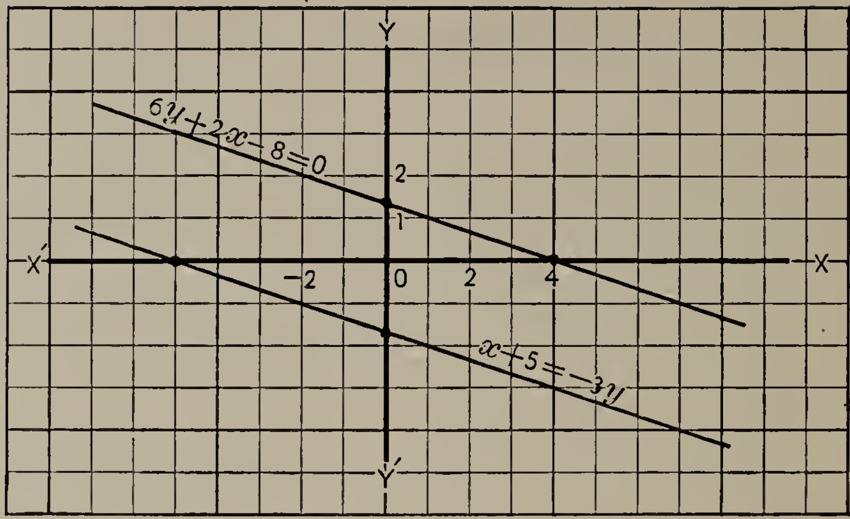
5. $x + 5 = -3y$

x	0	-5	7
y	$-\frac{5}{3}$	0	-4

$6y + 2x - 8 = 0$

x	0	4	-2
y	$\frac{4}{3}$	0	2

Inconsistent.



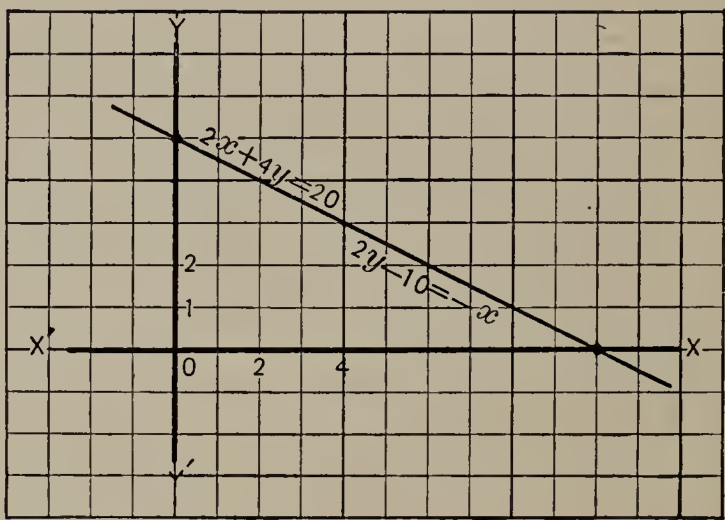
6. $2x + 4y = 20$

x	0	10	8
y	5	0	1

$2y - 10 = -x$

x	0	10	6
y	5	0	2

Infinite number of sets of roots.



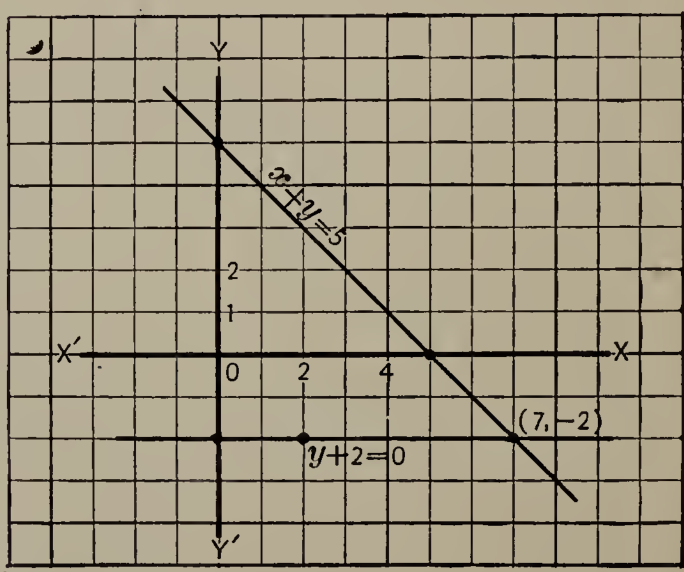
7. $x + y = 5$

x	0	5	2
y	5	0	3

$0x + y + 2 = 0$

x	0	2	4
y	-2	-2	-2

$x = 7, y = -2.$



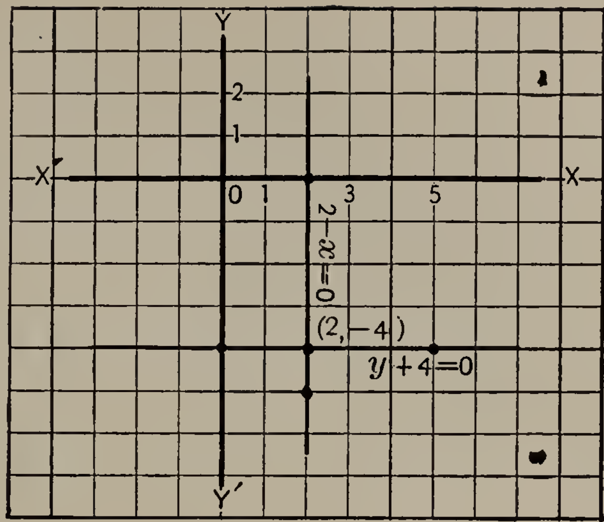
8. $0x + y + 4 = 0$

x	0	5	-2
y	-4	-4	-4

$0y + 2 - x = 0$

x	2	2	2
y	0	-5	-3

$x = 2,\ y = -4.$



9. $x + y = 4$

x	0	4	2
y	4	0	2

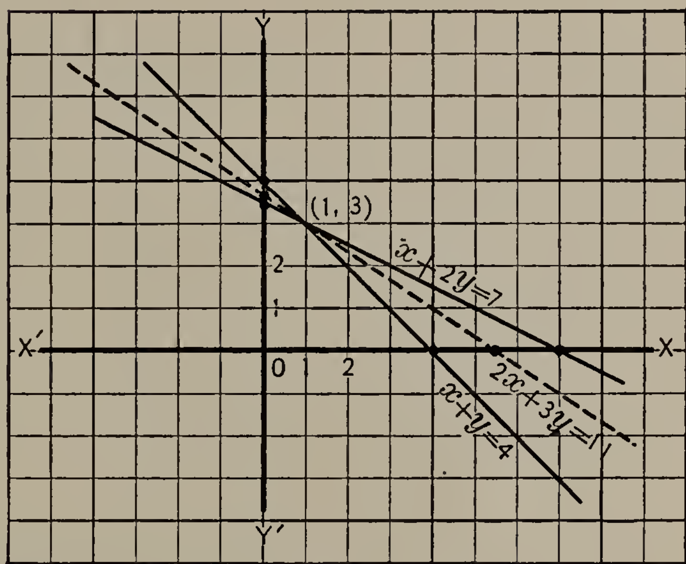
$x + 2y = 7$

x	0	7	1
y	$\frac{7}{2}$	0	3

$2x + 3y = 11$

x	0	$\frac{11}{2}$	4
y	$\frac{11}{3}$	0	1

$x = 1,\ y = 3.$



The third graph passes through the point of intersection of the other two.

10. $x - y = 5$

x	0	5	7
y	-5	0	2

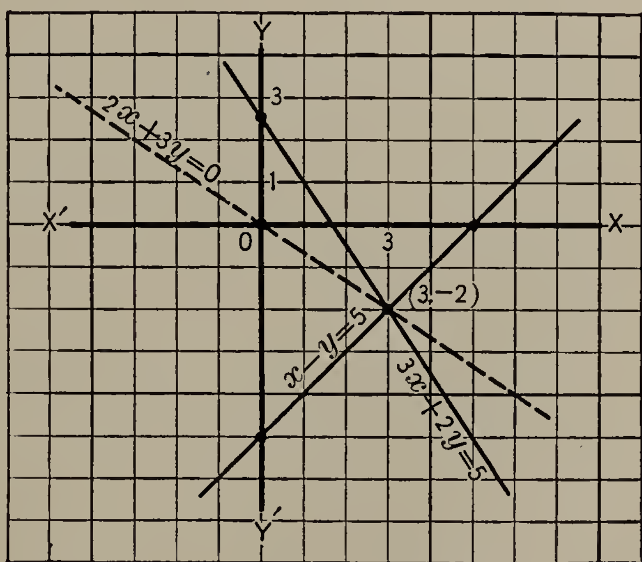
$3x + 2y = 5$

x	0	3	5
y	$\frac{5}{2}$	-2	-5

$2x + 3y = 0$

x	0	6	-3
y	0	-4	2

$x = 3,\ y = -2.$



The third graph passes through the point of intersection of the other two.

11. $3x - 4y = 12$

x	0	4	2
y	-3	0	$-\frac{3}{2}$

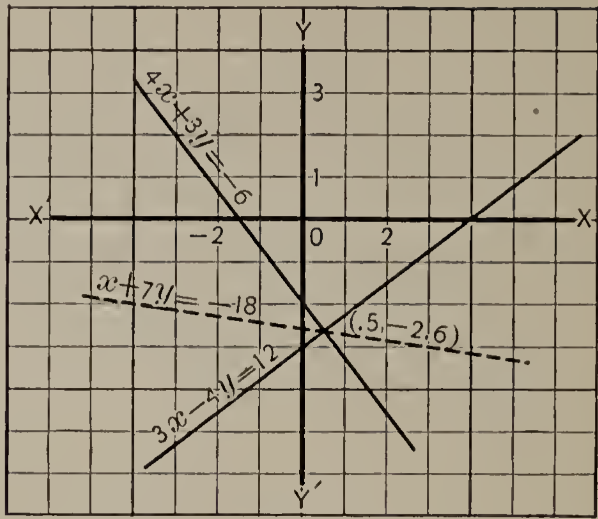
$4x + 3y = -6$

x	0	$-\frac{3}{2}$	3
y	-2	0	-6

$x + 7y = -18$

x	0	-4	3
y	$-\frac{18}{7}$	-2	-3

$x = .5,\ y = -2.6.$



The third graph passes through the point of intersection of the other two.

12. Yes. A linear equation obtained by adding two given linear equations or by subtracting one from the other is always satisfied by the set of values which satisfies the given equations because its graph passes through the point of intersection of the graphs of the given equations.

13. $x - 2y = 2$

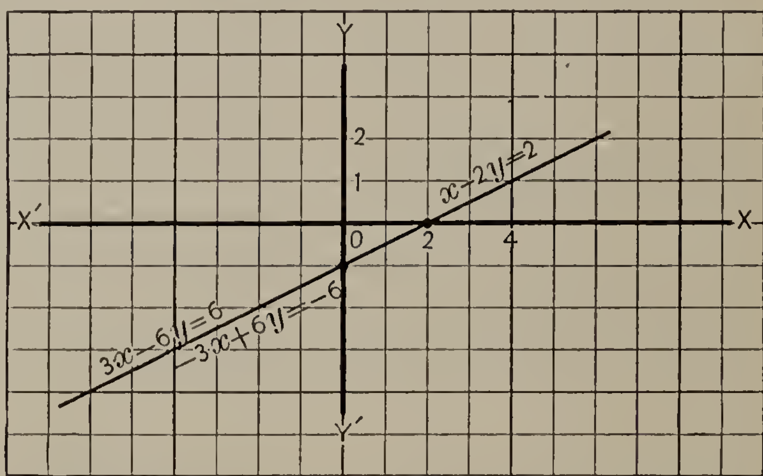
x	0	2	6
y	-1	0	2

$3x - 6y = 6$

x	0	2	4
y	-1	0	1

$-3x + 6y = -6$

x	0	2	-2
y	-1	0	-2



An infinite number of sets of values will satisfy. The three graphs are identical.

If one equation is obtained from another by multiplying both members of the given equation by a constant, the given and the resulting equations have identical graphs.

14. (0, 0).

15. Yes. If the graphs of two equations are parallel lines, the system has no graphical solution.

16. Yes, different scales could have been used on the two axes. The lines, however, must be plotted to the same scales on each axis, otherwise the point of intersection would have no meaning related to the two equations.

17. (*a*) $y = c$. (*b*) $x = d$.

18. $ax + by = 0$.

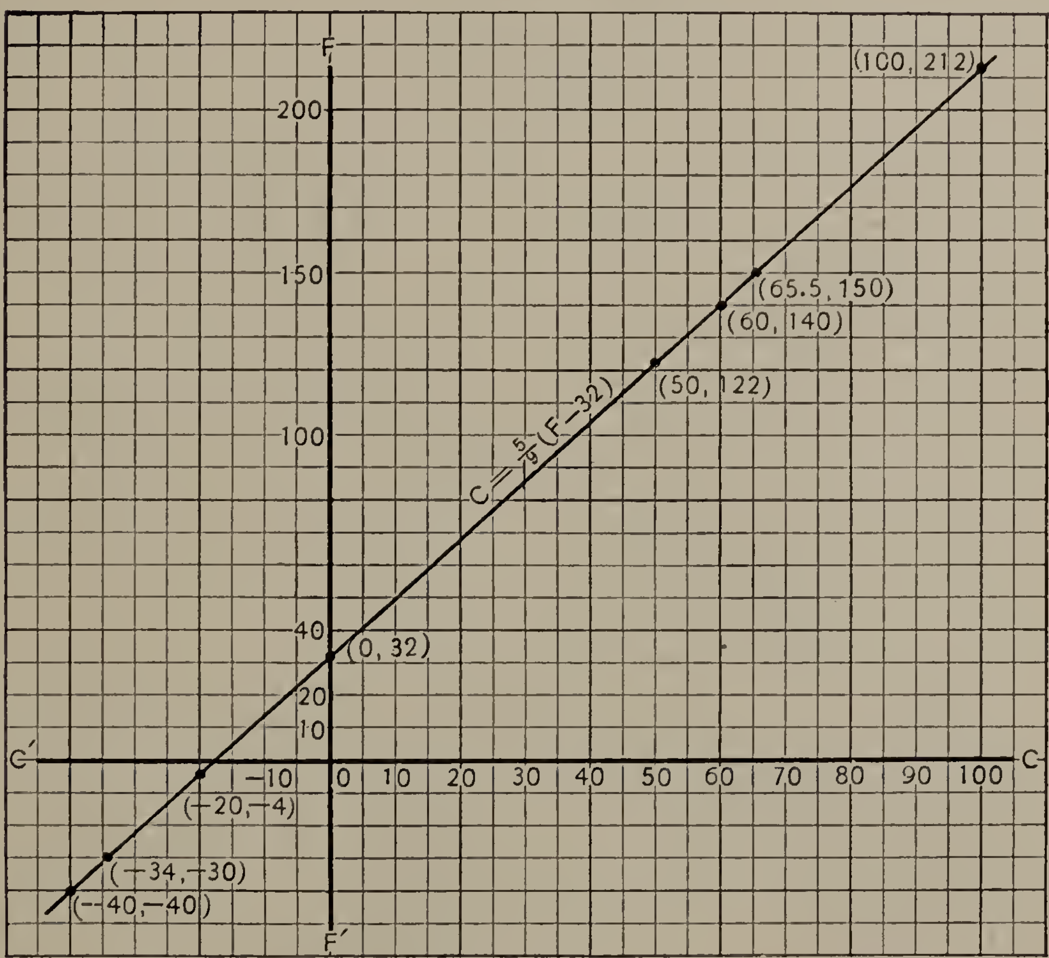
19. (*a*) $3x - y = 6$, $y - 3x = 18$.
(*b*) $3x - 4y = 12$, $6x - 8y = 24$.

20. The number of degrees from the freezing point of water to its boiling point is 100 on the Centigrade scale and 212 — 32, or 180, on the Fahrenheit. Therefore one degree on the Fahrenheit scale equals $\frac{5}{9}$ of a degree on the Centigrade scale. To get a reading above the freezing point on the Fahrenheit scale we must subtract 32 degrees from the given reading, while on the Centigrade scale all readings are made with reference to the freezing point. Hence, if C is degrees Centigrade and F degrees Fahrenheit, $C = \frac{5}{9}(F - 32)$.

$$C = \tfrac{5}{9}(F - 32).$$

Yes.

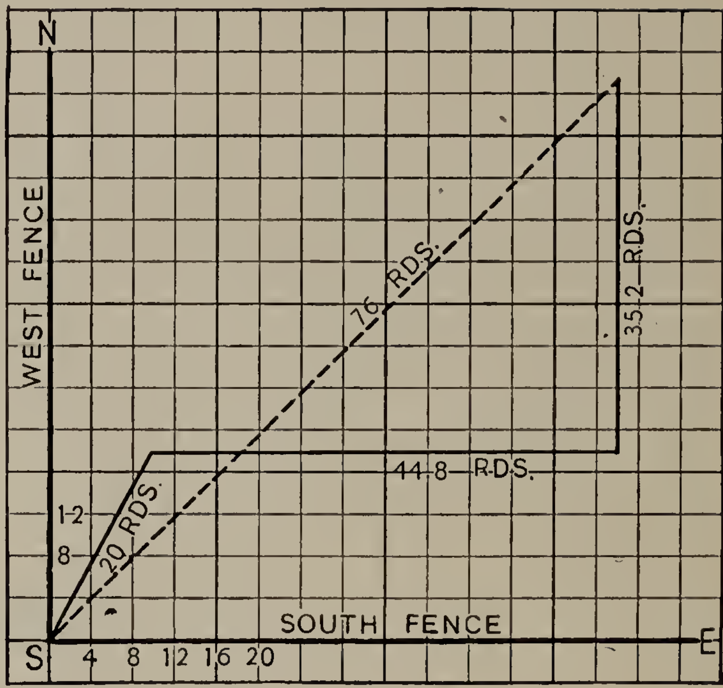
C	0	50	100
F	32	122	212



21. (*a*) 60° C = 140° F.
(*b*) 150° F = 65.5° C.
(*c*) − 20° C = − 4° F.
(*d*) − 30° F = − 34° C.

22. − 40° C = − 40° F.

23. First distance is 20 rods.
Second distance is 44.8 rods.
Third distance is 35.2 rods.
Distance from starting point is 76 rods.



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1. See pages 203 and 458.

2. For (a), (b), (c), (e), (f), and (g), see pages 203–204.
For (d) see page 308.
For (h) see page 206.

3. (a) A straight line.
(b) Two or more straight lines.
(c) Two or more straight lines.
(d) Two or more identical lines.
(e) Two or more intersecting lines.
(f) One straight line.
(g) Two coincident lines.
(h) Two parallel lines.

4. $ax + by = c.$

5. $ax + by = c,$
$ax + by = d.$

6.
$$ax + by = c,$$
$$kax + kby = kc.$$

7.
$$2x + 5y = 8, \qquad (1)$$
$$x - 10y = 9. \qquad (2)$$
(2) · 2, $\quad 2x - 20y = 18. \qquad (3)$
(1) – (3), $\quad 25y = -10.$
Then $\quad y = -\frac{2}{5}.$
Substituting $-\frac{2}{5}$ for y in (2),
$$x + 4 = 9.$$
Whence $\quad x = 5.$

8.
$$5x - 12y = -38, \qquad (1)$$
$$3x + 8y = 0. \qquad (2)$$
(1) · 2, $\quad 10x - 24y = -76. \qquad (3)$
(2) · 3, $\quad 9x + 24y = 0. \qquad (4)$
(3) + (4), $\quad 19x = -76.$
Whence $\quad x = -4.$
Substituting -4 for x in (2),
$$-12 + 8y = 0.$$
Whence $\quad y = \frac{3}{2}.$

9. $$9t - 2n = 18, \quad (1)$$
$$20t - 7n = 63. \quad (2)$$
(1) · 7, $$63t - 14n = 126. \quad (3)$$
(2) · 2, $$40t - 14n = 126. \quad (4)$$
(3) – (4), $$23t = 0.$$
Whence $$t = 0.$$
Substituting 0 for t in (1), $$n = -9.$$

10. $$11m + 18n = 10, \quad (1)$$
$$9m + 12n = -15. \quad (2)$$
[(1) · 2 – (2) · 3], $$-5m = +65.$$
$$\therefore m = -13.$$
Substituting – 13 for m in (2),
$$-117 + 12n = -15.$$
Whence $$n = 8\tfrac{1}{2}.$$

11. $$3x - 2y = 18, \quad (1)$$
$$5x - 8y = 30. \quad (2)$$
(1) · 4, $$12x - 8y = 72. \quad (3)$$
(2) – (3), $$-7x = -42.$$
$$\therefore x = 6.$$
Substituting 6 for x in (1), $$18 - 2y = 18.$$
Whence $$y = 0.$$

12. $$3r - 8s = 13, \quad (1)$$
$$r + 6s = 0. \quad (2)$$
From (2), $$r = -6s. \quad (3)$$
Substituting – $6s$ for r in (1),
$$-18s - 8s = 13, \text{ or } s = -\tfrac{1}{2}.$$
Substituting – $\frac{1}{2}$ for s in (2), $$r = 3.$$

13. $$2(x + y) + 3y = 4, \quad \text{or} \quad 2x + 5y = 4, \quad (1)$$
$$5 = x + y; \quad x + y = 5. \quad (2)$$
Substituting $5 - x$ for y from (2) in (1),
$$2x + 25 - 5x = 4.$$
Whence $$x = 7.$$
Substituting 7 for x in (2), $$y = -2.$$

14. $$16x + 7 = 15y, \quad (1)$$
$$4x + 5y = 0. \quad (2)$$
Substituting $-\dfrac{4x}{5}$ for y from (2) in (1),
$$16x + 7 = -12x. \quad (3)$$
Whence $$x = -\tfrac{1}{4}. \quad (4)$$
Then from (2) and (4), $$y = \tfrac{1}{5}.$$

15. $$6x+\frac{14}{3}-12y=0, \quad\text{or}\quad 18x-36y+14=0, \qquad (1)$$
$$7y-3x-4=0; \qquad 7y-3x-4=0. \qquad (2)$$

From (2), $$x=\frac{7y-4}{3}. \qquad (3)$$

Substituting $\frac{7y-4}{3}$ for x in (1),
$$42y-24-36y+14=0. \qquad (4)$$

Whence $$6y=10, \text{ or } y=\frac{5}{3}. \qquad (5)$$

Then from (3) and (5), $$x=\frac{23}{9}.$$

16. $$\frac{8m-3n}{2}+6n=-9, \quad\text{or}\quad 8m+9n=-18, \qquad (1)$$
$$4m-1=3n; \qquad 4m-1=3n. \qquad (2)$$

From (2), $$n=\frac{4m-1}{3}. \qquad (3)$$

Substituting $\frac{4m-1}{3}$ for n in (1),
$$8m+12m-3=-18. \qquad (4)$$

Whence $$20m=-15, \text{ or } m=-\tfrac{3}{4}. \qquad (5)$$

From (3) and (5), $$n=-\tfrac{4}{3}.$$

17. $$\frac{2x}{3}+y=-7, \quad\text{or}\quad 2x+3y=-21, \qquad (1)$$
$$5x-3y=105; \qquad 5x-3y=105. \qquad (2)$$

(1) + (2), $$7x=84. \qquad (3)$$

Whence $$x=12. \qquad (4)$$

From (1) and (4), $$y=-15.$$

18. $$\frac{3r}{4}-\frac{7}{2}=\frac{s}{12}, \qquad (1)$$
$$r+8=-2s. \qquad (2)$$

From (1),
$$9r-s=42. \qquad (3)$$

From (2),
$$r+2s=-8. \qquad (4)$$

(4) · 9, $9r+18s=-72.$ (5)

(3) − (5), $-19s=114.$
$$s=-6.$$

Substituting -6 for s in (4),
$$r=4.$$

19. $$\frac{3y+1}{13}-\frac{z+20}{12}=-3. \qquad (1)$$
$$\frac{z}{8}-\frac{2y}{9}=1. \qquad (2)$$

From (1), $36y-13z=-220.$ (3)

From (2), $-16y+9z=72.$ (4)

(3) · 4, $144y-52z=-880.$ (5)

(4) · 9, $-144y+81z=648.$ (6)

(5) + (6), $29z=-232.$
$$z=-8.$$

Substituting -8 for z in (4),
$$y=-9.$$

20.
$$5x + .7y = -1.25, \quad (1)$$
$$.12x - .08y = 3\tfrac{3}{5}. \quad (2)$$
$(1)\cdot 100,$
$$500x + 70y = -125. \quad (3)$$
$(2)\cdot 100,$ $12x - 8y = 360. \quad (4)$
$(3)\cdot 8,$ $4000x + 560y = -1000. \quad (5)$
$(4)\cdot 70,$
$$840x - 560y = 25{,}200. \quad (6)$$
$(5)+(6),$ $4840x = 24{,}200.$
$$x = 5.$$
Substituting 5 for x in (4),
$$60 - 8y = 360.$$
$$\therefore y = -37\tfrac{1}{2}.$$

21.
$$\frac{1}{x} + \frac{1}{y} = -\frac{1}{6}, \quad (1)$$
$$\frac{2}{x} - \frac{3}{y} = -\frac{4}{3}. \quad (2)$$
$(1)\cdot 2,$ $\dfrac{2}{x} + \dfrac{2}{y} = -\dfrac{1}{3}. \quad (3)$
$(2)-(3),$ $-\dfrac{5}{y} = -1.$
$$\therefore y = 5.$$
Substituting 5 for y in (1),
$$\frac{1}{x} + \frac{1}{5} = -\frac{1}{6}.$$
$$\therefore x = -\tfrac{30}{11}.$$

22.
$$\frac{2m + 3n - 2}{m + n + 6} = \frac{4}{3}, \quad (1)$$
$$\frac{1}{m} + \frac{1}{n} = \frac{9}{m}. \quad (2)$$
From (1), $2m + 5n = 30. \quad (3)$
From (2), $m - 8n = 0. \quad (4)$
$(4)\cdot 2,$ $2m - 16n = 0. \quad (5)$
$(3)-(5),$ $21n = 30.$
$$n = \tfrac{10}{7}.$$
Substituting $\tfrac{10}{7}$ for n in (4), $m = \tfrac{80}{7}.$

23.
$$\frac{.25h + 8 + .1k}{k - 10 + h} = 2.5, \quad (1)$$
$$\frac{1}{.8h - 2.2} + \frac{20}{35 - 5.5k} = 0. \quad (2)$$
From (1), $225h + 240k = 3300. \quad (3)$
From (2), $160h - 55k = 90. \quad (4)$
$(3)\cdot 32,$ $7200h + 7680k = 105{,}600. \quad (5)$
$(4)\cdot 45,$ $7200h - 2475k = 4{,}050. \quad (6)$
$(5)-(6),$ $10{,}155k = 101{,}550. \quad (7)$
$$k = 10. \quad (8)$$
From (4) and (8), $h = 4.$

24.
$$\tfrac{2}{3}(2h + 5k) = 39, \quad (1)$$
$$2h - \frac{3k}{4} = 3(k - h). \quad (2)$$
From (1), $4h + 10k = 117. \quad (3)$
From (2), $4h - 3k = 0. \quad (4)$
$(3)-(4),$ $13k = 117. \quad (5)$
$$\therefore k = 9. \quad (6)$$
From (6) and (4), $h = 6\tfrac{3}{4}.$

25. $\dfrac{3(x+y)}{\frac{3}{5}} + \dfrac{x-y}{-\frac{11}{5}} = 0, \quad (1)$

$2x + y = 7. \quad (2)$

From (1), $5x + 6y = 0. \quad (3)$

(2) · 6, $12x + 6y = 42. \quad (4)$

(4) − (3), $7x = 42. \quad (5)$

$\therefore x = 6. \quad (6)$

From (2) and (6), $y = -5$.

26. $\dfrac{m+5}{n} = \dfrac{m-5}{n-10}, \quad (1)$

$\dfrac{m - \frac{2}{5}(3m - 2n) - \frac{1}{3}}{m - 2n - 1} + \dfrac{1}{3} = 0. \quad (2)$

From (1), $-m + n = 5. \quad (3)$

From (2), $m + n = 5. \quad (4)$

(3) + (4), $2n = 10, \; n = 5.$

(3) − (4), $-2m = 0, \; m = 0.$

27. $2cx - y = 5ac, \quad (1)$

$\dfrac{2x}{3} - \dfrac{y}{c} = a. \quad (2)$

(1) · 1, $2cx - y = 5ac. \quad (3)$

From (2), $2cx - 3y = 3ac. \quad (4)$

(3) − (4), $2y = 2ac. \quad (5)$

$y = ac. \quad (6)$

From (3) and (6), $x = 3a$.

28. $\dfrac{a}{x} + \dfrac{3a}{y} = 1, \quad (1)$

$\dfrac{3a}{x} + \dfrac{a}{y} = \dfrac{1}{2}. \quad (2)$

(1) · 3, $\dfrac{3a}{x} + \dfrac{9a}{y} = 3. \quad (3)$

(2) − (3), $-\dfrac{8a}{y} = -\dfrac{5}{2}. \quad (4)$

$y = \dfrac{16a}{5}. \quad (5)$

From (1) and (5), $x = 16a$.

29. $\dfrac{x+y}{10a} = 2 - \dfrac{x-y}{4a}, \quad (1)$

$x = y. \quad (2)$

Substituting x for y in (1),

$\dfrac{2x}{10a} = 2 - 0. \quad (3)$

$x = 10a. \quad (4)$

From (2) and (4), $y = 10a$.

30. $ay + (a+b)x = xy, \quad (1)$

$\dfrac{a+b}{x} + \dfrac{b}{y} = \dfrac{4}{3}. \quad (2)$

(1) ÷ xy, $\dfrac{a}{x} + \dfrac{a+b}{y} = 1. \quad (3)$

(3) − (2), $-\dfrac{b}{x} + \dfrac{a}{y} = \dfrac{-1}{3}. \quad (4)$

(3) · b, $\dfrac{ab}{x} + \dfrac{ab+b^2}{x} = b. \quad (5)$

(4) · a, $-\dfrac{ab}{x} + \dfrac{a^2}{y} = \dfrac{-a}{3}. \quad (6)$

(5) + (6), $\dfrac{a^2 + ab + b^2}{y} = -\dfrac{a}{3} + b.$

$$y = \frac{3a^2 + 3ab + 3b^2}{-a + 3b}.$$

From (3), $\dfrac{a}{x} + (a+b) \cdot \dfrac{-a+3b}{3a^2 + 3ab - 3b^2} = 1.$

$$x = \frac{3a^2 + 3ab + 3b^2}{b + 4a}.$$

31.

$$(c+b)y+ax=1, \qquad (1)$$
$$ay+(c+b)x=1. \qquad (2)$$

$(1)\cdot a$, $$(ac+ab)y+a^2x=a. \qquad (3)$$

$(2)\cdot(b+c)$, $$(ac+ab)y+(c+b)^2x=b+c. \qquad (4)$$

$(4)-(3)$, $$[(b+c)^2-a^2]x=b+c-a. \qquad (5)$$

$$x=\frac{b+c-a}{(b+c)^2-a^2}=\frac{1}{b+c+a}. \qquad (6)$$

From (1) and (6), $$(c+b)y+\frac{a}{b+c+a}=1.$$

$$\therefore y=\frac{1}{b+c+a}.$$

32.

$$a^4\cdot a^{y-7}=a^{12}, \qquad (1)$$
$$c^4\cdot c^{x-3}=c^y. \qquad (2)$$

From (1), $$4+y-7=12. \qquad (3)$$

From (2), $$4+x-3=y. \qquad (4)$$

From (3), $$y=15. \qquad (5)$$

From (4) and (5), $$x=14.$$

33.

$$ax+by=c, \qquad (1)$$
$$dx+ey=f. \qquad (2)$$

$(1)\cdot d$, $$adx+bdy=cd. \qquad (3)$$

$(2)\cdot a$, $$adx+aey=af. \qquad (4)$$

$(3)-(4)$, $$(bd-ae)y=cd-af. \qquad (5)$$

From (5), $$y=\frac{cd-af}{bd-ae}. \qquad (6)$$

From (2) and (6), $$ax+b\left(\frac{cd-af}{bd-ae}\right)=c. \qquad (7)$$

From (7), $$x=\frac{bf-ce}{bd-ae}.$$

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1. $\begin{vmatrix}4 & 1\\ 3 & 5\end{vmatrix}=20-3=17.$

2. $\begin{vmatrix}6 & -2\\ 8 & 1\end{vmatrix}=6+16=22.$

3. $3\begin{vmatrix}2 & 3\\ 4 & -3\end{vmatrix}=3(-6-12)=-54.$

4. $\begin{vmatrix}-3 & 2\\ 7 & 8\end{vmatrix}=-24-14=-38.$

5. $3\begin{vmatrix}\frac{1}{2} & 6\\ \frac{1}{3} & 8\end{vmatrix}=3(4-2)=6.$

6. $4\begin{vmatrix}-3 & 5\\ 7 & 8\end{vmatrix}=4(-24-35)=-236.$

7. $\begin{vmatrix}2a & -10b\\ 5a & 3b\end{vmatrix}=6ab+50ab=56ab.$

8. $\dfrac{5}{2a}\begin{vmatrix} a^3 & 0 \\ a^2 & 2a \end{vmatrix} = \dfrac{5}{2a}(2a^4 - 0) = 5a^3.$

9. $\dfrac{7}{3}\begin{vmatrix} 3e & 0 \\ 9d & -14e^2 \end{vmatrix} = \dfrac{7}{3}(-42e^3 - 0) = -98e^3.$

10. $ax - cr = \begin{vmatrix} a & r \\ c & x \end{vmatrix}.$

11. $mz - 3d = \begin{vmatrix} m & d \\ 3 & z \end{vmatrix}.$

12. $\dfrac{8}{3} - ar = \begin{vmatrix} 8 & r \\ a & \frac{1}{3} \end{vmatrix}.$

13. $hk - c = \begin{vmatrix} h & c \\ 1 & k \end{vmatrix}.$

14. $ab + cd = \begin{vmatrix} a & d \\ -c & b \end{vmatrix}.$

15. $a - b = \begin{vmatrix} a & b \\ 1 & 1 \end{vmatrix}.$

16. $\dfrac{\begin{vmatrix} 5 & 1 \\ 3 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}} = \dfrac{-5-3}{-1-1} = 4.$

17. $\dfrac{\begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 4 & 3 \end{vmatrix}} = \dfrac{3-8}{9-8} = -5.$

18. $\dfrac{\begin{vmatrix} c & b \\ f & e \end{vmatrix}}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}} = \dfrac{ce - fb}{ae - db}.$

19. $\dfrac{\begin{vmatrix} c & (1-c) \\ 7c & (7-12c) \end{vmatrix}}{\begin{vmatrix} c & 3 \\ 7c & 36 \end{vmatrix}} = \dfrac{7c - 12c^2 - 7c + 7c^2}{36c - 21c} = \dfrac{-5c^2}{15c} = \dfrac{-c}{3}.$

20. $\dfrac{st - cd}{3cx - 5r} = \dfrac{\begin{vmatrix} s & d \\ c & t \end{vmatrix}}{\begin{vmatrix} 3c & r \\ 5 & x \end{vmatrix}}.$

21. $\dfrac{a + \dfrac{10}{c}}{a^2 - 12} = \dfrac{\begin{vmatrix} a & -\dfrac{1}{c} \\ 10 & 1 \end{vmatrix}}{\begin{vmatrix} a & 4 \\ 3 & a \end{vmatrix}}.$

22. $\dfrac{ax - 6}{2r - 5t} = \dfrac{\begin{vmatrix} a & 3 \\ 2 & x \end{vmatrix}}{\begin{vmatrix} 2 & t \\ 5 & r \end{vmatrix}}.$

23. $\dfrac{\dfrac{m}{3} - 7}{2m - 1} = \dfrac{\begin{vmatrix} m & 7 \\ 1 & \frac{1}{3} \end{vmatrix}}{\begin{vmatrix} 2 & 1 \\ 1 & m \end{vmatrix}}.$

24. $\dfrac{6x - hm}{3a - 3r} = \dfrac{\begin{vmatrix} 6 & m \\ h & x \end{vmatrix}}{\begin{vmatrix} 3 & r \\ 3 & a \end{vmatrix}}.$

25. $\dfrac{x^2 + 0}{\dfrac{a^2}{4} + \dfrac{15}{a}} = \dfrac{\begin{vmatrix} x & 0 \\ -3 & x \end{vmatrix}}{\begin{vmatrix} a^2 & -\dfrac{1}{a} \\ 15 & \dfrac{1}{4} \end{vmatrix}}.$

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1. $\begin{aligned} 2x + 3y &= 7, \\ 3x - 2y &= 4. \end{aligned}$ $\quad x = \dfrac{\begin{vmatrix} 7 & 3 \\ 4 & -2 \end{vmatrix}}{\begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix}} = \dfrac{-14-12}{-4-9} = 2.$

$\therefore\ y = \dfrac{\begin{vmatrix} 2 & 7 \\ 3 & 4 \end{vmatrix}}{-13} = \dfrac{8-21}{-13} = 1.$

2. $4x = 3y + 8,\quad 5y + 6 = 3x;$ or $4x - 3y = 8,\quad 3x - 5y = 6.$

$$x = \frac{\begin{vmatrix} 8 & -3 \\ 6 & -5 \end{vmatrix}}{\begin{vmatrix} 4 & -3 \\ 3 & -5 \end{vmatrix}} = \frac{-40 + 18}{-20 + 9} = 2.$$

$$\therefore y = \frac{\begin{vmatrix} 4 & 8 \\ 3 & 6 \end{vmatrix}}{-11} = \frac{24 - 24}{-11} = 0.$$

3. $5x + 4y = 10a + 4,\quad x - 2ay = 0.$

$$x = \frac{\begin{vmatrix} (10a + 4) & 4 \\ 0 & -2a \end{vmatrix}}{\begin{vmatrix} 5 & 4 \\ 1 & -2a \end{vmatrix}} = \frac{-20a^2 - 8a}{-10a - 4} = 2a.$$

$$\therefore y = \frac{\begin{vmatrix} 5 & (10a + 4) \\ 1 & 0 \end{vmatrix}}{-10a \quad -4} = \frac{-10a - 4}{-10a - 4} = 1.$$

4. $.3x + .02y = 185,\quad .5x + .04y = 335.$

$$x = \frac{\begin{vmatrix} 185 & .02 \\ 335 & .04 \end{vmatrix}}{\begin{vmatrix} .3 & .02 \\ .5 & .04 \end{vmatrix}} = \frac{7.4 - 6.7}{.012 - .01} = \frac{.7}{.002} = 350.$$

$$\therefore y = \frac{\begin{vmatrix} .3 & 185 \\ .5 & 335 \end{vmatrix}}{.002} = \frac{100.5 - 92.5}{.002} = 4000.$$

5. $4x + 3y = 6,\quad \frac{3x}{4} + \frac{3y}{4} = 3.$

$$x = \frac{\begin{vmatrix} 6 & 3 \\ 3 & \frac{3}{4} \end{vmatrix}}{\begin{vmatrix} 4 & 3 \\ \frac{3}{4} & \frac{3}{4} \end{vmatrix}} = \frac{4\frac{1}{2} - 9}{3 - 2\frac{1}{4}} = \frac{-\frac{9}{2}}{\frac{3}{4}} = -6.$$

$$\therefore y = \frac{\begin{vmatrix} 4 & 6 \\ \frac{3}{4} & 3 \end{vmatrix}}{\frac{3}{4}} = \frac{12 - 4\frac{1}{2}}{\frac{3}{4}} = \frac{\frac{15}{2}}{\frac{3}{4}} = 10.$$

6. $\frac{x}{2} - \frac{y}{3} = 6,\quad \frac{3x}{2} + \frac{2y}{3} = 8.$

$$x = \frac{\begin{vmatrix} 6 & -\frac{1}{3} \\ 8 & \frac{2}{3} \end{vmatrix}}{\begin{vmatrix} \frac{1}{2} & -\frac{1}{3} \\ \frac{3}{2} & \frac{2}{3} \end{vmatrix}} = \frac{4 + 2\frac{2}{3}}{\frac{1}{3} + \frac{1}{2}} = 8.$$

$$\therefore y = \frac{\begin{vmatrix} \frac{1}{2} & 6 \\ \frac{3}{2} & 8 \end{vmatrix}}{\frac{5}{6}} = \frac{4 - 9}{\frac{5}{6}} = -6.$$

7. $7x + 5y = 21c,$ $\quad \dfrac{x}{c} - \dfrac{y}{2c} = 3.$

$$x = \frac{\begin{vmatrix} 21c & 5 \\ 3 & -\dfrac{1}{2c} \end{vmatrix}}{\begin{vmatrix} 7 & 5 \\ \dfrac{1}{c} & -\dfrac{1}{2c} \end{vmatrix}} = \frac{-\dfrac{21}{2} - 15}{-\dfrac{7}{2c} - \dfrac{5}{c}} = \frac{-\dfrac{51}{2}}{-\dfrac{17}{2c}} = 3c.$$

$$y = \frac{\begin{vmatrix} 7 & 21c \\ \dfrac{1}{c} & 3 \end{vmatrix}}{\dfrac{-17}{2c}} = \frac{21 - 21}{\dfrac{-17}{2c}} = 0.$$

8. $\dfrac{x}{a} + \dfrac{y}{b} = \dfrac{a+b}{ab},$ $\quad x - y = \dfrac{a^2 - b^2}{ab}.$

$$x = \frac{\begin{vmatrix} \dfrac{a+b}{ab} & \dfrac{1}{b} \\ \dfrac{a^2-b^2}{ab} & -1 \end{vmatrix}}{\begin{vmatrix} \dfrac{1}{a} & \dfrac{1}{b} \\ 1 & -1 \end{vmatrix}} = \frac{\dfrac{-a-b}{ab} - \dfrac{a^2-b^2}{ab^2}}{-\dfrac{1}{a} - \dfrac{1}{b}} = \frac{a}{b}.$$

$$y = \frac{\begin{vmatrix} \dfrac{1}{a} & \dfrac{a+b}{ab} \\ 1 & \dfrac{a^2-b^2}{ab} \end{vmatrix}}{-\dfrac{1}{a} - \dfrac{1}{b}} = \frac{\dfrac{a^2-b^2}{a^2b} - \dfrac{a+b}{ab}}{-\dfrac{1}{a} - \dfrac{1}{b}} = \frac{b}{a}.$$

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1. $x - 2y + z = 6.$

Let $x = 0$ and $y = 0$, then $z = 6$.
Let $x = 0$ and $z = 0$, then $y = -3$.
Let $y = 0$ and $z = 0$, then $x = 6$.
Let $x = 1$ and $y = 5$, then $z = 15$.
Let $x = -3$ and $z = 7$, then $y = -1$.

2. $m + n - p = 8,$ (1)

$3m - 2n + 4p = 6.$ (2)

(1) · 2, $2m + 2n - 2p = 16.$ (3)

(3) + (2), $5m + 2p = 22.$ (4)

From (4), $p = \dfrac{22 - 5m}{2}.$ (5)

In (5), let m be 0, 2, 4 respectively; then $p = 11, 6, 1$.
Substituting these in (1), $n = 19, 12, 5$.

$m = 0, 2, 4$; $p = 11, 6, 1$; $n = 19, 12, 5$.

3.

$$2x + 3y + 4z = -14, \quad (1)$$
$$x - y + 3z = 0, \quad (2)$$
$$5x + 2y + z = 14. \quad (3)$$

(2) · 2, $2x - 2y + 6z = 0.$ (4)

(3) + (4), $7x + 7z = 14.$ (5)

(2) · 3, $3x - 3y + 9z = 0.$ (6)

(1) + (6), $5x + 13z = -14.$ (7)

(5) · $\frac{5}{7}$, $5x + 5z = 10.$ (8)

(7) − (8), $8z = -24.$ (9)

$\therefore z = -3.$ (10)

From (10) and (5), $x = 5.$ (11)

From (10), (11), and (2), $y = -4.$ (12)

4.

$$x + 2y + 3z = 14, \quad (1)$$
$$4x - 5y + 6z = 12, \quad (2)$$
$$x + 15y + 9z = 58. \quad (3)$$

(1) · 4, $4x + 8y + 12z = 56.$ (4)

(4) − (2), $13y + 6z = 44.$ (5)

(3) − (1), $13y + 6z = 44.$ (6)

(5) and (6) show that the system is indeterminate.

(3) really = (1) · 5 − (2). Hence (1), (2), and (3) are not independent.

5.

$$.4r + .3s - 8t = 4, \quad (1)$$
$$.5r + .8s + t = 1.2, \quad (2)$$
$$-r - .5s + 2.6t = -.3. \quad (3)$$

(3) · .4, $-.4r - .2s + 1.04t = -.12.$ (4)

(1) + (4), $.1s - 6.96t = 3.88.$ (5)

(3) · .5, $-.5r - .25s + 1.3t = -.15.$ (6)

(2) + (6), $.55s + 2.3t = 1.05.$ (7)

(5) · 5.5, $.55s - 38.28t = 21.34.$ (8)

(7) − (8), $40.58t = -20.29.$ (9)

$\therefore t = -\frac{1}{2}.$ (10)

From (5) and (10), $s = 4.$ (11)

From (3), (10), and (11), $r = -3.$

$$r = -3,\ s = 4,\ t = -\tfrac{1}{2}.$$

6.

$$.25x + .05y - .10z = -1, \quad (1)$$
$$.50x - .30y = 0, \quad (2)$$
$$.05y + .04z = 3. \quad (3)$$

(2) · $\frac{1}{2}$, $.25x - .15y = 0.$ (4)

(1) − (4), $.2y - .1z = -1.$ (5)

(3) · 4, $.2y + .16z = 12.$ (6)

(5) − (6), $-.26z = -13.$ (7)

$\therefore z = 50.$ (8)

From (3) and (8), $y = 20.$ (9)
From (2) and (9), $x = 12.$

7. $\frac{r}{a} + \frac{3s}{2a} - \frac{t}{3a} = 6,$ (1)

$7r + 4t = 6s,$ (2)

$\frac{t}{9} - \frac{s}{6} = 0.$ (3)

(1) · 6 a, $6r + 9s - 2t = 36a.$ (4)
(3) · 54, $-9s + 6t = 0.$ (5)
(4) + (5), $6r + 4t = 36a.$ (6)
(2) · 3, $21r - 18s + 12t = 0.$ (7)
(5) · 2, $-18s + 12t = 0.$ (8)
(7) − (8), $21r = 0.$ (9)
$\therefore r = 0.$ (10)
From (6) and (10), $t = 9a.$ (11)
From (5) and (11), $s = 6a.$ (12)

8. $h + 2k - l = 3b + c,$ (1)

$5h - 4k - 4l = a + b - 8c,$ (2)

$\frac{h}{2} - \frac{k}{3} = \frac{3a + b}{6} - \frac{2c}{6}.$ (3)

(1) · 4, $4h + 8k - 4l = 12b + 4c.$ (4)
(2) − (4), $h - 12k = a - 11b - 12c.$ (5)
(3) · 6, $3h - 2k = 3a + b - 2c.$ (6)
(5) · 3, $3h - 36k = 3a - 33b - 36c.$ (7)
(6) − (7), $34k = 34b + 34c.$ (8)
$k = b + c.$ (9)
From (9) and (6), $h = a + b.$ (10)
From (10), (9), and (1) $l = a + c.$

9. $x + 2y + z = -1,$ (1)

$2x - y + z = -20,$ (2)

$-x - y - 5z = 13.$ (3)

(1) + (3), $y - 4z = 12.$ (4)
[(2) + (3) · 2] ÷ 3, $-y - 3z = 2.$ (5)
(4) + (5), $z = -2.$ (6)
From (6) and (5), $y = 4.$ (7)
From (6), (7), and (1), $x = -7.$

10. $x + y = 10,$ (1)

$x + z = 12,$ (2)

$y + z = 14.$ (3)

(1) − (2), $y - z = -2.$ (4)

(3) + (4), $2y = 12.$ (5)

$\therefore y = 6.$ (6)

(3) − (4), $2z = 16.$ (7)

$z = 8.$ (8)

From (6) and (1), $x = 4.$

11. $\dfrac{a}{x} + \dfrac{b}{y} + \dfrac{c}{z} = 3,$ (1)

$\dfrac{a}{x} - \dfrac{b}{y} + \dfrac{2c}{z} = 2,$ (2)

$\dfrac{5a}{x} - \dfrac{3b}{y} - \dfrac{2c}{z} = 0.$ (3)

(2) · 5, $\dfrac{5a}{x} - \dfrac{5b}{y} + \dfrac{10c}{z} = 10.$ (4)

(1) − (2), $\dfrac{2b}{y} - \dfrac{c}{z} = 1.$ (5)

(3) − (4), $\dfrac{2b}{y} - \dfrac{12c}{z} = -10.$ (6)

(5) − (6), $\dfrac{11c}{z} = 11.$ (7)

$z = c.$ (8)

From (5) and (8), $\dfrac{2b}{y} - \dfrac{c}{c} = 1.$ (9)

$\therefore y = b.$ (10)

From (1), (8), and (10), $\dfrac{x}{a} + 1 + 1 = 3.$

$\therefore x = a.$

12. $r + s + t + u = 6,$ (1)

$2r - s + 3t - u = -16,$ (2)

$5r + 9s - 6t + 4u = 81,$ (3)

$r + 5s + 9t - 7u = -54.$ (4)

(4) · 2, $2r + 10s + 18t - 14u = -108.$ (5)

(4) · 5, $5r + 25s + 45t - 35u = -270.$ (6)

(1) − (4), $-4s - 8t + 8u = 60.$ (7)

(2) − (5), $-11s - 15t + 13u = 92.$ (8)

(3) − (6), $-16s - 51t + 39u = 351.$ (9)

(7) ÷ − 4, $s + 2t - 2u = -15.$ (10)

(10) · 11, $11s + 22t - 22u = -165.$ (11)

(10) · 16, $16s + 32t - 32u = -240.$ (12)

(8) + (11), $7t - 9u = -73.$ (13)

(9) + (12), $-19t + 7u = 111.$ (14)

(13) · 7, $49t - 63u = -511.$ (15)

(14) · 9, $-171t + 63u = 999.$ (16)

(15) + (16), $-122t = 488.$ (17)

$\therefore t = -4.$ (18)

From (18) and (13), $-28 - 9u = -73.$ (19)

$\therefore u = 5.$ (20)

From (20), (18), and (10), $s - 8 - 10 = -15.$ (21)

$\therefore s = 3.$ (22)

From (20), (18), (22), and (1), $r + 3 - 4 + 5 = 6.$

$\therefore r = 2.$

13. $4x - 3y + 2z = 20,$ (1)

$5x + 4y - 10z = 3,$ (2)

$-7x - 18y + 34z = 31.$ (3)

(1) · 5, $20x - 15y + 10z = 100.$ (4)

(4) + (2), $25x - 11y = 103.$ (5)

(1) · 17 − (3), $75x - 33y = 309.$ (6)

But (6) = 3 · (5). Therefore the system is indeterminate.

14. $4h - k + m = 0,$ (1)

$7k + 2m + x = 0,$ (2)

$8h + 4m + x = 0,$ (3)

$16h + 5k - x = 4.$ (4)

(2) − (3), $-8h + 7k - 2m = 0.$ (5)

(2) + (4), $16h + 12k + 2m = 4.$ (6)

(1) · 2, $8h - 2k + 2m = 0.$ (7)

(5) + (6), $8h + 19k = 4.$ (8)

(5) + (7), $5k = 0.$ (9)

$k = 0.$ (10)

From (10) and (8), $8h = 4.$ (11)

$\therefore h = \frac{1}{2}.$ (12)

From (10), (12), and (1), $2 + m = 0.$ (13)

$\therefore m = -2.$ (14)

From (10), (14), and (2), $-4 + x = 0.$

$\therefore x = 4.$

15. $ax + by + cz = p,$ (1)

$dx + ey + fz = q,$ (2)

$gx + hy + iz = r.$ (3)

(1) · f, $afx + bfy + cfz = pf.$ (4)

(2) · c, $cdx + cey + cfz = cq.$ (5)

(4) − (5), $x(af - cd) + y(bf - ce) = pf - cq.$ (6)

(2) · i, $dix + eiy + fiz = qi.$ (7)

(3) · f, $fgx + fhy + fiz = fr.$ (8)

$$(7)-(8),\quad x(di-fg)+y(ei-fh)=qi-fr. \tag{9}$$

$$(6)\cdot(ei-fh),\quad x(af-cd)(ei-fh)+y(bf-ce)(ei-fh) = (pf-cq)(ei-fh). \tag{10}$$

$$(9)\cdot(bf-ce),\quad x(di-fg)(bf-ce)+y(bf-ce)(ei-fh) = (qi-fr)(bf-ce). \tag{11}$$

$$(10)-(11),\quad x[(af-cd)(ei-fh)-(di-fg)(bf-ce)] = (pf-cq)(ei-fh)-(qi-fr)(bf-ce).$$

$$x(afci-af^2h+cdfh-bfdi+bf^2g-cefg) = pfei-pf^2h+cqfh-bfqi+bf^2r-cefr.$$

$$x=\frac{pf(ei-fh)+qf(ch-bi)+rf(bf-ce)}{af(ei-fh)+df(ch-bi)+gf(bf-ce)}.$$

$$=\frac{p(ei-fh)+q(ch-bi)+r(bf-ce)}{a(ei-fh)+d(ch-bi)+g(bf-ce)}.$$

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1. $\begin{vmatrix}1&1&1\\2&3&1\\2&4&1\end{vmatrix}=1\cdot3\cdot1+2\cdot4\cdot1+2\cdot1\cdot1-1\cdot3\cdot2-1\cdot4\cdot1-1\cdot2\cdot1$
$=3+8+2-6-4-2=1.$

2. $\begin{vmatrix}1&2&3\\4&5&6\\7&8&9\end{vmatrix}=1\cdot5\cdot9+4\cdot8\cdot3+7\cdot6\cdot2-3\cdot5\cdot7-6\cdot8\cdot1-9\cdot4\cdot2$
$=45+96+84-105-48-72=0.$

3. $\begin{vmatrix}1&-2&3\\3&2&1\\1&1&1\end{vmatrix}=1\cdot2\cdot1+3\cdot1\cdot3+1\cdot1\cdot(-2)-3\cdot2\cdot1-1\cdot1\cdot1-1\cdot3(-2)$
$=2+9-2-6-1+6=8.$

4. $\begin{vmatrix}a&b&c\\1&2&3\\a&b&c\end{vmatrix}=a\cdot2\cdot c+1\cdot b\cdot c+a\cdot3\cdot b-c\cdot2\cdot a-3\cdot b\cdot a-c\cdot1\cdot b$
$=2ac+bc+3ab-2ac-3ab-bc=0.$

5. $\begin{vmatrix}1&0&0\\2&4&5\\-3&1&1\end{vmatrix}=1\cdot4\cdot1+2\cdot1\cdot0+(-3)\cdot5\cdot0-0\cdot4\cdot(-3)-5\cdot1\cdot1-1\cdot2\cdot0$
$=4+0+0-0-5-0=-1.$

6. $\begin{vmatrix}1&1&1\\a&1&a\\-a&5&6\end{vmatrix}=1\cdot1\cdot6+a\cdot5\cdot1+(-a)\cdot a\cdot1-1\cdot1\cdot(-a)-a\cdot5\cdot1-6\cdot a\cdot1$
$=6+5a-a^2+a-5a-6a=-a^2-5a+6.$

7. $-\begin{vmatrix}2&-1&3\\-3&1&2\\4&5&1\end{vmatrix}=-[2\cdot1\cdot1+(-3)\cdot5\cdot3+4\cdot2(-1)-3\cdot1\cdot4$
$-2\cdot5\cdot2-1\cdot(-3)\cdot(-1)]$
$=-(2-45-8-12-20-3)=86.$

8. $\begin{vmatrix}a&2&7\\b&3&8\\c&4&9\end{vmatrix}=a\cdot3\cdot9+b\cdot4\cdot7+c\cdot8\cdot2-7\cdot3\cdot c-8\cdot4\cdot a-9\cdot b\cdot2$
$=27a+28b+16c-21c-32a-18b=10b-5a-5c.$

9. $\begin{vmatrix} x & y & 1 \\ 2 & 2 & 1 \\ 3 & 4 & 1 \end{vmatrix} = x \cdot 2 \cdot 1 + 2 \cdot 4 \cdot 1 + 3 \cdot 1 \cdot y - 1 \cdot 2 \cdot 3 - 1 \cdot 4 \cdot x - 1 \cdot 2 \cdot y$
$= 2x + 8 + 3y - 6 - 4x - 2y = y - 2x + 2.$

10. $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = aei + dhc + gbf - ceg - fha - idb.$

11. $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = xy_1 + x_1y_2 + x_2y - y_1x_2 - y_2x - x_1y.$

12. $c\begin{vmatrix} 5 & 1 & b \\ c & 0 & 0 \\ 8 & 6 & a \end{vmatrix} = c(5 \cdot 0 \cdot a + c \cdot 6 \cdot b + 8 \cdot 0 \cdot 1 - b \cdot 0 \cdot 8 - 0 \cdot 6 \cdot 5 - a \cdot c \cdot 1)$
$= c(0 + 0 + 6bc - 0 - 0 - ac) = 6bc^2 - ac^2.$

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1. $$x = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & -1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{vmatrix}} = \frac{1-2-3-3-1-2}{1-1-1-1-1-1} = \frac{-10}{-4} = \frac{5}{2}.$$

$$y = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & -1 \\ 1 & 3 & 1 \end{vmatrix}}{-4} = \frac{2+3-1-2+3-1}{-4} = \frac{4}{-4} = -1.$$

Substituting $\frac{5}{2}$ for x and -1 for y in the first equation, $\frac{5}{2} - 1 + z = 1$. Whence $z = -\frac{1}{2}$.

2. $$x = \frac{\begin{vmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 0 & 2 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & -1 \\ 1 & 2 & -1 \end{vmatrix}} = \frac{-1+0+0-0+2-0}{-1+4-2-1+2+4} = \frac{1}{6}.$$

$$y = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 2 & 0 & -1 \\ 1 & 0 & -1 \end{vmatrix}}{6} = \frac{0+0-1+0+0+2}{6} = \frac{1}{6}.$$

Substituting $\frac{1}{6}$ for x and $\frac{1}{6}$ for y in the first equation, $\frac{1}{6} + \frac{1}{3} + z = 1$. Whence $z = \frac{1}{2}$.

3. $$x = \frac{\begin{vmatrix} 5 & 1 & -1 \\ 6 & 0 & -2 \\ 0 & 3 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & -1 \\ 1 & 0 & -2 \\ -1 & 3 & 2 \end{vmatrix}} = \frac{0 - 18 + 0 - 0 + 30 - 12}{0 - 3 + 2 - 0 + 12 - 2} = \frac{0}{9} = 0.$$

Substituting 0 for x in $x - 2z = 6$, $z = -3$.

Substituting 0 for x and -3 for z in $3y + 2z = x$, $y = 2$.

4. $$x = \frac{\begin{vmatrix} 1 & 1 & 0 \\ 2 & 0 & 1 \\ 3 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix}} = \frac{0 + 0 + 3 - 0 - 1 - 2}{0 + 0 + 0 - 0 - 1 - 1} = \frac{0}{-2} = 0.$$

Substituting 0 for x in $x + y = 1$, $y = 1$.

Substituting 0 for x in $x + z = 2$, $z = 2$.

5. $$x = \frac{\begin{vmatrix} 3a & 1 & 0 \\ 4a & 0 & 1 \\ 5a & 1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix}} = \frac{0 + 0 + 5a - 0 - 3a - 4a}{0 + 0 + 0 - 0 - 1 - 1} = \frac{-2a}{-2} = a.$$

Substituting a for x in $x + y = 3a$, $y = 2a$.

Substituting a for x in $x + z = 4a$, $z = 3a$.

6. $$x = \frac{\begin{vmatrix} 9 & \frac{1}{2} & 0 \\ 8 & 0 & \frac{1}{3} \\ 13 & \frac{1}{3} & \frac{1}{2} \end{vmatrix}}{\begin{vmatrix} \frac{1}{3} & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{2} \end{vmatrix}} = \frac{0 + 0 + \frac{13}{6} - 0 - 1 - 2}{0 + 0 + 0 - 0 - \frac{1}{27} - \frac{1}{8}} = \frac{\frac{-5}{6}}{\frac{-35}{216}} = \frac{36}{7}.$$

Substituting $\frac{36}{7}$ for x in first equation, $\frac{12}{7} + \frac{y}{2} = 9$, or $y = \frac{102}{7}$.

Substituting $\frac{36}{7}$ for x in second equation, $\frac{18}{7} + \frac{z}{3} = 8$, or $z = \frac{114}{7}$.

7. $$x = \frac{\begin{vmatrix} 0 & b & 0 \\ 2bc & 0 & -b \\ b^2 & -c & a \end{vmatrix}}{\begin{vmatrix} a & b & 0 \\ c & 0 & -b \\ b & -c & a \end{vmatrix}} = \frac{0+0-b^4-0-0-2ab^2c}{0+0-b^3-0-abc-abc} = \frac{-b^4-2ab^2c}{-b^3-2abc} = b.$$

Substituting b for x in $ax + by = 0$, $y = -a$.

Substituting b for x in $cx - bz = 2bc$, $z = -c$.

8. $$x = \frac{\begin{vmatrix} 2hk & k & -l \\ 2kl & k & l \\ 2hl & -k & l \end{vmatrix}}{\begin{vmatrix} h & k & -l \\ -h & k & l \\ h & -k & l \end{vmatrix}} = \frac{2hk^2l + 2k^2l^2 + 2hkl^2 + 2hkl^2 + 2hk^2l - 2k^2l^2}{hkl - hkl + hkl + hkl + hkl + hkl}$$
$$= \frac{4hk^2l + 4hkl^2}{4hkl} = k + l.$$

$$y = \frac{\begin{vmatrix} h & 2hk & -l \\ -h & 2kl & l \\ h & 2hl & l \end{vmatrix}}{4hkl} = \frac{2hkl^2 + 2h^2l^2 + 2h^2kl + 2hkl^2 - 2h^2l^2 + 2h^2kl}{4hkl}$$
$$= \frac{4hkl^2 + 4h^2kl}{4hkl} = l + h.$$

Then from the first equation, $hk + hl + kl + hk - lz = 2kh$. Whence $z = h + k$.

9. $$x = \frac{\begin{vmatrix} 0 & m & 1 \\ (ma-a) & 1 & m \\ 4ma & -3m & 1 \end{vmatrix}}{\begin{vmatrix} m & m & 1 \\ m & 1 & m \\ m & -3m & 1 \end{vmatrix}} = \frac{0-3m^2a+3ma+4m^3a-4ma-0-m^2a+ma}{m-3m^2+m^3-m+3m^3-m^2}$$
$$= \frac{ma(4m^2-4m)}{m(4m^2-4m)} = a.$$

$$x_1 = \frac{\begin{vmatrix} m & 0 & 1 \\ m & (ma-a) & m \\ m & 4ma & 1 \end{vmatrix}}{m(4m^2-4m)} = \frac{m^2a - ma + 4m^2a + 0 - m^2a + ma - 4m^3a - 0}{m(4m^2-4m)}$$
$$= \frac{ma(-4m^2+4m)}{m(4m^2-4m)} = -a.$$

Then from the first equation, $ma - ma + x_2 = 0$. Whence $x_2 = 0$.

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1. Let $a =$ the number of days A requires alone,
and $b =$ the number of days B requires alone.

Then $$\frac{1}{a} + \frac{1}{b} = \frac{7}{24}, \qquad (1)$$

and $$\frac{2}{a} + \frac{2}{b} + \frac{5}{2a} = 1. \qquad (2)$$

(1) · 2, $$\frac{2}{a} + \frac{2}{b} = \frac{7}{12}. \qquad (3)$$

(2) – (3), $$\frac{5}{2a} = \frac{5}{12}. \qquad (4)$$

Whence $$a = 6.$$

From (1) and (4), $$\frac{1}{6} + \frac{1}{b} = \frac{7}{24}.$$

Whence $$b = 8.$$

2. Let $m =$ the number of days the man requires to do the work,
and $b =$ the number of days the boy requires to do the work.

Then $$\frac{1}{m} + \frac{1}{b} = \frac{1}{18}, \qquad (1)$$

$$\frac{5}{m} + \frac{9}{b} = \frac{1}{3}. \qquad (2)$$

(1) · 5, $$\frac{5}{m} + \frac{5}{b} = \frac{5}{18}. \qquad (3)$$

(2) – (3), $$\frac{4}{b} = \frac{1}{18}. \qquad (4)$$

Whence $$b = 72. \qquad (5)$$

From (1) and (5), $$\frac{1}{m} + \frac{1}{72} = \frac{1}{18}.$$

$$\therefore m = 24.$$

3. $$ax + by = 2.$$

$$2a + 3b = 2. \qquad (1)$$

$$6a + 5b = 2. \qquad (2)$$

(1) · 3, $$6a + 9b = 6. \qquad (3)$$

(3) – (2), $$4b = 4. \qquad (4)$$

Whence $$b = 1. \qquad (5)$$

From (1) and (5), $$2a + 3 = 2.$$

Whence $$a = -\tfrac{1}{2}.$$

4. Let $x =$ the number of hours required to go upstream.
Then $24 - x =$ the number of hours required to go downstream.
Therefore $$x \cdot 10 = 14(24 - x).$$

$$10x = 336 - 14x.$$

Whence $x = 14.$

Then $24 - x = 10.$

5. Let $x =$ in dollars the sum at 5%,

and $y =$ in dollars the sum at 6%.

Then $.05x + .06y = 100.$ (1)

$.06x + .05y = 98.$ (2)

(1) · 600, $30x + 36y = 60{,}000.$ (3)

(2) · 500, $30x + 25y = 49{,}000.$ (4)

(3) – (4), $11y = 11{,}000.$ (5)

Whence $y = 1000.$ (6)

From (3) and (6), $30x + 36{,}000 = 60{,}000.$

Whence $x = 800.$

6. Let $x =$ in dollars the sum at 5%,

and $y =$ in dollars the sum at 6%.

Then $x + y = 4000,$ (1)

and $.05\left(\frac{x}{.90}\right) + .06\left(\frac{y}{1.10}\right) = 220.$ (2)

(2) · 990, $55x + 54y = 217{,}800.$

(1) · 55, $55x + 55y = 220{,}000.$

Whence $y = 2200.$

From (1), $x + 2200 = 4000.$

Whence $x = 1800.$

7. Let $d =$ the distance in miles from M to P,

and $r =$ the usual rate in miles per hour.

Then $\frac{d}{r} =$ scheduled time in hours.

Therefore $\dfrac{d}{\frac{3r}{2}} = \dfrac{d}{r} - 2,$ (1)

and $\dfrac{d}{25} = \dfrac{d}{r} - 2 + \dfrac{4}{5}.$ (2)

From (1), $\dfrac{d}{3r} = 2$, or $d = 6r.$ (3)

From (3) and (2), $\dfrac{6r}{25} = 6 - 2 + \dfrac{4}{5}.$

Whence $r = 20.$

Then $d = 6r$, or 120.

8. Let $d =$ the distance from M to N in miles,
and $r =$ the usual rate from M to N in miles per hour.

Then $\dfrac{d}{r} =$ the scheduled time in hours.

Therefore
$$\frac{d}{\frac{6r}{5}} = \frac{d}{r} - \frac{1}{2} + \frac{1}{10}, \tag{1}$$

and
$$\frac{15}{r} + \frac{d-15}{\frac{6r}{5}} = \frac{d}{r} - \frac{1}{2} + \frac{1}{5}. \tag{2}$$

From (1),
$$\frac{d}{6r} = \frac{4}{10}, \text{ or } d = \frac{12r}{5}. \tag{3}$$

From (2) and (3), $d = 60$, and $r = 25$.

9. Let $p =$ the rate of the passenger train in feet per second,
and $f =$ the rate of the freight train in feet per second.

Then
$$\frac{1430 + 550}{p + f} = 18,$$

and
$$\frac{1430 + 550}{p - f} = 90.$$

Whence $p = 66$ feet per second, or 45 miles per hour,
and $f = 44$ feet per second, or 30 miles per hour.

10. Let $f =$ the rate of the first in feet per second,
and $s =$ the rate of the second in feet per second.

Then
$$\frac{1320}{s} = \frac{1120}{f} + 4,$$

and
$$\frac{1320}{s} = \frac{1200}{f}.$$

Whence $f = 20$, and $s = 22$.

11. Let $x =$ number of gallons of 95% alcohol,
and $y =$ number of gallons of 15% alcohol.

Then
$$\frac{.95x + .15y}{10} = \frac{45}{100}, \text{ and } x + y = 10.$$

Whence $x = 3\frac{3}{4}$, and $y = 6\frac{1}{4}$.

12. Let g and s in pounds be the respective weights of the gold and of the silver.

Then
$$g + s = 20,$$

and
$$\frac{18\frac{1}{4}}{19\frac{1}{4}}g + \frac{9\frac{1}{2}}{10\frac{1}{2}}s = 18\frac{3}{4}.$$

Whence $g = 15\frac{1}{8}$, and $s = 4\frac{7}{8}$.

13. Let x, y, and z be respectively the number of degrees in each angle.

Then
$$x = 2y,$$
$$x + y = z,$$
and
$$x + y + z = 180.$$
Whence
$$z = 90,\ x = 60,\ \text{and}\ y = 30.$$

14. Let f, s, and t equal respectively the three numbers.

Then
$$f + s + t = 217,$$
$$\frac{f}{s} = 5,$$
and
$$\frac{s}{t} = 5.$$
Whence
$$f = 175,\ s = 35,\ \text{and}\ t = 7.$$

15. Let h, t, and u be respectively the hundreds', tens', and units' digits of the number.

Then
$$100h + 10u + t = 100h + 10t + u - 27, \qquad (1)$$
$$100t + 10h + u = 100h + 10t + u - 180, \qquad (2)$$
and
$$h + t + u = 14. \qquad (3)$$
From (1),
$$u - t = -3. \qquad (4)$$
From (2),
$$h - t = 2. \qquad (5)$$
From (3), (4), and (5),
$$h = 7,\ t = 5,\ \text{and}\ u = 2.$$
Hence the number is 752.

16. Let f, s, and t be the number of hours respectively that the first, second, and third pipes require to fill the tank.

Then
$$\frac{1}{f} + \frac{1}{s} + \frac{1}{t} = \frac{7}{8},$$
$$\frac{1}{f} + \frac{1}{s} = \frac{3}{4},$$
and
$$\frac{1}{s} + \frac{1}{t} = \frac{3}{8}.$$
Whence
$$f = 2,\ s = 4,\ \text{and}\ t = 8.$$

17. Let f, s, t, and x in feet be the length of the sides of the quadrilateral.

Then
$$f + s = 80,\quad s + t = 108,\quad t + x = 116,\quad \text{and}\quad x - f = 24;\qquad \text{or}\ \begin{cases} f+s=80, \\ s+t=108, \\ t+x=116, \\ f-x=24. \end{cases}$$
Whence
$$f = 32,\quad s = 48,\quad t = 60,\quad \text{and}\quad x = 56;\qquad \text{or}\ \begin{cases} f = 56, \\ s = 24, \\ t = 84, \\ x = 32. \end{cases}$$

18. Let f = the cost of the first chair in cents,

and s = the cost of the second chair in cents.

Then
$$f + s = 100h,$$
and
$$f - s = m.$$
Whence
$$f = 50h + \frac{m}{2},$$
and
$$s = 50h - \frac{m}{2}.$$

19. Let $a =$ the number of dollars A has,
and $b =$ the number of dollars B has.

Then $$a + b = d,$$
and $$a - c + m = \tfrac{1}{3}(b + c - m).$$
Whence $$a = \frac{4c - 4m + d}{4}, \text{ and } b = \frac{3d - 4c + 4m}{4}.$$

20. Let $a =$ the number of days A requires to do the work,
and $b =$ the number of days B requires to do the work.

Then $$\frac{1}{a} + \frac{1}{b} = \frac{1}{m},$$
and $$b = \frac{a}{c}.$$
Whence $$a = m + cm, \text{ and } b = \frac{m}{c} + m.$$

21. Let $x =$ his rate in miles per hour in still water,
and $r =$ the rate in miles per hour of the river.

Then $$\frac{m}{x + r} = t,$$
and $$\frac{m}{x - r} = a.$$
Whence $$x = \frac{ma + mt}{2\,at}, \text{ and } r = \frac{ma - mt}{2\,at}.$$

22. Since each of two children has borrowed more than his share of the estate, he receives nothing. The note of each, however, is collectible up to the amount of his share in the estate, as will be clear from the following solution:

Let $x =$ the number of dollars in each child's share.
Then $5x =$ the number of dollars in the widow's share,
and $10x =$ the number of dollars in the entire estate.
But two children have received their share (and more).
Hence the estate equals $(1200 + 2x)$ dollars.
Therefore $10x = 1200 + 2x.$
Whence $x = 150.$
Then the widow receives \$750.
Each of the three children receives \$150.
Each of the two children receives nothing.

23. When $5x + 2y = 42$,

x	8	6	4	2
y	1	6	11	16

24. $7x + 2y = 36.$

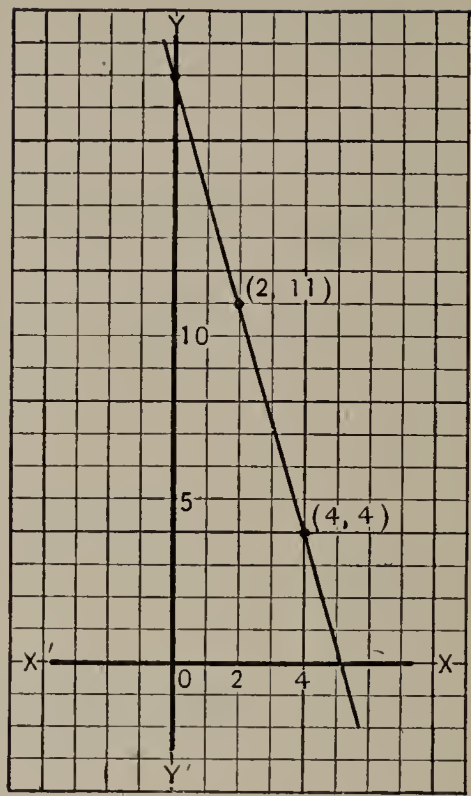
$$\therefore y = \frac{36 - 7x}{2} = 18 - \frac{7x}{2}.$$

Hence if x is any even number, y is an integer. Therefore the positive values which satisfy the equation are

x	4	2
y	4	11

For $7x + 2y = 36.$

x	0	$5\frac{1}{7}$	2	4
y	18	0	11	4



25. $5x + 2y = 73.$

$$y = \frac{73 - 5x}{2} = 36 + \frac{1 - 5x}{2}.$$

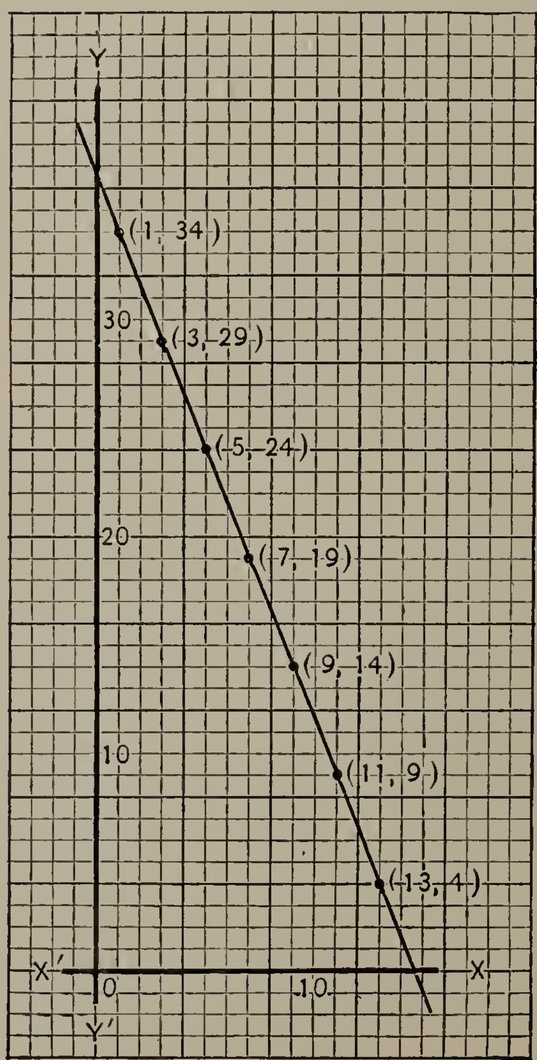
Hence if x is any odd number, y is an integer. Therefore the positive values which satisfy the equation are

x	13	11	9	7	5	3	1
y	4	9	14	19	24	29	34

For $5x + 2y = 73.$

x	14.6	13	9
y	0	4	14

etc.



26. Let c = the cost of one calf in dollars, and p = the cost of one pig in dollars.

Then $6c + 4p = 72.$

$$p = \frac{72 - 6c}{4} = 18 - \frac{3c}{2}.$$

Hence if c is any even number, p is an integer. Therefore the values which satisfy the equation are

c	2	4	6	8	10
p	15	12	9	6	3

27. Let q be a possible number of quarters and n be a corresponding number of nickels.

Then $$25q + 5n = 175,$$
and $$n = 35 - 5q.$$

Hence any integral value of q gives an integral value of n. Therefore the values which satisfy the equation are

q	1	2	3	4	5	6
n	30	25	20	15	10	5

Six ways.

28. Let c, p, and l represent the number of the calves, the pigs, and the lambs respectively.

Then $$6c + 3p + 4l = 126, \quad (1)$$
and $$c + p + l = 32. \quad (2)$$
(2) · 6, $$6c + 6p + 6l = 192. \quad (3)$$
(3) – (1), $$3p + 2l = 66. \quad (4)$$
Whence $$l = \frac{66 - 3p}{2} = 33 - \frac{3p}{2}.$$

Hence if p is any *even* integer, l is an integer. Therefore the values which satisfy (4) are

p	2	4	6	8	10	12	14	16	18	20
l	30	27	24	21	18	15	12	9	6	3

Substitution of these values in (2) gives respectively the following for c:

c	0	1	2	3	4	5	6	7	8	9

Rejecting $p = 2$, $l = 30$, and $c = 0$, there are nine possible sales.

29. Let n, d, and q represent the number of the nickels, the dimes, and the quarters respectively.

Then $$5n + 10d + 25q = 240, \quad (1)$$
and $$n = d + q. \quad (2)$$
(2) · 5, $$5n - 5d - 5q = 0. \quad (3)$$
(1) – (3), $$15d + 30q = 240. \quad (4)$$
(4) ÷ 15, $$d + 2q = 16.$$
Whence $$d = 16 - 2q.$$

If q is any integer whatever, d has an integral value. Therefore the values which satisfy (4) are

q	1	2	3	4	5	6	7
d	14	12	10	8	6	4	2

Substitution of these values in (2) gives respectively the following for n:

n	15	14	13	12	11	10	9

Hence there are seven possible groups, all different.

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1–3. See pages 101–103.

4. By extracting the square root of the square root.

5. See page 229. **6.** 2, 3.

7. (a) No. (b) No. The result in either case would be the sixth root.

8.

$$\begin{array}{l|l}
 & a^6 - 10a^4 - 4a^3 + 25a^2 + 20a + 4 \;\big|\underline{a^3 - 5a - 2} \\
 & \underline{a^6} \\
2a^3 - 5a & -10a^4 - 4a^3 \\
 & \underline{-10a^4 \qquad + 25a^2} \\
2a^3 - 10a - 2 & -4a^3 \qquad + 20a + 4 \\
 & \underline{-4a^3 \qquad + 20a + 4}
\end{array}$$

Therefore the square roots are $\pm (a^3 - 5a - 2)$.

9.

$$\begin{array}{l|l}
 & 4a^8 + 12a^4 - 7 - 24a^{-4} + 16a^{-8} \;\big|\underline{2a^4 + 3 - 4a^{-4}} \\
 & \underline{4a^8} \\
4a^4 + 3 & 12a^4 - 7 \\
 & \underline{12a^4 + 9} \\
4a^4 + 6 - 4a^{-4} & -16 - 24a^{-4} + 16a^{-8} \\
 & \underline{-16 - 24a^{-4} + 16a^{-8}}
\end{array}$$

Therefore the square roots are $\pm (2a^4 + 3 - 4a^{-4})$.

10.

$$\begin{array}{l|l}
 & 49c^{-6} - 28c^{-4} + 74c^{-2} - 20 + 25c^2 \;\big|\underline{7c^{-3} - 2c^{-1} + 5c} \\
 & \underline{49c^{-6}} \\
14c^{-3} - 2c^{-1} & -28c^{-4} + 74c^{-2} \\
 & \underline{-28c^{-4} + 4c^{-2}} \\
14c^{-3} - 4c^{-1} + 5c & 70c^{-2} - 20 + 25c^2 \\
 & \underline{70c^{-2} - 20 + 25c^2}
\end{array}$$

Therefore the square roots are $\pm (7c^{-3} - 2c^{-1} + 5c)$.

11.

$$\begin{array}{l|l}
 & 9x^4 - 6x^{\frac{7}{2}} + x^3 - 66x^{\frac{5}{2}} + 22x^2 + 121x \;\big|\underline{3x^2 - x^{\frac{3}{2}} - 11x^{\frac{1}{2}}} \\
 & \underline{9x^4} \\
6x^2 - x^{\frac{3}{2}} & -6x^{\frac{7}{2}} + x^3 \\
 & \underline{-6x^{\frac{7}{2}} + x^3} \\
6x^2 - 2x^{\frac{3}{2}} - 11x^{\frac{1}{2}} & -66x^{\frac{5}{2}} + 22x^2 + 121x \\
 & \underline{-66x^{\frac{5}{2}} + 22x^2 + 121x}
\end{array}$$

Therefore the square roots are $\pm (3x^2 - x^{\frac{3}{2}} - 11x^{\frac{1}{2}})$.

12. $16m^{-7}-8m^{-4}+m^{-1}+104m-26m^4+169m^9 \;\Big|\; \underline{4m^{-\frac{7}{2}}-m^{-\frac{1}{2}}+13m^{\frac{9}{2}}}$

$$\begin{array}{l|l}
 & 16m^{-7} \\
\hline
8m^{-\frac{7}{2}}-m^{-\frac{1}{2}} & -8m^{-4}+m^{-1} \\
 & -8m^{-4}+m^{-1} \\
\hline
8m^{-\frac{7}{2}}-2m^{-\frac{1}{2}}+13m^{\frac{9}{2}} & 104m-26m^4+169m^9 \\
 & 104m-26m^4+169m^9 \\
\hline
\end{array}$$

Therefore the square roots are $\pm\left(4m^{-\frac{7}{2}}-m^{-\frac{1}{2}}+13m^{\frac{9}{2}}\right)$.

13. $\dfrac{a^2}{9c^2}+\dfrac{4a}{3c}+\dfrac{20}{3}+\dfrac{16c}{a}+\dfrac{16c^2}{a^2} \;\Big|\; \underline{\dfrac{a}{3c}+2+\dfrac{4c}{a}}$

$$\begin{array}{l|l}
 & \dfrac{a^2}{9c^2} \\
\hline
\dfrac{2a}{3c}+2 & \dfrac{4a}{3c}+\dfrac{20}{3} \\
 & \dfrac{4a}{3c}+4 \\
\hline
\dfrac{2a}{3c}+4+\dfrac{4c}{a} & \dfrac{8}{3}+\dfrac{16c}{a}+\dfrac{16c^2}{a^2} \\
 & \dfrac{8}{3}+\dfrac{16c}{a}+\dfrac{16c^2}{a^2} \\
\hline
\end{array}$$

Therefore the square roots are $\pm\left(\dfrac{a}{3c}+2+\dfrac{4c}{a}\right)$.

14. $\dfrac{1}{4a^4}-\dfrac{3x}{a^3}+\dfrac{9x^2}{a^2}+\dfrac{1}{5x}-\dfrac{6a}{5}+\dfrac{a^4}{25x^2} \;\Big|\; \underline{\dfrac{1}{2a^2}-\dfrac{3x}{a}+\dfrac{a^2}{5x}}$

$$\begin{array}{l|l}
 & \dfrac{1}{4a^4} \\
\hline
\dfrac{1}{a^2}-\dfrac{3x}{a} & -\dfrac{3x}{a^3}+\dfrac{9x^2}{a^2} \\
 & -\dfrac{3x}{a^3}+\dfrac{9x^2}{a^2} \\
\hline
\dfrac{1}{a^2}-\dfrac{6x}{a}+\dfrac{a^2}{5x} & \dfrac{1}{5x}-\dfrac{6a}{5}+\dfrac{a^4}{25x^2} \\
 & \dfrac{1}{5x}-\dfrac{6a}{5}+\dfrac{a^4}{25x^2} \\
\hline
\end{array}$$

Therefore the square roots are $\pm\left(\dfrac{1}{2a^2}-\dfrac{3x}{a}+\dfrac{a^2}{5x}\right)$.

15.

$$\begin{array}{l|l}
 & 1+2x \;\big|\; 1+x-\dfrac{x^2}{2}+\dfrac{x^3}{2} \\
 & 1 \\ \hline
2+x & 2x \\
 & 2x+x^2 \\ \hline
2+2x-\dfrac{x^2}{2} & -x^2 \\
 & -x^2-x^3+\dfrac{x^4}{4} \\ \hline
2+2x-x^2 & x^3-\dfrac{x^4}{4}
\end{array}$$

The required terms of the square roots are $\pm\left(1+x-\dfrac{x^2}{2}+\dfrac{x^3}{2}\right)$.

16.

$$\begin{array}{l|l}
 & \dfrac{25}{9}+a^3 \;\Big|\; \dfrac{5}{3}+\dfrac{3a^3}{10}-\dfrac{27a^6}{1000}+\dfrac{50,000}{243a^9} \\
 & \dfrac{25}{9} \\ \hline
\dfrac{10}{3}+\dfrac{3a^3}{10} & a^3 \\
 & a^3+\dfrac{9a^6}{100} \\ \hline
\dfrac{10}{3}+\dfrac{3a^3}{5}-\dfrac{27a^6}{1000} & -\dfrac{9a^6}{100} \\
 & -\dfrac{9a^6}{100}-\dfrac{81a^9}{5000}+\dfrac{1,000,000}{729a^{12}} \\ \hline
\dfrac{10}{3}+\dfrac{8a^3}{5}-\dfrac{27a^6}{500} & \dfrac{81a^9}{5000}
\end{array}$$

The required terms of the square roots are

$$\pm\left(\frac{5}{3}+\frac{3a^3}{10}-\frac{27a^6}{1000}+\frac{243a^9}{50,000}\right).$$

17.

$$\begin{array}{l|l}
 & 7c^{-3}-2c^{-1}+5c \;\big|\; 7^{\frac{1}{2}}c^{-\frac{3}{2}}-7^{-\frac{1}{2}}c^{\frac{1}{2}}+17\cdot 7^{-\frac{3}{2}}c^{\frac{5}{2}} \\
 & 7c^{-3} \\ \hline
2\cdot 7^{\frac{1}{2}}c^{-\frac{3}{2}}-7^{-\frac{1}{2}}c^{\frac{1}{2}} & -2c^{-1}+5c \\
 & -2c^{-1}+\dfrac{c}{7} \\ \hline
2\cdot 7^{\frac{1}{2}}\cdot c^{-\frac{3}{2}}-2\cdot 7^{-\frac{1}{2}}c^{\frac{1}{2}}+17c^{\frac{5}{2}} & +\dfrac{34c}{7}
\end{array}$$

The required terms of the fourth roots are $\pm(7^{\frac{1}{2}}c^{-\frac{3}{2}}-7^{-\frac{1}{2}}c^{\frac{1}{2}}+17\cdot 7^{-\frac{3}{2}}c^{\frac{5}{2}})$.

Page 340 (First set)

1. (*a*) 4.4; (*b*) 6.7; (*c*) 7.7; (*d*) 8.3.
2. (*a*) 2.9; (*b*) 2.6; (*c*) 3.7; (*d*) 5.4; (*e*) 5.3.
3. (*a*) 5.7; (*b*) 37; (*c*) 63; (*d*) 69.
4. (*a*) 32.8; (*b*) 59; (*c*) 22; (*d*) 175; (*e*) 111.

Page 340 (Second set)

1.
```
      68′89|83
      64
  163|489
     |489   Ans. ± 83.
```

2.
```
    5′61′69|237
    4
  43|161
    |129
 467|3269
    |3269   Ans. ± 237.
```

3.
```
     .67′24|.82
      64
  162|324
     |324   Ans. ± .82.
```

4.
```
   1′.46′41|1.21
   1
  22|46
    |44
 241|241
    |241   Ans. ± 1.21.
```

5.
```
    4′.20′25|2.05
    4
 405|2025
    |2025   Ans. ± 2.05.
```

6.
```
    .04′02′80′49|.2007
     04
 4007|028049
     | 28049   Ans. ± .2007.
```

7.
```
    5′00′00′00′00|2.2360
    4
   42|100
     | 84
  443|1600
     |1329
 4466|27100
     |26796
44720|30400   Ans. ± 2.2360.
```

8.
```
    .07′00′00′00|.2645
     04
   46|300
     |276
  524|2400
     |2096
 5285|30400   Ans. ± .2645.
```

9.
```
 7|13
   1.85′71′42′85′|1.3627
   1
    23|85
      |69
   266|1671
      |1596
  2722|7542
      |5444
 27247|209885   Ans. ±1.3627.
```

10.
```
 93|237|2.54838709
    186
     510
     465
      450
      372
       780
       744
        360
        279
         810
         744
          660
          651
           900
```

```
   2.54′83′87′09′|1.5963
   1
    25|154
      |125
   309|2983
      |2781
  3186|20287
      |19116
 31923|117109   Ans. ± 1.5963.
```

11. $$\sqrt{\overline{183}^2 + \overline{264}^2} = 321.22, \text{ hypotenuse.}$$

12. $$\sqrt{\overline{90}^2 + \overline{90}^2} = 127.27.$$

13. $$\sqrt{\overline{207}^2 - \overline{83}^2} = 189.63 \text{ feet.}$$

14. $$\sqrt{\overline{292849}^2 - \overline{207000}^2} = 207{,}151 \text{ feet.}$$

15. Let x represent the altitude in inches.

Then $$x = \sqrt{(11)^2 - (\tfrac{11}{2})^2} = \tfrac{11}{2}\sqrt{3}.$$

Whence $$x = 9.5 \text{ inches.}$$

16. $$x^2 - \frac{x^2}{4} = 100.$$

$$x = \tfrac{20}{3}\sqrt{3} = 11.547 \text{ inches.}$$

17. $$\begin{aligned} \text{Area} &= \sqrt{37 \cdot 25 \cdot 10 \cdot 2} \\ &= \sqrt{37 \cdot 5^2 \cdot 5 \cdot 2^2} \\ &= 10\sqrt{185} \\ &= 136.014 \text{ square feet.} \end{aligned}$$

18. $$\begin{aligned} \text{Area} &= \sqrt{33 \cdot 11 \cdot 11 \cdot 11} \\ &= \sqrt{(11)^4 3} \\ &= 121\sqrt{3} = 209.57 \text{ square inches.} \end{aligned}$$

19. $$\pi r^2 = 40.$$

Then $$r = \sqrt{\frac{40}{\pi}} = 3.56 \text{ feet.}$$

20. Diagonal $= \sqrt{\overline{14}^2 + \overline{20}^2 + \overline{30}^2} = \sqrt{1496} = 38.67.$

21. Diagonal $= \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3} = 1.732$ feet.

22. Let $ABCD$ be the floor (AB being 24), DKC one end wall, and AKC an adjoining wall. Let DKC be folded back about DC, to the plane of the floor. Fold ADK back about AD. Then the required line is the shorter of the two lines BK.

BK (across end wall) $= \sqrt{\overline{24}^2 + (40 + 14)^2} = 59.09$ feet.

BK (across side wall) $= \sqrt{(24 + 14)^2 + \overline{40}^2} = 55.17$ feet, required length.

23. Let the integers be 2 and 3.

Then $$2^2 + 3^2 = 13, \quad 3^2 - 2^2 = 5, \qquad 5^2 + \overline{12}^2 = \overline{13}^2 = 169.$$

and $$2 \cdot 3 \cdot 2 = 12.$$

One can thus take any two integers and find three integers which will be the three sides of a right triangle.

24. $$28 = 2 \cdot 7 \cdot 2 = 2 \cdot 1 \cdot 14.$$

Now using 7 and 2, and 1 and 14, as in Exercise 23, we obtain:

$$7^2 + 2^2 = 53, \qquad \overline{14}^2 + 1^2 = 197,$$
$$7^2 - 2^2 = 45, \qquad \overline{14}^2 - 1^2 = 195,$$
$$\overline{28}^2 + \overline{45}^2 = \overline{53}^2, \qquad \overline{28}^2 + \overline{195}^2 = \overline{197}^2.$$

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1. $x^{\frac{3}{5}} = \sqrt[5]{x^3}$.
2. $(ac)^{\frac{2}{3}} = \sqrt[3]{a^2c^2}$.
3. $(4x)^{\frac{1}{3}} = \sqrt[3]{4x}$.
4. $4x^{\frac{1}{3}} = 4\sqrt[3]{x}$.
5. $6cx^{\frac{3}{4}} = 6c\sqrt[4]{x^3}$.
6. $a^{\frac{2}{3}}c^{\frac{3}{5}} = \sqrt[3]{a^2}\sqrt[5]{c^3}$.
7. $3a^{\frac{1}{2}}(c^2x)^{\frac{2}{3}} = 3\sqrt{a}\sqrt[3]{c^4x^2}$.
8. $\dfrac{4a^{\frac{2}{3}}(c-x)^{\frac{4}{3}}}{a^{\frac{1}{2}}(c-x)} = 4a^{\frac{1}{6}}(c-x)^{\frac{1}{3}}$
$= 4\sqrt[6]{a(c-x)^2}$.
9. $4^{\frac{1}{2}} = 2$.
10. $36^{\frac{1}{2}} = 6$.
11. $64^{\frac{1}{3}} = 4$.
12. $81^{\frac{1}{4}} = 3$.
13. $4^{\frac{3}{2}} = 2^3 = 8$.
14. $27^{\frac{2}{3}} = 3^2 = 9$.
15. $(-8)^{\frac{2}{3}} = (-2)^2 = 4$.
16. $(\frac{1}{9})^{\frac{1}{2}} = \frac{1}{3}$.
17. $(-32)^{\frac{3}{5}} = (-2)^3 = -8$.
18. $(\frac{4}{25})^{\frac{1}{2}} \cdot (\frac{125}{8})^{\frac{1}{3}} = \frac{2}{5} \cdot \frac{5}{2} = 1$.
19. $(-125)^{\frac{2}{3}} \cdot (\frac{1}{25})^{\frac{3}{2}} = 25 \cdot \frac{1}{125} = \frac{1}{5}$.
20. $(-243)^{\frac{2}{5}} \cdot (81)^{\frac{3}{4}} = 9 \cdot 27 = 243$.
21. $\sqrt[3]{x^4} = x^{\frac{4}{3}}$.
22. $\sqrt{ac^3} = a^{\frac{1}{2}}c^{\frac{3}{2}}$.
23. $3\sqrt{x^5} = 3x^{\frac{5}{2}}$.
24. $4\sqrt[3]{4x^2} = 2^{\frac{8}{3}}x^{\frac{2}{3}}$.
25. $5\sqrt[3]{8x^4} = 10x^{\frac{4}{3}}$.
26. $6\sqrt[3]{64x^5} = 24x^{\frac{5}{3}}$.
27. $5\sqrt[3]{-125c^2} = -25c^{\frac{2}{3}}$.
28. $c\sqrt{(x+a)^3} = (cx+ca)(x+a)^{\frac{1}{2}}$.
29. $\dfrac{5c\sqrt{ax^3}\cdot\sqrt[3]{5x^4}}{\sqrt[3]{5x^2}\sqrt{ax}} = 5c\sqrt{x^2}\cdot\sqrt[3]{x^2}$
$= 5cx^{\frac{5}{3}}$.
30. $\sqrt[n]{x^a}\cdot\sqrt[n]{c} = x^{\frac{a}{n}}c^{\frac{1}{n}}$.
31. $\sqrt[n]{x^{4a}}\cdot\sqrt[n]{x^{2a}} = x^{\frac{4a}{n}}x^{\frac{2a}{n}} = x^{\frac{6a}{n}}$.
32. (*a*) 5. (*g*) 3 in $\sqrt[3]{5}$.
(*b*) $3\sqrt{-2}$. (*h*) ax in $3\sqrt{ax}$.
(*c*) $\frac{3}{4}$. (*i*) $+2$ of $\sqrt[3]{8}$.
(*d*) $\sqrt{3}$. (*j*) 8 of $\sqrt{64}$.
(*e*) $\sqrt{7}$. (*k*) -3 of $\sqrt[3]{-27}$.
(*f*) $\sqrt{8}$.
33. See pp. 237–238.
34. The first four are rational; the last two are irrational.
35. See p. 238.
36. No. See pp. 238–239.
37. Yes. See pp. 238–239.
38. See p. 238.
39. $\sqrt{3}$, $\sqrt{\sqrt[3]{6}}$ only are surds; all are radicals.
40. 2, 2, -2.
41. Second, third, third, third, and fourth.
42. One; one.
43. Two; none.

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1. $\sqrt{18} = \sqrt{9\cdot 2} = 3\sqrt{2}$.
2. $\sqrt[3]{16} = \sqrt[3]{8\cdot 2} = 2\sqrt[3]{2}$.
3. $2\sqrt{75} = 10\sqrt{3}$.
4. $4\sqrt[3]{-54} = -12\sqrt[3]{2}$.
5. $\sqrt{\sqrt{4}} = \sqrt{2}$.
6. $\sqrt[3]{3\sqrt{9}} = \sqrt[3]{9}$.
7. $\sqrt{\frac{3}{5}} = \frac{1}{5}\sqrt{15}$.
8. $\sqrt{\frac{5}{8}} = \frac{1}{4}\sqrt{10}$.

9. $\sqrt[3]{-\frac{3}{4}} = \frac{1}{2}\sqrt[3]{-6}$.

10. $\sqrt[3]{\frac{1}{7}} = \frac{1}{7}\sqrt[3]{49}$.

11. $6\sqrt[3]{-\frac{1}{9}} = 2\sqrt[3]{-3}$.

12. $\sqrt{3^2 - (\frac{3}{2})^2} = \sqrt{9 - \frac{9}{4}} = \frac{3}{2}\sqrt{3}$.

13. $\sqrt{7^2 - (\frac{7}{2})^2} = \sqrt{49 - \frac{49}{4}} = \frac{7}{2}\sqrt{3}$.

14. $\sqrt{4 - 8\sqrt{3}} = \sqrt{4(1 - 2\sqrt{3})} = 2\sqrt{1 - 2\sqrt{3}}$.

15. $\sqrt{36 + 18\sqrt{5}} = \sqrt{9(4 + 2\sqrt{5})} = 3\sqrt{4 + 2\sqrt{5}}$.

16. $\sqrt[3]{81 - 3\sqrt{243}} = \sqrt[3]{81 - 27\sqrt{3}} = 3\sqrt[3]{3 - \sqrt{3}}$.

17. $\sqrt{R^2 - 3R^2\sqrt{5}} = R\sqrt{1 - 3\sqrt{5}}$.

18. $\sqrt{\dfrac{R^2 - R^2\sqrt{6}}{3}} = \dfrac{R}{3}\sqrt{3 - 3\sqrt{6}}$.

19. $\sqrt{R^2 - \left(\dfrac{R}{2}\right)^2} = \dfrac{R}{2}\sqrt{3}$.

20. $\sqrt{R^2 + \left(\dfrac{R}{3}\right)^2\sqrt{3}} = \sqrt{R^2 + \dfrac{R^2}{9}\sqrt{3}} = \dfrac{R}{3}\sqrt{9 + \sqrt{3}}$.

21. $3\sqrt{5} = \sqrt{45}$.

22. $2\sqrt[3]{8} = \sqrt[3]{64}$.

23. $2c\sqrt[3]{c^2} = \sqrt[3]{8c^5}$.

24. $4\sqrt[3]{\frac{1}{4}} = \sqrt[3]{16}$.

25. $\dfrac{a}{3}\sqrt[3]{\dfrac{9}{a^2}} = \sqrt[3]{\dfrac{a}{3}}$.

26. $(2a+1)\sqrt{\dfrac{2}{4a^2-1}} = \sqrt{\dfrac{4a+2}{2a-1}}$.

27. $\dfrac{x - 3a}{5}\sqrt[3]{\dfrac{125}{(x-3a)^2}} = \sqrt[3]{x - 3a}$.

28. $$\frac{c}{2}\sqrt{a^2 - \left(\frac{a^2+c^2-b^2}{2c}\right)^2} = \frac{c}{2}\sqrt{\left(a + \frac{a^2+c^2-b^2}{2c}\right)\left(a - \frac{a^2+c^2-b^2}{2c}\right)}$$
$$= \frac{c}{2}\sqrt{\left(\frac{a^2+2ac+c^2-b^2}{2c}\right)\left(\frac{-a^2+2ac-c^2+b^2}{2c}\right)}$$
$$= \frac{c}{2}\sqrt{\frac{(a+b+c)(a-b+c)(a+b-c)(-a+b+c)}{4c^2}}$$
$$= \sqrt{\frac{a+b+c}{2}\cdot\frac{a+b-c}{2}\cdot\frac{a-b+c}{2}\cdot\frac{-a+b+c}{2}}$$
$$= \sqrt{s(s-a)(s-b)(s-c)}.$$

29. $\sqrt{\sqrt{x}} = \sqrt{x^{\frac{1}{2}}} = x^{\frac{1}{4}} = \sqrt[4]{x}$.

30. $\sqrt{\sqrt[3]{x}} = \sqrt{x^{\frac{1}{3}}} = x^{\frac{1}{6}} = \sqrt[6]{x}$.

31. $\sqrt[3]{\sqrt{x}} = \sqrt[3]{x^{\frac{1}{2}}} = x^{\frac{1}{6}} = \sqrt[6]{x}$.

32. $\sqrt[3]{\sqrt{8a^2x}} = \sqrt[3]{(8a^2x)^{\frac{1}{2}}} = \sqrt[6]{8a^2x}$.

33. $\sqrt{3\sqrt{3}} = \sqrt{\sqrt{27}} = \sqrt[4]{27}$.

34. $\sqrt{3\sqrt{3\sqrt{3}}} = \sqrt{\sqrt{27\sqrt{3}}} = \sqrt{\sqrt{\sqrt{3^7}}} = \sqrt[8]{2187}$.

35. $2\sqrt[3]{2\sqrt[3]{2}} = 2\sqrt[3]{\sqrt[3]{16}} = 2\sqrt[9]{16}$.

36. $\sqrt[a]{\sqrt[n]{x^c}} = \sqrt[a]{x^{\frac{c}{n}}} = x^{\frac{c}{na}} = \sqrt[na]{x^c}$.

37. $\sqrt[\frac{m}{n}]{x^{\frac{a}{n}}} = x^{\left(\frac{a}{n} \div \frac{m}{n}\right)} = x^{\frac{a}{m}} = \sqrt[m]{x^a}$.

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1. $\sqrt{50}+\sqrt{98}-\sqrt{32}=5\sqrt{2}+7\sqrt{2}-4\sqrt{2}=8\sqrt{2}.$

2. $\sqrt[3]{192}-4\sqrt[3]{24}+\sqrt[3]{375}=4\sqrt[3]{3}-8\sqrt[3]{3}+5\sqrt[3]{3}=\sqrt[3]{3}.$

3. $10\sqrt{\frac{6}{5}}-\sqrt{\frac{3}{10}}+4\sqrt{\frac{15}{2}}=2\sqrt{30}-\frac{1}{10}\sqrt{30}+2\sqrt{30}=\frac{39}{10}\sqrt{30}.$

4. $3\sqrt{\frac{2}{7}}+3\sqrt{\frac{7}{2}}-2\sqrt{\frac{1}{14}}=\frac{3}{7}\sqrt{14}+\frac{3}{2}\sqrt{14}-\frac{1}{7}\sqrt{14}=\frac{25}{14}\sqrt{14}.$

5. $\sqrt{x^3}+\sqrt[4]{x^2}-12\sqrt[6]{x^3}=x\sqrt{x}+\sqrt{x}-12\sqrt{x}=(x-11)\sqrt{x}.$

6. $$\sqrt{\frac{3a}{x}}+\sqrt{\frac{3x}{a}}-\sqrt{\frac{ax}{3}}=\frac{1}{x}\sqrt{3ax}+\frac{1}{a}\sqrt{3ax}-\frac{1}{3}\sqrt{3ax}$$
$$=\frac{3a+3x-ax}{3ax}\sqrt{3ax}.$$

7. $$\sqrt{\frac{a}{x^3}}-\sqrt{\frac{a}{x^5}}+\sqrt{\frac{5x^3}{a}}=\frac{1}{x^2}\sqrt{ax}-\frac{1}{x^3}\sqrt{ax}+\frac{x}{a}\sqrt{5ax}$$
$$=\frac{x-1}{x^3}\sqrt{ax}+\frac{x}{a}\sqrt{5ax}.$$

8. $$\sqrt[4]{32x^5}+\sqrt[4]{1250x}-\sqrt[4]{512x}-\sqrt[4]{2x}$$
$$=2x\sqrt[4]{2x}+5\sqrt[4]{2x}-4\sqrt[4]{2x}-\sqrt[4]{2x}$$
$$=2x\sqrt[4]{2x}.$$

9. $$\sqrt[3]{(a-c)^4}+c\sqrt[6]{a^2-2ac+c^2}+(a+c)\sqrt[3]{a-c}$$
$$=(a-c)\sqrt[3]{a-c}+c\sqrt[3]{a-c}+(a+c)\sqrt[3]{a-c}$$
$$=(2a+c)\sqrt[3]{a-c}.$$

10. $$\sqrt{\frac{a}{c}}-\sqrt{\frac{c}{a}}+\sqrt{\frac{a^2+c^2}{ac}+2}-\sqrt{\frac{a^2+c^2}{ac}-2}$$
$$=\frac{1}{c}\sqrt{ac}-\frac{1}{a}\sqrt{ac}+\frac{a+c}{ac}\sqrt{ac}-\frac{a-c}{ac}\sqrt{ac}$$
$$=\frac{a-c+a+c-a+c}{ac}\sqrt{ac}=\frac{a+c}{ac}\sqrt{ac}.$$

11. $$\sqrt[3]{24}+\sqrt[3]{(3a+9)(a+3)^2}-\sqrt[3]{81}+a\sqrt[6]{9}-4\sqrt[3]{3}$$
$$=2\sqrt[3]{3}+(a+3)\sqrt[3]{3}-3\sqrt[3]{3}+a\sqrt[3]{3}-4\sqrt[3]{3}$$
$$=(2a-2)\sqrt[3]{3}.$$

12. $$2\sqrt{9a^3-9a^2b}-3\sqrt{9ab^2-9b^3}+\sqrt{(a^2-b^2)(a+b)}$$
$$=6a\sqrt{a-b}-9b\sqrt{a-b}+(a+b)\sqrt{a-b}$$
$$=(7a-8b)\sqrt{a-b}.$$

13. $(a-b)\sqrt{\dfrac{a+b}{a-b}}+\sqrt{25a^2-25b^2}+\dfrac{a+b}{a-b}\sqrt{\dfrac{36ab^2-36b^3}{a+b}}$

$=\sqrt{a^2-b^2}+5\sqrt{a^2-b^2}+\dfrac{6b}{a-b}\sqrt{a^2-b^2}$

$=6+\dfrac{6b}{a-b}\sqrt{a^2-b^2}=\dfrac{6a}{a-b}\sqrt{a^2-b^2}.$

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1. $\sqrt{3}\cdot\sqrt{27}=\sqrt{81}=9.$

2. $\sqrt{\frac{4}{5}}\cdot\sqrt{\frac{5}{6}}\cdot\sqrt{\frac{3}{4}}=\sqrt{\frac{1}{2}}=\frac{1}{2}\sqrt{2}.$

3. $\sqrt{3}\cdot\sqrt[3]{2}=3^{\frac{1}{2}}\cdot 2^{\frac{1}{3}}=3^{\frac{3}{6}}\cdot 2^{\frac{2}{6}}=\sqrt[6]{3^3\cdot 2^2}=\sqrt[6]{108}.$

4. $\sqrt[4]{6}\cdot\sqrt{2}=\sqrt[4]{6}\cdot\sqrt[4]{4}=\sqrt[4]{24}.$

5. $\sqrt[3]{12}\cdot\sqrt{\frac{1}{8}}=\sqrt[6]{(12)^2(\frac{1}{8})^3}=\frac{1}{2}\sqrt[6]{18}.$

6. $\sqrt[3]{a^2}\cdot\sqrt{a^3}=\sqrt[6]{a^4\cdot a^9}=a^2\sqrt[6]{a}.$

7. $\sqrt[3]{\dfrac{x}{a}}\cdot\sqrt{\dfrac{a}{x}}=\sqrt[6]{\dfrac{x^2}{a^2}\cdot\dfrac{a^3}{x^3}}=\sqrt[6]{\dfrac{a}{x}}=\dfrac{1}{x}\sqrt[6]{ax^5}.$

8. $\sqrt[4]{2x^3}\cdot\sqrt{3x}=\sqrt[4]{18x^5}=x\sqrt[4]{18x}.$

9. $(\sqrt{x-3})^2=x-3.$

10. $(2\sqrt{3x-1})^2=12x-4.$

11. $(3\sqrt[4]{x-2})^2=9\sqrt{x-2}.$

12. $3\sqrt{x-3}\cdot\sqrt{4x-8}=6\sqrt{x^2-5x+6}.$

13. $(\sqrt{x}-\sqrt{x-3})^2=x+x-3-2\sqrt{x^2-3x}=2x-3-2\sqrt{x^2-3x}.$

14. $(\sqrt{x-3}-\sqrt{4x-7})^2=5x-10-2\sqrt{4x^2-19x+21}.$

15. $(\sqrt{x}-\sqrt{3x})4\sqrt{x}=4x-4x\sqrt{3}.$

16. $\left(\dfrac{6-2\sqrt{5}}{3}\right)^2\dfrac{[(\sqrt{5}+1)(\sqrt{5}+1)]^2}{2}\cdot\dfrac{(\sqrt{2}+1)(9\sqrt{2}-9)}{16}$

$=\dfrac{56-24\sqrt{5}}{9}(28+12\sqrt{5})\dfrac{9}{16}=(14-6\sqrt{5})(7+3\sqrt{5})=8.$

18. $\sqrt[3]{6}=\sqrt[6]{6^2}=\sqrt[6]{36}.$
$\sqrt{3}=\sqrt[6]{3^3}=\sqrt[6]{27}.$ $\therefore\ \sqrt[3]{6}>\sqrt{3}.$

19. $\sqrt[3]{19}=\sqrt[6]{(19)^2}=\sqrt[6]{361}.$
$\sqrt{7}=\sqrt[6]{7^3}=\sqrt[6]{343}.$ $\therefore\ \sqrt[3]{19}>\sqrt{7}.$

20. $\sqrt{3}=\sqrt[4]{9}.$
$\sqrt[4]{6}=\sqrt[4]{6}.$ $\therefore\ \sqrt{3}>\sqrt[4]{6}.$

21. $2\sqrt{5}=\sqrt{20}=\sqrt[6]{8000}.$
$\sqrt[3]{89}=\sqrt[6]{(89)^2}=\sqrt[6]{7921}.$ $\therefore\ 2\sqrt{5}>\sqrt[3]{89}.$

22. $3\sqrt{3} = \sqrt{27} = \sqrt[6]{(27)^3} = \sqrt[6]{19683}.$
$2\sqrt[3]{10} = \sqrt[3]{80} = \sqrt[6]{(80)^2} = \sqrt[6]{6400}.$ $\quad \therefore\ 3\sqrt{3} > 2\sqrt[3]{10}.$

23. $\sqrt[4]{48} = 2\sqrt[4]{3} = 2\sqrt[20]{243}.$
$\sqrt[5]{64} = 2\sqrt[5]{2} = 2\sqrt[20]{16}.$ $\quad \therefore\ \sqrt[4]{48} > \sqrt[5]{64}.$

24. $\sqrt{3} = \sqrt[6]{3^3} = \sqrt[6]{27}.$
$\sqrt[3]{6} = \sqrt[6]{6^2} = \sqrt[6]{36}.$ $\quad \therefore\ \sqrt[9]{125} < \sqrt{3} < \sqrt[3]{6}.$
$\sqrt[9]{125} = \sqrt[3]{5} = \sqrt[6]{25}.$

25. $4\sqrt[4]{6} = 4\sqrt[12]{216} = \sqrt[12]{(4^{12})\,216}.$
$3\sqrt[6]{25} = 3\sqrt[12]{625} = \sqrt[12]{(3^{12})\,625}.$
$4\sqrt[9]{64} = 4\sqrt[3]{4} = 4\sqrt[12]{4^4} = \sqrt[12]{(4^{12})\,256}.$
$(3^{12})\,625 < (4^{12})\,216 < (4^{12})\,256.$
$\therefore\ 3\sqrt[6]{25} < 4\sqrt[4]{6} < 4\sqrt[9]{64}.$

26. $\sqrt{3} = \sqrt[6]{3^3} = \sqrt[6]{27}.$
$\sqrt[3]{3x^2} = \sqrt[6]{(3x^2)^2} = \sqrt[6]{9x^4}.$

27. $\sqrt[3]{a+b} = \sqrt[6]{(a+b)^2}.$
$\sqrt{a-b} = \sqrt[6]{(a-b)^3}.$

28. $2x\sqrt[3]{5xy} = 2x\sqrt[12]{625x^4y^4}.$
$5x\sqrt[4]{3xy} = 5x\sqrt[12]{27x^3y^3}.$

29. $\sqrt[3]{xy} = \sqrt[60]{x^{20}y^{20}}.$
$\sqrt[4]{xy^2} = \sqrt[60]{x^{15}y^{30}}.$
$\sqrt[5]{x^2y} = \sqrt[60]{x^{24}y^{12}}.$

30. $(\sqrt[3]{3})^2 = \sqrt[3]{9}.$

31. $(2\sqrt[3]{4})^2 = 4\sqrt[3]{16} = 8\sqrt[3]{2}.$

32. $\left(\sqrt{5-\sqrt{5}}\right)^2 = 5 - \sqrt{5}.$

33. $\left(4\sqrt{3-\sqrt{5}}\right)^2 = 48 - 16\sqrt{5}.$

34. $\left(\sqrt[3]{4-4\sqrt{3}}\right)^3 = \sqrt[3]{64-32\sqrt{3}} = 2\sqrt[3]{8-4\sqrt{3}}.$

35. $\left(\sqrt[4]{6-3\sqrt{2}}\right)^2 = \sqrt{6-3\sqrt{2}}.$

36. $(3\sqrt{5})^3 = 27\sqrt{125} = 135\sqrt{5}.$

37. $(3\sqrt[3]{2} - 2\sqrt{3})^3 = 54 - 54\sqrt[6]{432} + 108\sqrt[3]{2} - 24\sqrt{3}.$

38. $\left(\left(\sqrt[4]{1-\sqrt{2}}\right)^3\right)^3 = \left(\sqrt[4]{7-5\sqrt{2}}\right)^3 = \sqrt[4]{1393-985\sqrt{2}}.$

39. $(5\sqrt{5} + 9\sqrt{3} - \sqrt{7} + 2\sqrt{105})(\sqrt{3} + \sqrt{5} - \sqrt{7})$
$= 5\sqrt{15} + 27 - \sqrt{21} + 6\sqrt{35} + 25 + 9\sqrt{15} - \sqrt{35}$
$+ 10\sqrt{21} - 5\sqrt{35} - 9\sqrt{21} + 7 - 14\sqrt{15} = 59.$

40. $(\sqrt{a} - \sqrt[4]{ac} + \sqrt{c})(\sqrt{a} + \sqrt[4]{ac} + \sqrt{c})(a + c + \sqrt{ac})$
$= (a + \sqrt{ac} + c)(a + \sqrt{ac} + c)$
$= a^2 + (a+c)\,2\sqrt{ac} + 3ac + c^2.$

41. $(\sqrt{2x-1} - \sqrt{5})(2\sqrt{2x-1} + \sqrt{45})(4x - \sqrt{10x-5} - 17)$
$= [2(2x-1) - 2\sqrt{10x-5} + 3\sqrt{10x-5} - 15](4x - \sqrt{10x-5} - 17)$
$= (4x + \sqrt{10x-5} - 17)(4x - \sqrt{10x-5} - 17).$
$= 16x^2 - 146x + 294.$

42. $$\sqrt{R^2-\left(\frac{R}{3}\sqrt{5}-\frac{R}{3}\right)^2}=\sqrt{R^2-\frac{5R^2-2R^2\sqrt{5}+R^2}{9}}$$
$$=\sqrt{\frac{2R^2\sqrt{5}+3R^2}{9}}=\frac{R}{3}\sqrt{2\sqrt{5}+3}.$$

43. $$\left[\left(R-\frac{R}{6}\sqrt{3}\right)^2+\left(\frac{R}{6}\right)^2\right]^{\frac{1}{2}}=\sqrt{R^2-\frac{2R^2\sqrt{3}}{6}+\frac{3R^2}{36}+\frac{R^2}{36}}$$
$$=\sqrt{\frac{40R^2-12R^2\sqrt{3}}{36}}=\frac{R}{3}\sqrt{10-3\sqrt{3}}.$$

44. $$\sqrt{R^2-\left(\frac{R\sqrt{4-\sqrt{2}}}{2}\right)^2}=\sqrt{R^2-\frac{4R^2-R^2\sqrt{2}}{4}}$$
$$=\sqrt{\frac{4R^2-4R^2+R^2\sqrt{2}}{4}}$$
$$=\frac{R}{2}\sqrt{\sqrt{2}}=\frac{R\sqrt[4]{2}}{2}.$$

45. $$\left(\frac{R}{2}\sqrt{3+\sqrt{3}}\right)\left(\frac{R}{2}\sqrt{3-\sqrt{3}}\right)=\frac{R^2}{4}\sqrt{9-3}=\frac{R^2}{4}\sqrt{6}.$$

46. $$\sqrt{\left(e^x+\frac{2}{e^x}\right)^2-(e^x-2e^{-x})(e^x-2e^{-x})+e^{2x}+e^{-2x}-6}$$
$$=\sqrt{e^{2x}+4+4e^{-2x}-e^{2x}+4-4e^{-2x}+e^{2x}+e^{-2x}-6}$$
$$=\sqrt{e^{2x}+2+e^{-2x}}=e^x+e^{-x}.$$

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1. $\sqrt{7}$.
2. $\sqrt[3]{2}$.
3. $\sqrt[4]{2}$.
4. $\sqrt[3]{4}$.
5. $\sqrt{5}+7$.
6. $3\sqrt{7}+2\sqrt{13}$.
7. $\sqrt{3a}+\sqrt{2x}$.
8. $\sqrt{x-c}+\sqrt{a}$.

9. $(\sqrt{2}+\sqrt{3}+\sqrt{5})\sqrt{6}$, or $2\sqrt{3}+3\sqrt{2}+\sqrt{30}$.

10. $\sqrt{8}\div\sqrt{24}=\sqrt{\frac{1}{3}}=\frac{1}{3}\sqrt{3}$.

11. $$8\div4\sqrt{3}=\frac{8\sqrt{3}}{4\sqrt{3}\sqrt{3}}=\frac{2}{3}\sqrt{3}.$$

12. $$24\div3\sqrt{3}=\frac{24\sqrt{3}}{3\sqrt{3}\sqrt{3}}=\frac{8}{3}\sqrt{3}.$$

13. $$a\div c\sqrt{x}=\frac{a\sqrt{x}}{c\sqrt{x}\sqrt{x}}=\frac{a\sqrt{x}}{cx}.$$

14. $$(\sqrt{12}-\sqrt{18})\div2\sqrt{3}=\frac{(\sqrt{12}-\sqrt{18})\sqrt{3}}{2\sqrt{3}\sqrt{3}}=\frac{\sqrt{36}-\sqrt{54}}{6}=1-\frac{\sqrt{6}}{2}.$$

15. $$(12-3\sqrt{6}-4\sqrt{24})\div3\sqrt{2}=\frac{(12-3\sqrt{6}-4\sqrt{24})\sqrt{2}}{3\sqrt{2}\cdot\sqrt{2}}$$
$$=2\sqrt{2}-\sqrt{3}-\tfrac{8}{3}\sqrt{3}=2\sqrt{2}-\tfrac{11}{3}\sqrt{3}.$$

16. $\dfrac{\sqrt{6}}{\sqrt[3]{2}}=\dfrac{\sqrt{6}\cdot\sqrt[3]{4}}{\sqrt[3]{2}\cdot\sqrt[3]{4}}$
$=\dfrac{\sqrt[6]{3^3\cdot 2^3\cdot 2^4}}{2}$
$=\sqrt[6]{3^3\cdot 2}=\sqrt[6]{54}.$

17. $\dfrac{\sqrt[4]{8}}{\sqrt{2}}=\dfrac{\sqrt[4]{8}}{\sqrt[4]{4}}=\sqrt[4]{2}.$

18. $\dfrac{\sqrt{32}}{\sqrt[4]{2}}=\dfrac{\sqrt[4]{32^2}}{\sqrt[4]{2}}$
$=\sqrt[4]{512}=4\sqrt[4]{2}.$

19. $\dfrac{\sqrt[3]{\frac{1}{4}}}{\sqrt{\frac{1}{2}}}=\dfrac{\sqrt[6]{(\frac{1}{4})^2}}{\sqrt[6]{(\frac{1}{2})^3}}=\sqrt[6]{\dfrac{1}{16}\cdot 8}=\sqrt[6]{\dfrac{1}{2}}=\dfrac{1}{2}\sqrt[6]{32}.$

20. $\dfrac{3}{2-\sqrt{3}}=\dfrac{3(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}=\dfrac{6+3\sqrt{3}}{4-3}=6+3\sqrt{3}.$

21. $\dfrac{\sqrt{3}}{\sqrt{2}+\sqrt{3}}=\dfrac{\sqrt{3}(\sqrt{2}-\sqrt{3})}{(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})}=\dfrac{\sqrt{6}-3}{2-3}=3-\sqrt{6}.$

22. $\dfrac{\sqrt{7}}{\sqrt{2-\sqrt{3}}}=\dfrac{\sqrt{7}(\sqrt{2+\sqrt{3}})}{(\sqrt{2-\sqrt{3}})(\sqrt{2+\sqrt{3}})}=\dfrac{\sqrt{14+7\sqrt{3}}}{\sqrt{4-3}}=\sqrt{14+7\sqrt{3}}.$

23. $\dfrac{\sqrt{2-\sqrt{3}}}{\sqrt{3-\sqrt{2}}}=\dfrac{\sqrt{2-\sqrt{3}}\sqrt{3+\sqrt{2}}}{\sqrt{3-\sqrt{2}}\sqrt{3+\sqrt{2}}}=\dfrac{\sqrt{6-3\sqrt{3}+2\sqrt{2}-\sqrt{6}}}{\sqrt{9-2}}$
$=\dfrac{\sqrt{6-3\sqrt{3}+2\sqrt{2}-\sqrt{6}}}{\sqrt{7}}=\dfrac{\sqrt{42-21\sqrt{3}+14\sqrt{2}-7\sqrt{6}}}{7}.$

24. $\dfrac{\sqrt{7}+\sqrt{5}}{2\sqrt{7}-\sqrt{5}}=\dfrac{(\sqrt{7}+\sqrt{5})(2\sqrt{7}+\sqrt{5})}{(2\sqrt{7}-\sqrt{5})(2\sqrt{7}+\sqrt{5})}=\dfrac{14+3\sqrt{35}+5}{28-5}=\dfrac{19+3\sqrt{35}}{23}.$
$\dfrac{19+3\sqrt{35}}{23(19-3\sqrt{35})}=\dfrac{(19+3\sqrt{35})^2}{23(19-3\sqrt{35})(19+3\sqrt{35})}$
$=\dfrac{361+114\sqrt{35}+315}{23(361-315)}=\dfrac{676+114\sqrt{35}}{23\cdot 46}=\dfrac{338+57\sqrt{35}}{529}.$

25. (*a*) $6+3\sqrt{3}=6+3(1.73205)=11.1961.$
(*b*) $3-\sqrt{6}=3-2.44948+=.55051+.$
(*c*) $\sqrt{14+7\sqrt{3}}=\sqrt{14+7(1.73205)}=\sqrt{26.12435}=5.1111.$

26. $\dfrac{\sqrt{3}}{\sqrt{2}+\sqrt{3}}=\dfrac{1.73205}{1.41421+1.73205}=\dfrac{1.73205}{3.14626}=.55050.$

There is a difference of one one-hundred-thousandth.

27. In general the result will be far more easily obtained and will be more nearly correct if rationalization is used.

28. $\dfrac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}=\dfrac{(\sqrt{5}+\sqrt{2})(\sqrt{5}+\sqrt{2})}{(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})}=\dfrac{7+2\sqrt{10}}{3}.$

29. $$\frac{2\sqrt{5}+3\sqrt{7}}{3\sqrt{5}-2\sqrt{7}}=\frac{(2\sqrt{5}+3\sqrt{7})(3\sqrt{5}+2\sqrt{7})}{(3\sqrt{5}-2\sqrt{7})(3\sqrt{5}+2\sqrt{7})}=\frac{30+42+13\sqrt{35}}{17}$$
$$=\frac{72+13\sqrt{35}}{17}.$$

30. $$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}=\frac{(\sqrt{a}+\sqrt{b})(\sqrt{a}+\sqrt{b})}{(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})}=\frac{a+b+2\sqrt{ab}}{a-b}.$$

31. $$\frac{\sqrt{x}-2\sqrt{c}}{\sqrt{x}+\sqrt{c}}=\frac{(\sqrt{x}-2\sqrt{c})(\sqrt{x}-\sqrt{c})}{(\sqrt{x}+\sqrt{c})(\sqrt{x}-\sqrt{c})}=\frac{x+2c-3\sqrt{cx}}{x-c}.$$

32. $$\frac{\sqrt{x-3}+\sqrt{3}}{\sqrt{x-3}-\sqrt{3}}=\frac{(\sqrt{x-3}+\sqrt{3})(\sqrt{x-3}+\sqrt{3})}{(\sqrt{x-3}-\sqrt{3})(\sqrt{x-3}+\sqrt{3})}=\frac{x+2\sqrt{3x-9}}{x-6}.$$

33. $$\frac{4}{\sqrt[4]{2-\sqrt{2}}}=\frac{4\sqrt[4]{2+\sqrt{2}}}{\sqrt[4]{2-\sqrt{2}}\,\sqrt[4]{2+\sqrt{2}}}=\frac{4\sqrt[4]{2+\sqrt{2}}}{\sqrt[4]{2}}=\frac{4\sqrt[4]{2+\sqrt{2}}\sqrt[4]{8}}{\sqrt[4]{2}\cdot\sqrt[4]{8}}$$
$$=2\sqrt[4]{16+8\sqrt{2}}.$$

34. $$\frac{\sqrt{10}-\sqrt{5}}{\sqrt{10}+\sqrt{5}}=\frac{(\sqrt{10}-\sqrt{5})(\sqrt{10}-\sqrt{5})}{(\sqrt{10}+\sqrt{5})(\sqrt{10}-\sqrt{5})}=\frac{15-10\sqrt{2}}{5}=3-2\sqrt{2}.$$

35. $$\frac{x-\sqrt{c}}{x-3\sqrt{c}}=\frac{(x-\sqrt{c})(x+3\sqrt{c})}{(x-3\sqrt{c})(x+3\sqrt{c})}=\frac{x^2-3c+2x\sqrt{c}}{x^2-9c}.$$

36. $$\frac{\sqrt{a+c}-\sqrt{x}}{\sqrt{a+c}+\sqrt{x}}=\frac{(\sqrt{a+c}-\sqrt{x})(\sqrt{a+c}-\sqrt{x})}{(\sqrt{a+c}+\sqrt{x})(\sqrt{a+c}-\sqrt{x})}$$
$$=\frac{a+c+x-2\sqrt{ax+cx}}{a+c-x}.$$

37. $$\frac{\sqrt{3}+\sqrt{2}}{2-\sqrt{3}+\sqrt{2}}=\frac{(\sqrt{3}+\sqrt{2})(2+\sqrt{3}+\sqrt{2})}{(2-\sqrt{3}+\sqrt{2})(2+\sqrt{3}+\sqrt{2})}$$
$$=\frac{2\sqrt{3}+2\sqrt{2}+5+2\sqrt{6}}{3+4\sqrt{2}}=\frac{(2\sqrt{3}+2\sqrt{2}+5+2\sqrt{6})(3-4\sqrt{2})}{(3+4\sqrt{2})(3-4\sqrt{2})}$$
$$=\frac{10\sqrt{3}+14\sqrt{2}+1+2\sqrt{6}}{23}.$$

38. $$\frac{\sqrt{5}-\sqrt{7}}{\sqrt{5}+\sqrt{7}-\sqrt{2}}=\frac{(\sqrt{5}-\sqrt{7})(\sqrt{5}+\sqrt{7}+\sqrt{2})}{(\sqrt{5}+\sqrt{7}-\sqrt{2})(\sqrt{5}+\sqrt{7}+\sqrt{2})}$$
$$=\frac{\sqrt{10}-\sqrt{14}-2}{10+2\sqrt{35}}=\frac{(\sqrt{10}-\sqrt{14}-2)(10-2\sqrt{35})}{(10+2\sqrt{35})(10-2\sqrt{35})}$$
$$=\frac{24\sqrt{10}-20\sqrt{14}-20+4\sqrt{35}}{-40}=\frac{5\sqrt{14}+5-\sqrt{35}-6\sqrt{10}}{10}.$$

39. No. **40.** Yes. **41.** No. **42.** Yes.

Page 349 (First set)

1. $6 - 2\sqrt{8} = 4 - 2\sqrt{8} + 2 = (2 - \sqrt{2})^2.$

2. $7 + 2\sqrt{10} = 5 + 2\sqrt{10} + 2 = (\sqrt{5} + \sqrt{2})^2.$

3. $13 + \sqrt{48} = 12 + 2\sqrt{12} + 1 = (2\sqrt{3} + 1)^2.$

4. $8 - \sqrt{60} = 8 - 2\sqrt{15} = 5 - 2\sqrt{15} + 3 = (\sqrt{5} - \sqrt{3})^2.$

5. $11 - 4\sqrt{7} = 7 - 2\sqrt{28} + 4 = (\sqrt{7} - 2)^2.$

6. $17 + 12\sqrt{2} = 8 + 2\sqrt{72} + 9 = (2\sqrt{2} + 3)^2.$

7. $11 - 3\sqrt{8} = 9 - 2\sqrt{18} + 2 = (3 - \sqrt{2})^2.$

8. $65x - 20\sqrt{3x^2} = 60x - 2\sqrt{300x^2} + 5x = (2\sqrt{15x} - \sqrt{5x})^2.$

9. $126a - 10a\sqrt{5} = a - 2\sqrt{125a^2} + 125a = (5\sqrt{5a} - \sqrt{a})^2.$

10. $\dfrac{13a}{4} - \sqrt{3a^2} = 3a - 2\sqrt{\dfrac{3a^2}{4}} + \dfrac{a}{4} = (\sqrt{3a} - \tfrac{1}{2}\sqrt{a})^2.$

11. $2x + 2\sqrt{x^2 - 49} = x - 7 + 2\sqrt{x^2 - 49} + x + 7 = (\sqrt{x-7} + \sqrt{x+7})^2.$

12. $a + \sqrt{a^2 - 1} = \dfrac{a+1}{2} + 2\sqrt{\dfrac{a^2-1}{4}} + \dfrac{a-1}{2}$
$= (\tfrac{1}{2}\sqrt{2a+2} + \tfrac{1}{2}\sqrt{2a-2})^2.$

13. $\sqrt{9 + 3\sqrt{8}} = \sqrt{6 + 3\sqrt{8} + 3} = \sqrt{6} + \sqrt{3}.$

14. $\sqrt{15 - 5\sqrt{8}} = \sqrt{10 - 2\sqrt{50} + 5} = (\sqrt{10} - \sqrt{5}).$

15. $\sqrt{m^2 + m + 2n + 2m\sqrt{m+2n}} = \sqrt{m^2 + 2m\sqrt{m+2n} + m + 2n}$
$= (m + \sqrt{m+2n}).$

Page 349 (Second set)

1. $x^2 - 11 = (x + \sqrt{11})(x - \sqrt{11}).$

2. $3x^2 - 16 = (x\sqrt{3} + 4)(x\sqrt{3} - 4).$

3. $x^3 + 2 = (x + \sqrt[3]{2})(x^2 - x\sqrt[3]{2} + \sqrt[3]{4}).$

4. $x^3 - 12 = (x - \sqrt[3]{12})(x^2 + x\sqrt[3]{12} + 2\sqrt[3]{18})$.

5. $3x^3 - 27 = 3(x^3 - 9) = 3(x - \sqrt[3]{9})(x^2 + x\sqrt[3]{9} + 3\sqrt[3]{3})$.

6. $5x^3 + 125 = 5(x^3 + 25) = 5(x + \sqrt[3]{25})(x^2 - x\sqrt[3]{25} + 5\sqrt[3]{5})$.

7. $\dfrac{2\sqrt{b}}{a-b} + \dfrac{2}{\sqrt{a}+\sqrt{b}} = \dfrac{2\sqrt{b}}{a-b} + \dfrac{2\sqrt{a} - 2\sqrt{b}}{a-b} = \dfrac{2\sqrt{a}}{a-b}$.

8. $$\frac{x+c}{\sqrt{x}-\sqrt{c}} - \frac{x^{\frac{3}{2}} + c^{\frac{3}{2}}}{x-c} = \frac{x^{\frac{3}{2}} + cx^{\frac{1}{2}} + xc^{\frac{1}{2}} + c^{\frac{3}{2}} - x^{\frac{3}{2}} - c^{\frac{3}{2}}}{x-c}$$
$$= \frac{cx^{\frac{1}{2}} + c^{\frac{1}{2}}x}{x-c}.$$

9. $x^2 - 5 = 0$.

$(x - \sqrt{5})(x + \sqrt{5}) = 0$. $\therefore x = \sqrt{5}$ and $-\sqrt{5}$.

10. $2x^2 - 3 = 0$.

$(x\sqrt{2} + \sqrt{3})(x\sqrt{2} - \sqrt{3}) = 0$.

$\therefore x = -\dfrac{\sqrt{3}}{\sqrt{2}}$ and $\dfrac{\sqrt{3}}{\sqrt{2}}$ or $-\dfrac{1}{2}\sqrt{6}$ and $\dfrac{1}{2}\sqrt{6}$.

11. $x^4 + 144 = 26x^2$.

$x^4 - 26x^2 + 144 = 0$.

$(x^2 - 8)(x^2 - 18) = 0$. $\therefore x = 2\sqrt{2}, -2\sqrt{2}, 3\sqrt{2}, -3\sqrt{2}$.

12. $4x^4 + c = x^2 + 4cx^2$ or $4x^4 - 4cx^2 - x^2 + c = 0$.

$(4x^2 - 1)(x^2 - c) = 0$. $(2x - 1)(2x + 1)(x + \sqrt{c})(x - \sqrt{c}) = 0$.

$x = \frac{1}{2}, -\frac{1}{2}, -\sqrt{c}, \sqrt{c}$.

Page 350

1. $\overline{12}^2 - 6^2 = a^2$. $\therefore a = 6\sqrt{3}$.

2. $s^2 - \left(\dfrac{s}{2}\right)^2 = a^2$. $\therefore a = \dfrac{s}{2}\sqrt{3}$. Area $= \dfrac{s \cdot \frac{s}{2}\sqrt{3}}{2} = \dfrac{s^2}{4}\sqrt{3}$.

3. $\overline{20}^2 = s^2 - \left(\dfrac{s}{2}\right)^2$. $\therefore s = \dfrac{40}{3}\sqrt{3}$. Area $= \dfrac{20 \cdot \frac{40}{3}\sqrt{3}}{2} = \dfrac{400}{3}\sqrt{3}$.

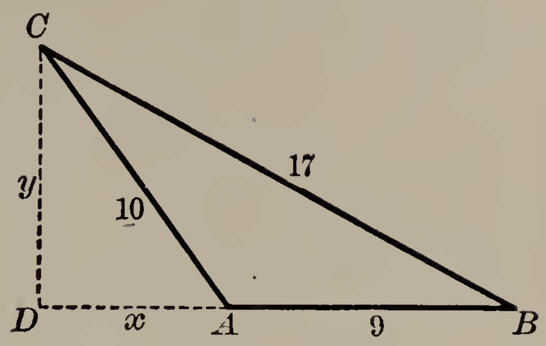
4. $s^2 - \left(\dfrac{s}{2}\right)^2 = a^2$. Whence $s = \dfrac{2a}{3}\sqrt{3}$.

5. $x^2 + y^2 = 100. \qquad (1)$

$(x+9)^2 + y^2 = 289. \qquad (2)$

$(2)-(1),$

$$18x + 81 = 189.$$
$$18x = 108.$$
$$x = 6.$$
$$\therefore y = \sqrt{100-36} = 8.$$
$$\text{Area} = \frac{8 \cdot 9}{2} = 36.$$

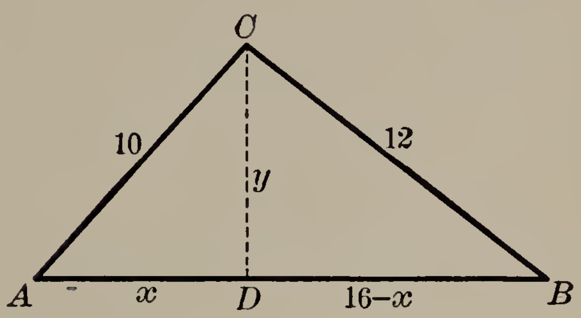


6. $y^2 + x^2 = 100. \qquad (1)$

$y^2 + (16-x)^2 = 144. \qquad (2)$

$(2)-(1),$

$$-32x + 256 = 44.$$
$$32x = 212.$$
$$x = 6\tfrac{5}{8}.$$
$$\therefore y = \sqrt{100 - (6\tfrac{5}{8})^2}$$
$$= \sqrt{\tfrac{3591}{64}} = \tfrac{3}{8}\sqrt{399}.$$



7. (*a*) In AOB, $AO = 15$ and $AK = \frac{15}{2}$.
Therefore $OK = \frac{15}{2}\sqrt{3}$ and the area of triangle AOB is $\frac{225}{4}\sqrt{3}$.
Since the hexagon contains six such triangles, its area $= \frac{675}{2}\sqrt{3}$.

(*b*) As in (*a*), $OK = \dfrac{s}{2}\sqrt{3}$, the area of $\triangle AOB = \dfrac{s^2}{4}\sqrt{3}$, and the area of the hexagon $= \dfrac{3s^2}{2}\sqrt{3}$.

8. (*a*) Let $AO = x$. Then $AK = \dfrac{x}{2}$.

$$x^2 - \left(\frac{x}{2}\right)^2 = 625. \quad \text{Whence } x = \frac{50}{3}\sqrt{3}.$$

$$\therefore \text{the area of } AOB = \frac{25 \cdot \frac{50}{3}\sqrt{3}}{2} = \frac{625}{3}\sqrt{3}.$$

$$\therefore \text{the area of hexagon} = \frac{6 \cdot 625}{3}\sqrt{3} = 1250\sqrt{3}.$$

(*b*) As in (*a*), $x^2 - \dfrac{x^2}{4} = h^2. \quad \therefore x = \dfrac{2h}{3}\sqrt{3}.$

$$\text{Then the area of } AOB = \frac{h \cdot \frac{2h}{3}}{2}\sqrt{3} = \frac{h^2}{3}\sqrt{3}.$$

$$\therefore \text{the area of the hexagon} = 6 \cdot \frac{h^2}{3}\sqrt{3} = 2h^2\sqrt{3}.$$

9. Let $ABCD$ be the base of the pyramid, K the vertex, and R the point of intersection of the diagonals of the base. Then $AR = 5\sqrt{2}$. Therefore

$$KR = \sqrt{20^2 - (5\sqrt{2})^2} = 5\sqrt{14}.$$

Hence the volume is $\dfrac{100 \cdot 5\sqrt{14}}{3} = \dfrac{500}{3}\sqrt{14}.$

10. As in Exercise 9, $AR = \sqrt{97}$, and therefore

$$KR = \sqrt{\overline{16}^2 - (\sqrt{97})^2} = \sqrt{159}.$$

11. Let $a =$ the altitude. Then $a^2 = \overline{18}^2 - (\frac{18}{2})^2$. Whence $a = 9\sqrt{3}$. Then $\frac{2}{3} \cdot 9\sqrt{3} = 6\sqrt{3}$, and $\frac{1}{3} \cdot 9\sqrt{3} = 3\sqrt{3}$.

12. As in Exercise 11, $CR = 6\sqrt{3}$, $CK = 4\sqrt{3}$, and therefore

$$DK = \sqrt{12^2 - (4\sqrt{3})^2} = \sqrt{144 - 48} = \sqrt{96} = 4\sqrt{6}.$$

13. As in Exercise 12, $CR = \frac{15}{2}\sqrt{3}$, $CK = 5\sqrt{3}$, $DK = 5\sqrt{6}$, and area of $ABC = \frac{225}{4}\sqrt{3}$. Therefore the volume of the pyramid

$$= \frac{5\sqrt{6} \cdot \frac{225}{4}\sqrt{3}}{3} = \frac{1125}{4}\sqrt{2}.$$

14. As in Exercise 12, $CR = \dfrac{e}{2}\sqrt{3}$, $CK = \dfrac{e}{3}\sqrt{3}$.

$$\therefore\ DK = \sqrt{e^2 - \left(\frac{e}{3}\sqrt{3}\right)^2} = \frac{e}{3}\sqrt{6}.$$

Area of $ABC = \dfrac{e^2}{4}\sqrt{3}$. Volume $= \dfrac{\frac{e}{3}\sqrt{6} \cdot \frac{e^2}{4}\sqrt{3}}{3} = \dfrac{e^3}{12}\sqrt{2}.$

15. Let $ABCDEF$ be the base, V the vertex, and OK the apothem. Then $VO = \sqrt{\overline{16}^2 - \overline{10}^2} = 2\sqrt{39}$. As in Exercise 7, area $ABCDEF = 150\sqrt{3}$. Therefore the volume of the pyramid

$$= \frac{2\sqrt{39} \cdot 150\sqrt{3}}{3} = 300\sqrt{13}.$$

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1. $3^{-2} = \dfrac{1}{3^2} = \dfrac{1}{9}.$

2. $4^{-3} = \dfrac{1}{4^3} = \dfrac{1}{64}.$

3. $2^{-4} \cdot 3^0 = \dfrac{1}{2^4} \cdot 1 = \dfrac{1}{16}.$

4. $2^{-2} \cdot 3^{-4} = \dfrac{1}{2^2 \cdot 3^4} = \dfrac{1}{324}.$

5. $7 \cdot 7^0 \cdot 0 = 0.$

6. $\left(\dfrac{1}{2}\right)^{-3} = \dfrac{1}{(\frac{1}{2})^3} = \dfrac{1}{\frac{1}{8}} = 8.$

7. $\left(\frac{2}{3}\right)^{-2} \cdot 4^0 = \frac{1}{(\frac{2}{3})^2} = \frac{1}{\frac{4}{9}} = \frac{9}{4}.$

8. $\left(\frac{4}{5}\right)^{-3}\left(\frac{10}{3}\right)^{-2} = \frac{1}{(\frac{4}{5})^3(\frac{10}{3})^2}$

$= \frac{1}{\frac{64 \cdot 100}{125 \cdot 9}} = \frac{1}{\frac{256}{45}} = \frac{45}{256}.$

9. $\frac{2}{3^{-2}} = 2 \cdot 3^2 = 18.$

10. $\frac{3}{3^0} = \frac{3}{1} = 3.$

11. $\frac{12}{4^{-1}} = 12 \cdot 4^1 = 48.$

12. $5 \cdot 2^0 - (5 \cdot 2)^0 = 5 - 1 = 4.$

13. $\frac{4^{-2} \cdot 3^{-2}}{6^{-2}} = \frac{6^2}{4^2 \cdot 3^2} = \frac{2^2}{4^2} = \frac{1}{4}.$

14. $(m - n)^0 = 1$ if $m \neq n$.

15. $32^{-\frac{2}{5}} = \frac{1}{32^{\frac{2}{5}}} = \frac{1}{2^2} = \frac{1}{4}.$

16. $0^5 \cdot 5^0 = 0.$

17. $4^{-\frac{1}{2}} = \frac{1}{4^{\frac{1}{2}}} = \frac{1}{2}.$

18. $8^{-\frac{1}{3}} = \frac{1}{8^{\frac{1}{3}}} = \frac{1}{2}.$

19. $16^{-\frac{1}{4}} = \frac{1}{16^{\frac{1}{4}}} = \frac{1}{2}.$

20. $8^{-\frac{2}{3}} = \frac{1}{8^{\frac{2}{3}}} = \frac{1}{2^2} = \frac{1}{4}.$

21. $16^{-\frac{3}{2}} = \frac{1}{16^{\frac{3}{2}}} = \frac{1}{4^3} = \frac{1}{64}.$

22. $25^{1.5} = 25^{\frac{3}{2}} = 5^3 = 125.$

23. $0^3 \cdot 0^{\frac{3}{4}} = 0.$

24. $(-8)^{-\frac{2}{3}} = \frac{1}{(-8)^{\frac{2}{3}}} = \frac{1}{(-2)^2} = \frac{1}{4}.$

25. $(-64)^{-\frac{2}{3}} = \frac{1}{(-64)^{\frac{2}{3}}} = \frac{1}{(-4)^2} = \frac{1}{16}.$

26. $(-32)^{\frac{1}{5}} = -2.$

27. $(32)^{-.4} = \frac{1}{(32)^{\frac{2}{5}}} = \frac{1}{2^2} = \frac{1}{4}.$

28. $(-125)^{-\frac{2}{3}} = \frac{1}{(-125)^{\frac{2}{3}}} = \frac{1}{(-5)^2} = \frac{1}{25}.$

29. $\sqrt[3]{27^{-2}} = \sqrt[3]{3^{-6}} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}.$

30. $\sqrt[3]{8^{-2}} = \sqrt[3]{2^{-6}} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}.$

31. $(\sqrt[3]{-8})^2 = (-2)^2 = 4.$

32. $\left(\frac{1}{2}\right)^{-4} \cdot \left(\frac{1}{3}\right)^{-3} \cdot \left(\frac{1}{2}\right)^0 = \frac{1}{(\frac{1}{2})^4(\frac{1}{3})^3} = \frac{1}{\frac{1}{16 \cdot 27}} = 432.$

33. $\left(\frac{1}{6}\right)^{-2} = \frac{1}{(\frac{1}{6})^2} = \frac{1}{\frac{1}{36}} = 36.$

34. $(.04)^{\frac{3}{2}} = (.2)^3 = .008.$

35. $(.027)^{-\frac{2}{3}} = \dfrac{1}{(.027)^{\frac{2}{3}}} = \dfrac{1}{(.3)^2} = \dfrac{1}{.09} = \dfrac{100}{9}.$

36. $(.064)^{-\frac{1}{3}} = \dfrac{1}{(.064)^{\frac{1}{3}}} = \dfrac{1}{.4} = \dfrac{5}{2}.$

37. $(.00032)^{\frac{2}{5}} = (.2)^2 = .04.$

38. $\dfrac{\sqrt[3]{9^{-3}} \cdot \sqrt[2]{9^{-2}}}{3^{-4}} = \dfrac{9^{-1} \cdot 9^{-1}}{3^{-4}} = \dfrac{3^4}{9 \cdot 9} = 1.$

39. $\dfrac{2^{-1}}{2^{-2} - 2^{-3}} = \dfrac{\frac{1}{2}}{\dfrac{1}{2^2} - \dfrac{1}{2^3}} = \dfrac{\frac{1}{2}}{\frac{1}{8}} = 4.$

40. $\dfrac{3^{-2} - 2^{-2}}{3^{-1} - 2^{-1}} = \dfrac{\dfrac{1}{3^2} - \dfrac{1}{2^2}}{\frac{1}{3} - \frac{1}{2}} = \dfrac{1}{3} + \dfrac{1}{2} = \dfrac{5}{6}.$

41. $\dfrac{2^{-1} + 3^{-1}}{2^{-3} + 3^{-3}} = \dfrac{\frac{1}{2} + \frac{1}{3}}{\dfrac{1}{2^3} + \dfrac{1}{3^3}} = \dfrac{1}{\dfrac{1}{2^2} - \dfrac{1}{2} \cdot \dfrac{1}{3} + \dfrac{1}{3^2}} = \dfrac{1}{\frac{7}{36}} = \dfrac{36}{7}.$

42. $\dfrac{3^{-3} - 2^{-3}}{3^{-1} - 2^{-1}} = \dfrac{\dfrac{1}{3^3} - \dfrac{1}{2^3}}{\frac{1}{3} - \frac{1}{2}} = \dfrac{1}{3^2} + \dfrac{1}{3} \cdot \dfrac{1}{2} + \dfrac{1}{2^2} = \dfrac{19}{36}.$

43. $m^{-3} = \dfrac{1}{m^3}.$

44. $2a^{-3} = \dfrac{2}{a^3}.$

45. $3ab^{-2} = \dfrac{3a}{b^2}.$

46. $7x^2y^{-2} = \dfrac{7x^2}{y^2}.$

47. $x^{-1}y^{-2}z = \dfrac{z}{xy^2}.$

48. $4a^3b^{-2}c^2 = \dfrac{4a^3c^2}{b^2}.$

49. $\dfrac{3}{a^{-2}} = 3a^2.$

50. $\dfrac{4x}{y^{-3}} = 4xy^3.$

51. $\dfrac{4c^0}{xy^{-2}} = \dfrac{4y^2}{x}.$

52. $\dfrac{4a^{-2}b^0}{y^{-5}} = \dfrac{4y^5}{a^2}.$

53. $\dfrac{5^{-1}(ab)^0}{10^{-2}b^2} = \dfrac{10^2}{5b^2} = \dfrac{20}{b^2}.$

54. $\dfrac{3a^3b^{-2}c}{4a^{-2}b^0} = \dfrac{3a^5c}{4b^2}.$

55. $\dfrac{12x^2y^{-1}}{2yx^{-1}} = \dfrac{6x^3}{y^2}.$

56. $\dfrac{10^{-1}a}{bc^3} = \dfrac{a}{10bc^3}.$

57. $\dfrac{4a^{-2}bc^{-3}}{6a^{-2}b^{-3}c^0} = \dfrac{2b^4}{3c^3}.$

58. $\dfrac{4^{-3} \cdot r^{-6} \cdot s^6}{s^{-2}r^{-2}t^3} = \dfrac{s^8}{4^3 \cdot r^4 \cdot t^3} = \dfrac{s^8}{64r^4t^3}.$

59. $\dfrac{2s^{-1}}{m^na^{-b}} = \dfrac{2a^b}{m^ns}.$

60. $\dfrac{5x^2y^a}{x^{-m}y^{-4}} = 5x^{(m+2)}y^{(a+4)}.$

61. $\dfrac{2}{a^{-2}-b^{-2}}=\dfrac{2}{\dfrac{1}{a^2}-\dfrac{1}{b^2}}=\dfrac{2}{\dfrac{b^2-a^2}{a^2b^2}}=\dfrac{2a^2b^2}{b^2-a^2}.$

62. $\dfrac{3}{a^{-1}+b^{-1}}=\dfrac{3}{\dfrac{1}{a}+\dfrac{1}{b}}=\dfrac{3}{\dfrac{b+a}{ab}}=\dfrac{3ab}{a+b}.$

63. $\dfrac{a}{a^{-2}-b^{-2}}=\dfrac{a}{\dfrac{1}{a^2}-\dfrac{1}{b^2}}=\dfrac{a}{\dfrac{b^2-a^2}{a^2b^2}}=\dfrac{a^3b^2}{b^2-a^2}.$

64. $\dfrac{5se^2}{s^{-2}+e^{-2}}=\dfrac{5se^2}{\dfrac{1}{s^2}+\dfrac{1}{e^2}}=\dfrac{5se^2}{\dfrac{e^2+s^2}{s^2e^2}}=\dfrac{5s^3e^4}{e^2+s^2}.$

65. $\dfrac{a^{-2}}{a^{-2}+b^{-2}}=\dfrac{1}{a^2\left(\dfrac{1}{a^2}+\dfrac{1}{b^2}\right)}=\dfrac{1}{\dfrac{a^2+b^2}{b^2}}=\dfrac{b^2}{a^2+b^2}.$

66. $\dfrac{a^{-3}b^{-3}}{a^{-3}+b^{-3}}=\dfrac{1}{a^3b^3\left(\dfrac{1}{a^3}+\dfrac{1}{b^3}\right)}=\dfrac{1}{b^3+a^3}.$

67. $\dfrac{a^{-2}-b^{-2}}{a^{-1}+b^{-1}}=\dfrac{\dfrac{1}{a^2}-\dfrac{1}{b^2}}{\dfrac{1}{a}+\dfrac{1}{b}}=\dfrac{1}{a}-\dfrac{1}{b}=\dfrac{b-a}{ab}.$

68. $\dfrac{a^{-4}-b^{-4}}{a^{-2}-b^{-2}}=\dfrac{\dfrac{1}{a^4}-\dfrac{1}{b^4}}{\dfrac{1}{a^2}-\dfrac{1}{b^2}}=\dfrac{1}{a^2}+\dfrac{1}{b^2}=\dfrac{b^2+a^2}{a^2b^2}.$

69. $\dfrac{a^{-1}+b^{-1}}{a^{-3}+b^{-3}}=\dfrac{\dfrac{1}{a}+\dfrac{1}{b}}{\dfrac{1}{a^3}+\dfrac{1}{b^3}}=\dfrac{1}{\dfrac{1}{a^2}-\dfrac{1}{ab}+\dfrac{1}{b^2}}=\dfrac{1}{\dfrac{b^2-ab+a^2}{a^2b^2}}=\dfrac{a^2b^2}{b^2-ab+a^2}.$

70. $\dfrac{a^{-3}-27^{-1}}{a^{-1}-3^{-1}}=a^{-2}+a^{-1}3^{-1}+3^{-2}=\dfrac{1}{a^2}+\dfrac{1}{3a}+\dfrac{1}{9}=\dfrac{a^2+3a+9}{9a^2}.$

71. $\dfrac{2xy}{z^2}=2xyz^{-2}.$ 72. $\dfrac{4a^3}{b^3}=4a^3b^{-3}.$ 73. $\dfrac{3x}{a^{-2}b^4}=3a^2b^{-4}x.$

74. $\dfrac{4sc^{-3}}{2^{-1}s^{-2}}=2^3\cdot s^3c^{-3}=8s^3c^{-3}.$ 76. $\dfrac{7x^{-1}y^2}{2^{-1}y^3}=14x^{-1}y^{-1}.$

75. $\dfrac{12a^2b^3}{4xy^2}=3a^2b^3x^{-1}y^{-2}.$ 77. $\dfrac{5ac^{-2}}{c(x-y)^2}=5ac^{-3}(x-y)^{-2}.$

78. $\dfrac{7\,m^{-3}n^{\frac{1}{2}}}{m^{\frac{2}{3}}(m-n)^0} = 7\,m^{-\frac{11}{3}}n^{\frac{1}{2}}.$

79. $\dfrac{1}{5\,a^2(c+d)^{-3}} = 5^{-1}a^{-2}(c+d)^3.$

80. $\dfrac{a\,(x-y)^{-2}}{bcx\,(x-y)} = ab^{-1}c^{-1}x^{-1}(x-y)^{-3}.$

81. $\dfrac{42\,m^{-n}n^{2m}}{56\,m^{-2n}n^{-3n}} = 3\cdot 4^{-1}m^n n^{2m+3n}.$

82. $\dfrac{r^{-1}s^2}{r^{-2}s^3(s-r)^3} = rs^{-1}(s-r)^{-3}.$

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1. $x^4\cdot x^{-2} = x^2.$
2. $x^{\frac{1}{3}}\cdot x^{\frac{1}{2}} = x^{\frac{5}{6}}.$
3. $x^{\frac{1}{2}}\cdot x^{\frac{1}{3}}\cdot x^0 = x^{\frac{1}{2}+\frac{1}{3}} = x^{\frac{5}{6}}.$
4. $7\,x^0\cdot x^{\frac{2}{3}}\cdot x^{\frac{3}{2}} = 7\,x^{\frac{13}{6}}.$
5. $\sqrt[3]{x}\cdot x^{\frac{1}{4}} = x^{\frac{1}{3}+\frac{1}{4}} = x^{\frac{7}{12}}.$
6. $\sqrt[3]{x}\cdot\sqrt[4]{x^3} = x^{\frac{1}{3}+\frac{3}{4}} = x^{\frac{13}{12}}.$
7. $a\sqrt{ax}\cdot a^{\frac{1}{2}}\sqrt{ax^{-1}}$
   $= aa^{\frac{1}{2}}x^{\frac{1}{2}}a^{\frac{1}{2}}a^{\frac{1}{2}}x^{-\frac{1}{2}} = a^{\frac{5}{2}}.$
8. $e^x\cdot e^{-x} = e^0 = 1.$
9. $e^{a-3}\cdot e^3 = e^a.$
10. $e^{3-2a}\cdot e^{2+3a} = e^{5+a}.$
11. $x\cdot x^a\cdot x^b\cdot x^0\cdot x^{2b-3a}$
    $= x^{3b-2a+1}.$
12. $(2^3)^3 = 2^9 = 512.$
13. $(2^{-3})^{-2} = 2^6 = 64.$
14. $(x^2)^3 = x^6.$
15. $(x^{\frac{1}{3}})^{\frac{1}{2}} = x^{\frac{1}{6}}.$
16. $(3\,x^{-2})^3 = 27\,x^{-6} = \dfrac{27}{x^6}.$
17. $(25\,a^4b^6)^{-\frac{1}{2}} = 5^{-1}a^{-2}b^{-3} = \dfrac{1}{5\,a^2b^3}.$
18. $(a^2b)^3(a+b) = a^6b^3(a+b) = a^7b^3 + a^6b^4.$
19. $(a^3)^{2x}(a^2)^{3x} = a^{6x}\cdot a^{6x} = a^{12x}.$
20. $(a^3)^{x+y}(a^2)^{y-x} = a^{3x+3y}\cdot a^{2y-2x} = a^{x+5y}.$
21. $(x^2 - x^{-2})\,x^3 = x^5 - x.$
22. $(x^4 - a^4)\,x^{-2}a^{-2} = x^2a^{-2} - x^{-2}a^2 = \dfrac{x^2}{a^2} - \dfrac{a^2}{x^2}.$
23. $(x^{\frac{1}{3}} + y^{\frac{1}{3}})\,x^{\frac{2}{3}}y^{\frac{1}{3}} = xy^{\frac{1}{3}} + x^{\frac{2}{3}}y^{\frac{2}{3}}.$
24. $(x^{\frac{1}{3}} + y^{\frac{1}{3}})(x^{\frac{1}{3}} - y^{\frac{1}{3}}) = x^{\frac{2}{3}} - y^{\frac{2}{3}}.$
25. $(a^{-2}+3)(a^{-2}-5) = a^{-4} - 2\,a^{-2} - 15 = \dfrac{1}{a^4} - \dfrac{2}{a^2} - 15.$
26. $(a^{-1} - a)^2 = a^{-2} - 2\,a^{-1}a + a^2 = \dfrac{1}{a^2} - 2 + a^2.$
27. $(a^3 - 2\,a^{-2})^3 = a^9 - 3\,(a^3)^2(2\,a^{-2}) + 3\,(a^3)(2\,a^{-2})^2 - (2\,a^{-2})^3$
    $= a^9 - 6\,a^4 + 12\,a^{-1} - 8\,a^{-6}$
    $= a^9 - 6\,a^4 + \dfrac{12}{a} - \dfrac{8}{a^6}.$

28. $(a^{-1}-2a+3a^{-2})^2 = a^{-2}+4a^2+9a^{-4}-4+6a^{-3}-12a^{-1}$
$$= \frac{1}{a^2}+4a^2+\frac{9}{a^4}-4+\frac{6}{a^3}-\frac{12}{a}.$$

29. $(e^x+e^{-x})^2 = e^{2x}+2+e^{-2x} = e^{2x}+2+\dfrac{1}{e^{2x}}.$

30. $(a^{-1}+b^{-2})\left(\dfrac{1}{a}-\dfrac{1}{b^2}\right)=\left(\dfrac{1}{a}+\dfrac{1}{b^2}\right)\left(\dfrac{1}{a}-\dfrac{1}{b^2}\right)=\dfrac{1}{a^2}-\dfrac{1}{b^4}.$

31. $(e^{2x}-2+e^{-2x})^2 = e^{4x}+4+e^{-4x}-4e^{2x}+2-4e^{-2x}$
$$= e^{4x}+6+\frac{1}{e^{4x}}-4e^{2x}-\frac{4}{e^{2x}}.$$

32. $(3a^{-\frac{1}{2}}+2a^{\frac{2}{3}})^2 = 9a^{-1}+12a^{\frac{1}{6}}+4a^{\frac{4}{3}}$
$$= \frac{9}{a}+12a^{\frac{1}{6}}+4a^{\frac{4}{3}}.$$

33. $(a^{\frac{4}{5}}-2a^{\frac{2}{5}}x+4x^2)(a^{\frac{2}{5}}+2x) = a^{\frac{6}{5}}+8x^3.$

34. $(x^{\frac{1}{3}}+2y^{\frac{1}{3}})(x^{\frac{2}{3}}-2x^{\frac{1}{3}}y^{\frac{1}{3}}+4y^{\frac{2}{3}}) = x+8y.$

35. $(a^{\frac{1}{2}}+a^{\frac{1}{4}}b^{\frac{1}{4}}+b^{\frac{1}{2}})(a^{\frac{1}{2}}-a^{\frac{1}{4}}b^{\frac{1}{4}}+b^{\frac{1}{2}}) = a+a^{\frac{1}{2}}b^{\frac{1}{2}}+b.$

36. $(x-x^{\frac{1}{2}}y^{-\frac{1}{2}}+y^{-1})(x+x^{\frac{1}{2}}y^{-\frac{1}{2}}+y^{-1}) = x^2+xy^{-1}+y^{-2}$
$$= x^2+\frac{x}{y}+\frac{1}{y^2}.$$

37. $(\sqrt[3]{a^5}-3\sqrt{a})(\sqrt[3]{a^5}-3\sqrt{a}) = \sqrt[3]{a^{10}}-6\sqrt[3]{a^5}\cdot\sqrt{a}+9a$
$$= a^{\frac{10}{3}}-6a^{\frac{13}{6}}+9a.$$

38. $(5\sqrt{c^{-5}}\sqrt[3]{d^{-3}}-a\sqrt[5]{c^{-2}})^3 = (5c^{-\frac{5}{2}}d^{-1}-ac^{-\frac{2}{5}})^3$
$$= \frac{125}{c^{\frac{15}{2}}d^3}-\frac{75a}{c^{\frac{27}{5}}d^2}+\frac{15a^2}{c^{\frac{33}{10}}d}-\frac{a^3}{c^{\frac{6}{5}}}.$$

39. $\left(\sqrt{a}+\dfrac{\sqrt{b}}{c}\right)\left(\sqrt{a}+\dfrac{\sqrt{b}}{c}\right)^2=\left(a^{\frac{1}{2}}+\dfrac{b^{\frac{1}{2}}}{c}\right)^3=a^{\frac{3}{2}}+\dfrac{3ab^{\frac{1}{2}}}{c}+\dfrac{3a^{\frac{1}{2}}b}{c^2}+\dfrac{b^{\frac{3}{2}}}{c^3}.$

40.
$$\begin{array}{llll}
m^{n-1}-5m^{n-2}a^n+25a^{2n} & & & \\
\underline{m+5a^n} & & & \\
m^n-5m^{n-1}a^n+25ma^{2n} & & & \\
\quad\quad 5m^{n-1}a^n & & -25m^{n-2}a^{2n}+125a^{3n} & \\
\hline
m^n & +25ma^{2n} & -25m^{n-2}a^{2n}+125a^{3n} &
\end{array}$$

41.
$$\begin{array}{lll}
25x^{-4}+15x^{-2}y^{-8}+9y^{-16} & & \\
\underline{5x^{-2}-3y^{-8}} & & \\
125x^{-6}+75x^{-4}y^{-8}+45x^{-2}y^{-16} & & \\
\quad\quad -75x^{-4}y^{-8}-45x^{-2}y^{-16} & -27y^{-24} & \\
\hline
125x^{-6} & -27y^{-24} & = \dfrac{125}{x^6}-\dfrac{27}{y^{24}}.
\end{array}$$

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1. $x^4 \div x^6 = x^{-2} = \dfrac{1}{x^2}.$

2. $x^3 \div x^{\frac{1}{3}} = x^{\frac{8}{3}}.$

3. $x^{\frac{1}{2}} \div x^2 = x^{-\frac{3}{2}} = \dfrac{1}{x^{\frac{3}{2}}}.$

4. $\dfrac{ax^{\frac{2}{3}}}{a^{\frac{1}{2}}x^{\frac{1}{2}}} = a^{\frac{1}{2}}x^{\frac{1}{6}}.$

5. $\dfrac{ax - a^2x^2}{a^2x^{\frac{1}{2}}} = \dfrac{x^{\frac{1}{2}}}{a} - x^{\frac{3}{2}}.$

6. $(x^a - 2\,x^{2a-1} + 3\,x^{3a-2}) \div x^{2a-1} = x^{1-a} - 2 + 3\,x^{a-1}.$

7. $(6\,a^{3+4n} - 9\,a^{n-2} + 5\,a^{2-n}) \div 3\,a^{n-2} = 2\,a^{5+3n} - 3 + \frac{5}{3}\,a^{4-2n}.$

8. $(x - y) \div (x^{\frac{1}{2}} - y^{\frac{1}{2}}) = x^{\frac{1}{2}} + y^{\frac{1}{2}}.$

9.
$$
\begin{array}{l|l}
x - 8\,y & x^{\frac{1}{3}} - 2\,y^{\frac{1}{3}} \\ \cline{2-2}
\underline{x - 2\,x^{\frac{2}{3}}y^{\frac{1}{3}}} & \underline{x^{\frac{2}{3}} + 2\,x^{\frac{1}{3}}y^{\frac{1}{3}} + 4\,y^{\frac{2}{3}}} \\
\quad 2\,x^{\frac{2}{3}}y^{\frac{1}{3}} & \\
\quad \underline{2\,x^{\frac{2}{3}}y^{\frac{1}{3}} - 4\,x^{\frac{1}{3}}y^{\frac{2}{3}}} & \\
\qquad\qquad 4\,x^{\frac{1}{3}}y^{\frac{2}{3}} - 8\,y & \\
\qquad\qquad \underline{4\,x^{\frac{1}{3}}y^{\frac{2}{3}} - 8\,y} &
\end{array}
$$

10.
$$
\begin{array}{l|l}
16\,x^2 - 81\,y^2 & 2\,x^{\frac{1}{2}} + 3\,y^{\frac{1}{2}} \\ \cline{2-2}
\underline{16\,x^2 + 24\,x^{\frac{3}{2}}y^{\frac{1}{2}}} & \underline{8\,x^{\frac{3}{2}} - 12\,xy^{\frac{1}{2}} + 18\,x^{\frac{1}{2}}y - 27\,y^{\frac{3}{2}}} \\
\quad -24\,x^{\frac{3}{2}}y^{\frac{1}{2}} & \\
\quad \underline{-24\,x^{\frac{3}{2}}y^{\frac{1}{2}} - 36\,xy} & \\
\qquad\qquad 36\,xy & \\
\qquad\qquad \underline{36\,xy + 54\,x^{\frac{1}{2}}y^{\frac{3}{2}}} & \\
\qquad\qquad\qquad -54\,x^{\frac{1}{2}}y^{\frac{3}{2}} - 81\,y^2 & \\
\qquad\qquad\qquad \underline{-54\,x^{\frac{1}{2}}y^{\frac{3}{2}} - 81\,y^2} &
\end{array}
$$

11.
$$
\begin{array}{l|l}
a^3 - b^2 & a^{\frac{1}{2}} + b^{\frac{1}{3}} \\ \cline{2-2}
\underline{a^3 + a^{\frac{5}{2}}b^{\frac{1}{3}}} & \underline{a^{\frac{5}{2}} - a^2b^{\frac{1}{3}} + a^{\frac{3}{2}}b^{\frac{2}{3}} - ab + a^{\frac{1}{2}}b^{\frac{4}{3}} - b^{\frac{5}{3}}} \\
\quad -a^{\frac{5}{2}}b^{\frac{1}{3}} & \\
\quad \underline{-a^{\frac{5}{2}}b^{\frac{1}{3}} - a^2b^{\frac{2}{3}}} & \\
\qquad\qquad a^2b^{\frac{2}{3}} & \qquad\qquad ab^{\frac{4}{3}} - b^2 \\
\qquad\qquad \underline{a^2b^{\frac{2}{3}} + a^{\frac{3}{2}}b} & \qquad\qquad \underline{ab^{\frac{4}{3}} + a^{\frac{1}{2}}b^{\frac{5}{3}}} \\
\qquad\qquad\qquad -a^{\frac{3}{2}}b & \qquad\qquad\qquad -a^{\frac{1}{2}}b^{\frac{5}{3}} - b^2 \\
\qquad\qquad\qquad \underline{-a^{\frac{3}{2}}b - ab^{\frac{4}{3}}} & \qquad\qquad\qquad \underline{-a^{\frac{1}{2}}b^{\frac{5}{3}} - b^2}
\end{array}
$$

12.
$$\begin{array}{l|l}
a^2 + ab^{-1} + b^{-2} & a - a^{\frac{1}{2}}b^{-\frac{1}{2}} + b^{-1} \\ \cline{2-2}
a^2 - a^{\frac{3}{2}}b^{-\frac{1}{2}} + ab^{-1} & a + a^{\frac{1}{2}}b^{-\frac{1}{2}} + b^{-1} \\ \hline
\quad a^{\frac{3}{2}}b^{-\frac{1}{2}} & \\
\quad a^{\frac{3}{2}}b^{-\frac{1}{2}} - ab^{-1} + a^{\frac{1}{2}}b^{-\frac{3}{2}} & \\ \hline
\qquad ab^{-1} - a^{\frac{1}{2}}b^{-\frac{3}{2}} + b^{-2} & \\
\qquad ab^{-1} - a^{\frac{1}{2}}b^{-\frac{3}{2}} + b^{-2} & \\ \hline
\end{array}$$

13.
$$\begin{array}{l|l}
e^{2x} - 2 + e^{-2x} & e^x - e^{-x} \\ \cline{2-2}
e^{2x} - 1 & e^x - e^{-x} \\ \hline
\quad -1 + e^{-2x} & \\
\quad -1 + e^{-2x} & \\ \hline
\end{array}$$

14.
$$\begin{array}{l|l}
e^{3x} + 3e^x + \frac{3}{e^x} + \frac{1}{e^{3x}} & e^x + e^{-x} \\ \cline{2-2}
e^{3x} + e^x & e^{2x} + 2 + \frac{1}{e^{2x}} \\ \hline
\quad 2e^x + \frac{3}{e^x} & \\
\quad 2e^x + \frac{2}{e^x} & \\ \hline
\qquad \frac{1}{e^x} + \frac{1}{e^{3x}} & \\
\qquad \frac{1}{e^x} + \frac{1}{e^{3x}} & \\ \hline
\end{array}$$

15.
$$\begin{array}{l|l}
a + a^{\frac{3}{5}}b^{\frac{2}{3}} + 2a^{\frac{2}{5}}b^{\frac{1}{3}} + 2b & a^{\frac{3}{5}} + 2b^{\frac{1}{3}} \\ \cline{2-2}
a + 2a^{\frac{2}{5}}b^{\frac{1}{3}} & a^{\frac{2}{5}} + b^{\frac{2}{3}} \\ \hline
\quad a^{\frac{3}{5}}b^{\frac{2}{3}} + 2b & \\
\quad a^{\frac{3}{5}}b^{\frac{2}{3}} + 2b & \\ \hline
\end{array}$$

16.
$$\begin{array}{l|l}
m^4 + 7m^2 + 8 - 7m^{-2} + m^{-4} & m^2 + 5 - m^{-2} \\ \cline{2-2}
m^4 + 5m^2 - 1 & m^2 + 2 - m^{-2} \\ \hline
\quad 2m^2 + 9 - 7m^{-2} & \\
\quad 2m^2 + 10 - 2m^{-2} & \\ \hline
\qquad -1 - 5m^{-2} + m^{-4} & \\
\qquad -1 - 5m^{-2} + m^{-4} & \\ \hline
\end{array}$$

17.
$$\begin{array}{l|l}
2x^{2n-1} - x^{3n-2} + 9x^{5n-4} & 2x^{n-1} + 3x^{2n-2} \\ \cline{2-2}
2x^{2n-1} + 3x^{3n-2} & x^n - 2x^{2n-1} + 3x^{3n-2} \\ \hline
\quad -4x^{3n-2} + 9x^{5n-4} & \\
\quad -4x^{3n-2} - 6x^{4n-3} & \\ \hline
\qquad 6x^{4n-3} + 9x^{5n-4} & \\
\qquad 6x^{4n-3} + 9x^{5n-4} & \\ \hline
\end{array}$$

18.
$$\begin{array}{l|l}
-16a^{\frac{19}{5}}b^{\frac{3}{4}}+40ab-25a^{-\frac{9}{5}}b^{\frac{5}{4}} & -4a^{\frac{12}{5}}b^{-\frac{5}{8}}+5a^{-\frac{2}{5}}b^{-\frac{3}{8}} \\ \cline{2-2}
\underline{-16a^{\frac{19}{5}}b^{\frac{3}{4}}+20ab} & 4a^{\frac{7}{5}}b^{\frac{11}{8}}-5a^{-\frac{7}{5}}b^{\frac{13}{8}} \\
\qquad\qquad\quad 20ab-25a^{-\frac{9}{5}}b^{\frac{5}{4}} & \\
\qquad\qquad\quad \underline{20ab-25a^{-\frac{9}{5}}b^{\frac{5}{4}}} &
\end{array}$$

19.
$$\begin{array}{l|l}
x^{3a}+11x^{2a}+38x^{a}+33-28x^{-a}+2x^{-2a} & x^{a}+4-2x^{-a} \\ \cline{2-2}
\underline{x^{3a}+\ \ 4x^{2a}-\ \ 2x^{a}} & x^{2a}+7x^{a}+12-x^{-a} \\
\qquad\quad 7x^{2a}+40x^{a}+33 & \\
\qquad\quad \underline{7x^{2a}+28x^{a}-14} & \\
\qquad\qquad\qquad 12x^{a}+47-28x^{-a} & \\
\qquad\qquad\qquad \underline{12x^{a}+48-24x^{-a}} & \\
\qquad\qquad\qquad\qquad -1-4x^{-a}+2x^{-2a} & \\
\qquad\qquad\qquad\qquad \underline{-1-4x^{-a}+2x^{-2a}} &
\end{array}$$

20.
$$\begin{array}{l|l}
25m^{3}-4mn^{\frac{4}{3}}+4mn^{2}-mn^{\frac{8}{3}} & 5m^{\frac{3}{2}}-2n^{\frac{2}{3}}m^{\frac{1}{2}}+m^{\frac{1}{2}}n^{\frac{4}{3}} \\ \cline{2-2}
\underline{25m^{3}-10m^{2}n^{\frac{2}{3}}+5m^{2}n^{\frac{4}{3}}} & 5m^{\frac{3}{2}}+2m^{\frac{1}{2}}n^{\frac{2}{3}}-m^{\frac{1}{2}}n^{\frac{4}{3}} \\
\qquad +10m^{2}n^{\frac{2}{3}}-5m^{2}n^{\frac{4}{3}}-4mn^{\frac{4}{3}}+4mn^{2} & \\
\qquad \underline{+10m^{2}n^{\frac{2}{3}}\qquad\qquad\ \ -4mn^{\frac{4}{3}}+2mn^{2}} & \\
\qquad\qquad\quad -5m^{2}n^{\frac{4}{3}}\qquad\qquad +2mn^{2}-mn^{\frac{8}{3}} & \\
\qquad\qquad\quad \underline{-5m^{2}n^{\frac{4}{3}}\qquad\qquad +2mn^{2}-mn^{\frac{8}{3}}} &
\end{array}$$

21.
$$\begin{array}{l|l}
e^{3x}-3e^{x}+3e^{-x}-e^{-3x} & e^{x}-e^{-x} \\ \cline{2-2}
\underline{e^{3x}-\ \ e^{x}} & e^{2x}-2+e^{-2x} \\
\quad -2e^{x} & \\
\quad \underline{-2e^{x}+2e^{-x}} & \\
\qquad\qquad e^{-x}-e^{-3x} & \\
\qquad\qquad \underline{e^{-x}-e^{-3x}} &
\end{array}$$

22.
$$\begin{array}{l|l}
e^{5x}-5e^{3x}+10e^{x}-10e^{-x}+5e^{-3x}-e^{-5x} & e^{2x}-2+e^{-2x} \\ \cline{2-2}
\underline{e^{5x}-2e^{3x}+\quad e^{x}} & e^{3x}-3e^{x}+3e^{-x}-e^{-3x} \\
\quad -3e^{3x}+9e^{x}-10e^{-x} & \\
\quad \underline{-3e^{3x}+6e^{x}-\ \ 3e^{-x}} & \\
\qquad\qquad 3e^{x}-7e^{-x}+5e^{-3x} & \\
\qquad\qquad \underline{3e^{x}-6e^{-x}+3e^{-3x}} & \\
\qquad\qquad\quad -e^{-x}+2e^{-3x}-e^{-5x} & \\
\qquad\qquad\quad \underline{-e^{-x}+2e^{-3x}-e^{-5x}} &
\end{array}$$

23. The equation $e^{2x}=1$ is equivalent to the two equations $e^{x}=-1$ and $e^{x}=+1$. While $x=0$ is one root of $e^{2x}=1$, it is not a root of $e^{x}=-1$. Therefore the error consists in setting the positive square root of a number equal to the negative square root of the same number.

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1. $a^3 \cdot a^2 = a^n$. $\therefore n = 5$.

2. $2^3 \cdot 2^2 = 2^n$. $\therefore n = 5$.

3. $2^4 \cdot 2^3 = 2^n$. $\therefore n = 7$.

4. $2^5 \cdot 2^n = 2^{11}$. $\therefore n = 6$.

5. $3^2 \cdot 2^4 = 9 \cdot 2^n$.
   $2^4 = 2^n$. $\therefore n = 4$.

6. $4^2 \cdot 2^2 = 2^n$.
   $2^4 \cdot 2^2 = 2^n$. $\therefore n = 6$.

7. $4^3 \cdot 2^3 = 2^n$.
   $2^6 \cdot 2^3 = 2^n$. $\therefore n = 9$.

8. $4^2 \cdot 2^4 = 2^{2n}$.
   $2^4 \cdot 2^4 = 2^{2n}$. $\therefore n = 4$.

9. $3^3 \cdot 9^3 = 3^n$.
   $3^3 \cdot 3^6 = 3^n$. $\therefore n = 9$.

10. $9^2 \cdot 3^n = 3^5$.
    $3^4 \cdot 3^n = 3^5$. $\therefore n = 1$.

11. $2^2 \cdot 2^n = 32 = 2^5$. $\therefore n = 3$.

12. $8^2 \cdot 4^3 = 2^n$.
    $2^6 \cdot 2^6 = 2^n$. $\therefore n = 12$.

13. $8^3 \cdot 4^2 = 2^n$.
    $2^9 \cdot 2^4 = 2^n$. $\therefore n = 13$.

14. $9^3 \cdot 27^2 = 3^n$.
    $3^6 \cdot 3^6 = 3^n$. $\therefore n = 12$.

15. $27^n \cdot 9^2 = 3^{10}$.
    $3^{3n} \cdot 3^4 = 3^{10}$. $\therefore n = 2$.

16. $8^n \cdot 4^{2n} = 2^{14}$.
    $2^{3n} \cdot 2^{4n} = 2^{14}$. $\therefore n = 2$.

17. $3^6 \cdot 9^n = 81^2$.
    $9^3 \cdot 9^n = 9^4$. $\therefore n = 1$.

18. $9^n \cdot 3^3 = 27^n$.
    $3^{2n} \cdot 3^3 = 3^{3n}$. $\therefore n = 3$.

19. $2^{2n+2} \cdot 4^{n+2} = 8^{2n}$.
    $2^{2n+2} \cdot 2^{2n+4} = 2^{6n}$.
    $4n + 6 = 6n$. $\therefore n = 3$.

20. $81 \cdot 27^n = (9^n)^{\frac{n}{2}}$.
    $3^4 \cdot 3^{3n} = 3^{n^2}$.
    $n^2 = 3n + 4$. $\therefore n = 4$ and -1

21. $(25^n)^n = \dfrac{5^{7n}}{(125)^2}$, or $5^{2n^2} = \dfrac{5^{7n}}{5^6}$. $2n^2 = 7n - 6$. $\therefore n = 2$ and $\frac{3}{2}$.

22. $2^{6n+3} \cdot 4^{3n+6} = (8^n)^n$, or $2^{6n+3} \cdot 2^{6n+12} = 2^{3n^2}$.
    $\therefore 3n^2 = 12n + 15$. Whence $n = 5$ and -1.

23. $x^{\frac{2}{3}} = 8$.
    $x^{\frac{1}{3}} = \pm\sqrt{8}$.
    $x = (\pm\sqrt{8})^3 = \pm 16\sqrt{2}$.

24. $x^{-\frac{1}{3}} = 5$.
    $\sqrt[3]{x} = \frac{1}{5}$.
    $x = \frac{1}{125}$.

25. $x^{-\frac{4}{3}} = 256$.
    $x^{\frac{4}{3}} = \frac{1}{256}$.
    $x^{\frac{1}{3}} = \pm\frac{1}{4}$.
    $x = \pm\frac{1}{64}$.

26. $x^{\frac{3}{2}} = -343$.
    $x^{\frac{1}{2}} = -7$.
    $x = 49$.

27. $\frac{1}{2}x^{-\frac{2}{3}} = 2$.
    $x^{-\frac{2}{3}} = 4$.
    $x^{\frac{2}{3}} = \frac{1}{4}$.
    $x^{\frac{1}{3}} = \pm\frac{1}{2}$.
    $x = \frac{1}{8}, -\frac{1}{8}$.

28. $(x^{-\frac{1}{2}})^{-4} = 49.$

$x^2 = 49.$

$x = \pm 7.$

29. $(ax^{\frac{1}{2}})^{-6} = 27.$

$x^{-3} = 27\,a^6.$

$x = \dfrac{1}{3\,a^2}.$

30. $\dfrac{\sqrt[3]{x^{\frac{7}{5}}}}{\sqrt[3]{x^{\frac{1}{5}}}} = \dfrac{\sqrt[5]{25}}{\sqrt[5]{16}}.\ x^{\frac{2}{5}} = \dfrac{25^{\frac{1}{5}}}{16^{\frac{1}{5}}}$ or $x^2 = \frac{25}{16}$. Whence $x = \pm\,\frac{5}{4}$.

31. $(\sqrt[3]{-27\,x^9})^{-2} = \dfrac{1}{(-3\,x^3)^2} = \dfrac{1}{9\,x^6}.$

32. $(\sqrt[4]{16\,a^4x^8})^{-2} = \dfrac{1}{(2\,ax^2)^2} = \dfrac{1}{4\,a^2x^4}.$

33. $\sqrt{16^{\frac{3}{4}}x^9} = (16)^{\frac{3}{8}}x^{\frac{9}{2}} = 2^{\frac{3}{2}}x^{\frac{9}{2}}.$

34. $(x\sqrt[3]{x^{-2}})^3 = x^3 \cdot x^{-2} = x.$

35. $\left(x^{\frac{3}{2}}\sqrt{x^{-4}}\right)^{-\frac{2}{3}} = x^{-1}x^{\frac{4}{3}} = x^{\frac{1}{3}}.$

36. $\left(x^{-2}\sqrt{x^3\sqrt{4}}\right)^4 = \dfrac{4\,x^6}{x^8} = \dfrac{4}{x^2}.$

37. $\left(x^{-2}\sqrt{x^3\sqrt[3]{x^2}}\right)^{-\frac{1}{2}} = x \cdot x^{-\frac{3}{4}} \cdot x^{-\frac{1}{6}} = x^{\frac{1}{12}}.$

38. $\left[(\sqrt[3]{8\,x^4})^{-2}\right]^{\frac{4}{3}} = (8^{\frac{1}{3}}x^{\frac{4}{3}})^{-\frac{8}{3}} = \dfrac{1}{2^{\frac{8}{3}}x^{\frac{32}{9}}}.$

39. $\left(\sqrt{\sqrt[4]{16\,a^6}}\right)^4 = (16\,a^6)^{\frac{4}{8}} = 4\,a^3.$

40. $\left[(27\,x^3)^{-\frac{1}{3}} \cdot \dfrac{1}{5\,x^{-4}}\right]^{-3} = 27\,x^3 \cdot \dfrac{125}{x^{12}} = \dfrac{3375}{x^9}.$

41. $\left(a^{\frac{1}{2}}x^{-\frac{1}{3}}\sqrt{ax^{-\frac{1}{4}}\sqrt[3]{x^{\frac{5}{3}}}}\right)^{\frac{1}{2}} = a^{\frac{1}{4}}x^{-\frac{1}{6}}a^{\frac{1}{4}}x^{-\frac{1}{16}}x^{\frac{5}{36}} = \dfrac{a^{\frac{1}{2}}}{x^{\frac{13}{144}}}.$

42. $\sqrt{\sqrt{a}} = a^{\frac{1}{4}}.$

43. $\sqrt[3]{\sqrt{x^4}} = x^{\frac{2}{3}}.$

44. $\sqrt[3]{\sqrt[4]{x^5}} = x^{\frac{5}{12}}.$

45. $\sqrt{2\,a^4\sqrt[5]{x^3}} = 2^{\frac{1}{2}}\,a^2x^{\frac{3}{10}}.$

46. $\dfrac{\left[\sqrt{(25x^2)^2}\right]^{-\frac{1}{3}}}{\sqrt[3]{5x}} = \dfrac{5^{-\frac{2}{3}}x^{-\frac{2}{3}}}{5^{\frac{1}{3}}x^{\frac{1}{3}}} = \dfrac{1}{5x}.$

47. $(32)^{.4} = 32^{\frac{2}{5}} = 4.$

48. $16^{-1.5} = 16^{-\frac{3}{2}} = \frac{1}{64}.$

49. $4 \cdot 2^5 \cdot 2^{n-5} = 2^{n+2}.$

50. $\left(\dfrac{2}{3}\right)^{-2} \cdot \left(\dfrac{3}{2}\right)^0 \cdot 2^2 - \dfrac{2\,(\frac{1}{8})^{\frac{1}{3}}}{8^{-\frac{2}{3}}} = 9 - 4 = 5.$

51. $5 \cdot 2^0 - 1^5 + (5 \cdot 2)^0 = 5.$

52. $\dfrac{4^{-3} - 2^{-3}}{4^{-1} + 2^{-1}} = \dfrac{-\frac{7}{64}}{\frac{3}{4}} = -\dfrac{7}{48}.$

53. $\dfrac{3^e \cdot 3 - 3^{4+e}}{9 \cdot 3^{2+e}} = 3^{-3} - 1 = -\dfrac{26}{27}.$

54. $$\frac{a^{-1}+2x^{-1}}{a^{-3}+8x^{-3}} = \frac{\frac{1}{a}+\frac{2}{x}}{\frac{1}{a^3}+\frac{8}{x^3}} = \frac{x+2a}{ax}\cdot\frac{a^3x^3}{x^3+8a^3} = \frac{a^2x^2}{x^2-2xa+4a^2}.$$

55. $$\frac{x^{-4}+x^{-2}+1}{x^{-2}-x^{-1}+1} = \frac{\frac{1}{x^4}+\frac{1}{x^2}+1}{\frac{1}{x^2}-\frac{1}{x}+1} = \frac{1+x^2+x^4}{x^4}\cdot\frac{x^2}{1-x+x^2} = \frac{1+x+x^2}{x^2}.$$

56. $$(a^{n-1})(a^{3-n})^3(a^{2n+5})^{-1} = a^{n-1}a^{9-3n}a^{-2n-5} = a^{3-4n}.$$

57. $$a^{\frac{n+2}{n+1}} \div a^{\frac{1}{n+1}} = a^{\frac{n+1}{n+1}} = a.$$

58. $$a^{\frac{n}{n-1}} \div a^{\frac{n}{n+1}} = a^{\frac{n^2+n-n^2+n}{n^2-1}} = a^{\frac{2n}{n^2-1}}.$$

59. $$a^{n^2-m^2} \div a^{n-m} = a^{n^2-m^2+m-n}.$$

60. $$\left((a^{n+1})^{\frac{1}{n^2-1}}\right)^{n^2-n} = \left(a^{\frac{1}{n-1}}\right)^{n^2-n} = a^n.$$

61. $$\left(\sqrt[n]{x^{n^2}} \div \sqrt[3]{x}\right)^{\frac{3}{3n-1}} = \left(x^n \div x^{\frac{1}{3}}\right)^{\frac{3}{3n-1}} = \left(x^{\frac{3n-1}{3}}\right)^{\frac{3}{3n-1}} = x.$$

62. $$\frac{a^2\cdot 16^{\frac{1}{2}}a^{-\frac{3}{4}}}{a^{-\frac{7}{4}}+\frac{4}{a^{\frac{3}{4}}}} = \frac{4a^{\frac{5}{4}}}{\frac{a^{-1}+4}{a^{\frac{3}{4}}}} = \frac{4a^2}{\frac{1}{a}+4} = \frac{4a^3}{4a+1}.$$

63. $$[(x-y)^2]^0 = 1.$$

64. $$\left(x^{\frac{1}{n^2}-1}\right)^{\frac{1}{n+1}} = \left(x^{\frac{1-n^2}{n^2}}\right)^{\frac{1}{n+1}} = x^{\frac{1-n}{n^2}}.$$

65. $$\left[32^{-\frac{3}{5}}\cdot 9^{\frac{3}{2}}\right]^{-\frac{1}{3}} = 32^{\frac{1}{5}}\cdot 9^{-\frac{1}{2}} = \tfrac{2}{3}.$$

66. $$\left(\frac{8x^3}{125y^{-3}}\right)^{-\frac{1}{3}}\left(\frac{m^{-3}n^2}{x^3y^{-2}}\right)^{-2}\left(\frac{2^4x^{-6}}{x^2y^8}\right)^{\frac{1}{2}}\left(\frac{2y^3}{m^2}\right)^{-3}\left(\frac{n^6}{x^3}\right)^{\frac{1}{3}}$$
$$= \frac{2^{-1}x^{-1}}{5^{-1}y}\cdot\frac{m^6n^{-4}}{x^{-6}y^4}\cdot\frac{2^2x^{-3}}{xy^4}\cdot\frac{2^{-3}y^{-9}}{m^{-6}}\cdot\frac{n^2}{x} = \frac{5}{4}y^{-18}m^{12}n^{-2}.$$

67. $$\frac{4^{n+1}}{2^n(4^{n-1})^n} \div \frac{8^{n+1}}{(4^{n+1})^{n-1}} + 3 + 9x^0 - (9x)^0 - 1^{9x} + 9^0$$
$$= \frac{2^{2n+2}}{2^{2n^2-n}}\cdot\frac{2^{2n^2-2}}{2^{3n+3}} + 3 + 9 - 1 - 1 + 1$$
$$= \frac{2^{2n^2+2n}}{2^{2n^2+2n+3}} + 11 = \frac{1}{2^3} + 11 = 11\tfrac{1}{8}.$$

68. $$[((x^{-m})^n)^{-p}] \div [((x^{-n})^m)^p] = x^{mnp} \div x^{-mnp} = x^{2mnp}.$$

69. $$\left[(2x^2)^0\cdot(2x)^{\frac{1}{2}}\cdot(2x)^{-\frac{1}{2}}\right]^{-\frac{1}{5}} = 1.$$

70. $$\left[(5a)^{2n}\cdot(5a)^{p-3n}\right]^{\frac{2}{3}} = ((5a)^{p-n})^{\frac{2}{3}} = (5a)^{\frac{2p-2n}{3}}.$$

71. $\left(a^{\frac{2}{3}} \cdot a^{\frac{5}{11}}\right)^{\frac{33}{5}} = a^{\frac{37}{5}}.$

72. $\left(r^{-\frac{1}{2}} s^{\frac{2}{3}} \sqrt{r^3 s^{-\frac{4}{3}}}\right)^{\frac{2}{5}} = (rs^0)^{\frac{2}{5}} = r^{\frac{2}{5}}.$

73. $\sqrt[5]{x^{10}} + 3\,(x^{-6})^{-\frac{1}{2}} = x^2 + 3\,x^3.$

74. $\dfrac{3\,a^{-b}c^{-3}}{7\,x^{-4}y^{-d}} \cdot \dfrac{21\,x^{-2}y^{5-d}}{6\,a^{b}c^{d-3}} = \dfrac{3\,x^2y^5}{2\,a^{2b}c^{d}}.$

75. $\dfrac{2^{2n} \cdot 3^{n+1} \cdot 6^{n+\frac{1}{2}}}{(2 \cdot 3)^n \cdot 3^{n-\frac{1}{2}}} \div \dfrac{2^{\frac{3}{2}-n} \cdot 12^{2n+2}}{18^{n+2}}$

$$= \frac{2^{2n} \cdot 3^{n+1} \cdot 2^{n+\frac{1}{2}} \cdot 3^{n+\frac{1}{2}} \cdot 3^{2n+4} \cdot 2^{n+2}}{2^n \cdot 3^n \cdot 3^{n-\frac{1}{2}} \cdot 2^{\frac{3}{2}-n} 2^{4n+4} \cdot 3^{2n+2}}$$

$$= \frac{2^{4n+\frac{5}{2}} \cdot 3^{4n+\frac{11}{2}}}{2^{4n+\frac{11}{2}} \cdot 3^{4n+\frac{3}{2}}} = \frac{3^4}{2^3} = \frac{81}{8}.$$

76. $\left(\dfrac{x^{-2}y}{x^{\frac{3}{2}}y^{-\frac{5}{2}}}\right)^{\frac{2}{7}} \div \left(\dfrac{xy^{-\frac{1}{2}}}{\sqrt{xy^{-1}}}\right)^{-2} = \dfrac{x^{-\frac{4}{7}}y^{\frac{2}{7}}}{x^{\frac{3}{7}}y^{-\frac{5}{7}}} \cdot \dfrac{x^{-1}y^2}{x^{-2}y} = y^2.$

77. $\dfrac{m^7n}{r^{-r}s^{-1}} \cdot \dfrac{m^s r^{r+2}}{n^{-5}s^{-3}} = m^{7+s}\,r^{2r+2}\,s^4n^6.$

78. $\dfrac{ab^{\frac{1}{2}}c\sqrt{ab}\sqrt[3]{c}}{\sqrt[3]{a^2}b^{-\frac{1}{2}}a^{\frac{1}{3}}b^2c^{\frac{4}{3}}} = \dfrac{a \cdot b^{\frac{1}{2}} \cdot c \cdot a^{\frac{1}{2}} \cdot b \cdot c^{\frac{1}{3}}}{a^{\frac{2}{3}}b^{-\frac{1}{2}}a^{\frac{1}{3}}b^2c^{\frac{4}{3}}} = \dfrac{a^{\frac{3}{2}}b^{\frac{3}{2}}c^{\frac{4}{3}}}{ab^{\frac{3}{2}}c^{\frac{4}{3}}} = a^{\frac{1}{2}}.$

79. $\dfrac{xy^{\frac{1}{3}}z\sqrt{xy}\sqrt[5]{z}}{\sqrt{x^3} \cdot y^{-1}zy^3\sqrt[5]{z^6}\,x^{-2}} = \dfrac{xy^{\frac{1}{3}}zx^{\frac{1}{2}}y^{\frac{1}{2}}z^{\frac{1}{5}}}{x^{\frac{3}{2}}y^{-1}zy^3z^{\frac{6}{5}}x^{-2}} = \dfrac{x^{\frac{3}{2}}y^{\frac{5}{6}}z^{\frac{6}{5}}}{x^{-\frac{1}{2}}y^2z^{\frac{11}{5}}} = x^2y^{-\frac{7}{6}}z^{-1}.$

80.
$$\begin{array}{l|l|l}
 & x^6 - 4\,x^{\frac{9}{2}}y^{\frac{1}{3}} + 4\,x^3y^{\frac{2}{3}} + 6\,x^3 - 12\,x^{\frac{3}{2}}y^{\frac{1}{3}} + 9 & x^3 - 2\,x^{\frac{3}{2}}y^{\frac{1}{3}} + 3 \\
 & x^6 & \pm(x^3 - 2\,x^{\frac{3}{2}}y^{\frac{1}{3}} + 3) \\
2\,x^3 - 2\,x^{\frac{3}{2}}y^{\frac{1}{3}} & - 4\,x^{\frac{9}{2}}y^{\frac{1}{3}} & \\
 & - 4\,x^{\frac{9}{2}}y^{\frac{1}{3}} + 4\,x^3y^{\frac{2}{3}} & \\
2\,x^3 - 4\,x^{\frac{3}{2}}y^{\frac{1}{3}} + 3 & 6\,x^3 - 12\,x^{\frac{3}{2}}y^{\frac{1}{3}} + 9 & \\
 & 6\,x^3 - 12\,x^{\frac{3}{2}}y^{\frac{1}{3}} + 9 &
\end{array}$$

81. $\dfrac{x^2}{y^2} + 4 + \dfrac{4\,y^2}{x^2} + \dfrac{3\,x^2}{y^2} - 20 + \dfrac{12\,y^2}{x^2} = \dfrac{4\,x^2}{y^2} - 16 + \dfrac{16\,y^2}{x^2}.$

$$\begin{array}{l|l|l}
 & \dfrac{4\,x^2}{y^2} - 16 + \dfrac{16\,y^2}{x^2} & \dfrac{2\,x}{y} - \dfrac{4\,y}{x} \\
 & \dfrac{4\,x^2}{y^2} & \pm\left(\dfrac{2\,x}{y} - \dfrac{4\,y}{x}\right) \\
\dfrac{4\,x}{y} - \dfrac{4\,y}{x} & - 16 + \dfrac{16\,y^2}{x^2} & \\
 & - 16 + \dfrac{16\,y^2}{x^2} &
\end{array}$$

82. $[(e^x - e^{-x})^2 + 4]^{\frac{1}{2}} = [e^{2x} + 2 + e^{-2x}]^{\frac{1}{2}} = e^x + e^{-x}.$

83. $[(e^x + 2^{-1}e^{-x})^2 - 2]^{\frac{1}{2}} = [e^{2x} - 1 + 2^{-2}e^{-2x}]^{\frac{1}{2}} = e^x - 2^{-1}e^{-x}.$

84.

$$
\begin{array}{l|l|l}
 & e^{2x} + 4e^x + 2 - 4e^{-x} + e^{-2x} & e^x + 2 - e^{-x} \\
 & e^{2x} & \\
\hline
2e^x + 2 & 4e^x & \\
 & 4e^x + 4 & \\
\hline
2e^x + 4 - e^{-x} & -2 - 4e^{-x} + e^{-2x} & \\
 & -2 - 4e^{-x} + e^{-2x} & \\
\hline
\end{array}
$$

85.

$$
\begin{array}{l|l|l}
 & 16x^2 - 24 + 17x^{-2} - 6x^{-4} + x^{-6} & 4x - 3x^{-1} + x^{-3} \\
 & 16x^2 & \\
\hline
8x - 3x^{-1} & -24 & \\
 & -24 + 9x^{-2} & \\
\hline
8x - 6x^{-1} + x^{-3} & 8x^{-2} & \\
 & 8x^{-2} - 6x^{-4} + x^{-6} & \\
\hline
\end{array}
$$

86.

$$
\begin{array}{l|l|l}
 & x^{-1} - 4x^{-\frac{3}{4}} + 6x^{-\frac{1}{2}} - 4x^{-\frac{1}{4}} + 1 & x^{-\frac{1}{2}} - 2x^{-\frac{1}{4}} + 1 \\
 & x^{-1} & \\
\hline
2x^{-\frac{1}{2}} - 2x^{-\frac{1}{4}} & -4x^{-\frac{3}{4}} & \\
 & -4x^{-\frac{3}{4}} + 4x^{-\frac{1}{2}} & \\
\hline
2x^{-\frac{1}{2}} - 4x^{-\frac{1}{4}} + 1 & 2x^{-\frac{1}{2}} & \\
 & 2x^{-\frac{1}{2}} - 4x^{-\frac{1}{4}} + 1 & \\
\hline
\end{array}
$$

$\sqrt{x^{-\frac{1}{2}} - 2x^{-\frac{1}{4}} + 1} = x^{-\frac{1}{4}} - 1$

87.

$$
\begin{array}{l|l|l}
 & \frac{4a^{-8}b^8}{9} + a^{-6}b^6 + \frac{9a^{-4}b^4}{16} - \frac{16a^{-4}b^4}{15} - \frac{6a^{-2}b^2}{5} + \frac{16}{25} & \frac{2a^{-4}b^4}{3} + \frac{3a^{-2}b^2}{4} - \frac{4}{5} \\
 & \frac{4a^{-8}b^8}{9} & \\
\hline
\frac{4a^{-4}b^4}{3} + \frac{3a^{-2}b^2}{4} & a^{-6}b^6 & \\
 & a^{-6}b^6 + \frac{9a^{-4}b^4}{16} & \\
\hline
\frac{4a^{-4}b^4}{3} + \frac{3a^{-2}b^2}{2} - \frac{4}{5} & -\frac{16a^{-4}b^4}{15} & \\
 & -\frac{16a^{-4}b^4}{15} - \frac{6a^{-2}b^2}{5} + \frac{16}{25} & \\
\hline
\end{array}
$$

88. $$\frac{\dfrac{x^3 \cdot a \cdot x^0 - a \cdot 3 \cdot x^2}{(x^3)^2}}{\dfrac{a}{x^3}} = \frac{ax^3 - 3ax^2}{x^6} \cdot \frac{x^3}{a} = \frac{x-3}{x}.$$

89. $$\frac{\dfrac{x^4(3x^2)-(x^3+5)4x^3}{(x^4)^2}}{\dfrac{x^3+20}{x^4}}=\frac{3x^6-4x^6-20x^3}{x^8}\cdot\frac{x^4}{x^3+20}$$
$$=\frac{-x^6-20x^3}{x^4(x^3+20)}=-\frac{1}{x}.$$

90. $$\frac{\dfrac{(x^2-1)ax^{a-1}-x^a\cdot 2x}{(x^2-1)^2}}{\dfrac{x^a}{x^2-1}}=\frac{ax^{a+1}-ax^{a-1}-2x^{a+1}}{(x^2-1)^2}\cdot\frac{x^2-1}{x^a}$$
$$=\frac{ax-ax^{-1}-2x}{x^2-1}.$$

91. $$\frac{\dfrac{x^2(ax^{a-1})-(x^a+1)2x}{(x^2)^2}}{\dfrac{x^a+1}{x^2}}=\frac{ax^{a+1}-2x^{a+1}-2x}{x^4}\cdot\frac{x^2}{x^a+1}$$
$$=\frac{ax^a-2x^a-2}{x(x^a+1)}.$$

92. $$\frac{\dfrac{x^{2a}nx^{n-1}-x^n\cdot 2ax^{2a-1}}{(x^{2a})^2}}{\dfrac{x^n}{x^{2a}}}=\frac{nx^{2a+n-1}-2ax^{2a+n-1}}{x^{4a}}\cdot\frac{x^{2a}}{x^n}=\frac{n-2a}{x}.$$

93. $$\frac{\dfrac{x^{-5}(-2x^{-1})-(x^2+3)(-5x^{-4})}{(x^{-5})^2}}{\dfrac{x^{-2}+3}{x^{-5}}}=\frac{-2x^{-6}+5x^{-2}+15x^{-4}}{x^{-10}}\cdot\frac{x^{-5}}{x^{-2}+3}$$
$$=\frac{-2x^{-1}+5x^3+15x}{x^{-2}+3}$$
$$=\frac{5x^5+15x^3-2x}{1+3x^2}.$$

94. $$\frac{\dfrac{e^{-nx}(ne^{nx})-(e^{nx}+1)(-ne^{-nx})}{(e^{-nx})^2}}{\dfrac{2e^{nx}+1}{e^{-nx}}}=\frac{n+n+ne^{-nx}}{e^{-2nx}}\cdot\frac{e^{-nx}}{2e^{nx}+1}$$
$$=\frac{2n+ne^{-nx}}{e^{-nx}(2e^{nx}+1)}=\frac{2ne^{nx}+n}{2e^{nx}+1}=n.$$

95. $$\frac{\dfrac{(\sqrt[3]{x}+1)\frac{1}{3}x^{-\frac{2}{3}}-\sqrt[3]{x}\left(\frac{1}{3}x^{-\frac{2}{3}}\right)}{(\sqrt[3]{x}+1)^2}}{\dfrac{\sqrt[3]{x}}{\sqrt[3]{x}+1}}=\frac{\dfrac{x^{-\frac{1}{3}}}{3}+\dfrac{x^{-\frac{2}{3}}}{3}-\dfrac{x^{-\frac{1}{3}}}{3}}{\left(x^{\frac{1}{3}}+1\right)^2}\cdot\frac{x^{\frac{1}{3}}+1}{x^{\frac{1}{3}}}=\frac{\dfrac{x^{-1}}{3}}{x^{\frac{1}{3}}+1}$$
$$=\frac{1}{3\left(x^{\frac{1}{3}}+1\right)x}=\frac{1}{3x^{\frac{4}{3}}+3x}.$$

96. $$\frac{\dfrac{\sqrt{ax-x^2}\cdot b - bx(ax-x^2)^{-\frac{1}{2}}\cdot(a-2x)}{(\sqrt{ax-x^2})^2}}{\dfrac{bx}{\sqrt{ax-x^2}}}$$

$$= \frac{abx - bx^2 - abx + 2bx^2}{(ax-x^2)^{\frac{3}{2}}}\cdot\frac{(ax-x^2)^{\frac{1}{2}}}{bx} = \frac{x}{ax-x^2} = \frac{1}{a-x}.$$

97. $$\frac{\dfrac{(\sqrt{x^{2n}-1})\,2nx^{2n-1} - x^{2n}(x^{2n}-1)^{-\frac{1}{2}}(nx^{n-1})}{(\sqrt{x^{2n}-1})^2}}{\dfrac{x^{2n}}{\sqrt{x^{2n}-1}}}$$

$$= \frac{2nx^{4n-1} - 2nx^{2n-1} - nx^{3n-1}}{(x^{2n}-1)^{\frac{3}{2}}}\cdot\frac{(x^{2n}-1)^{\frac{1}{2}}}{x^{2n}}$$

$$= \frac{2nx^{2n-1} - 2nx^{-1} - nx^{n-1}}{x^{2n}-1} = \frac{2nx^n - 2n - nx^n}{x^{2n+1}-x}.$$

98. $$\frac{\dfrac{\sqrt{x^2-10x}\cdot 5 - 5x\cdot\frac{1}{2}(x^2-10x)^{-\frac{1}{2}}(2x-10)}{(\sqrt{x^2-10x})^2}}{\dfrac{5x}{(x^2-10x)^{\frac{1}{2}}}}$$

$$= \frac{5x^2 - 50x - 5x^2 + 25x}{(x^2-10x)^{\frac{3}{2}}}\cdot\frac{(x^2-10x)^{\frac{1}{2}}}{5x}$$

$$= \frac{-5}{x^2-10x} = \frac{5}{10x-x^2}.$$

99. $$\frac{\sqrt{m-n}}{\sqrt{m}+\sqrt{n}} = \frac{\sqrt{m}}{\sqrt{m-n}} - \frac{\sqrt{n}}{\sqrt{m-n}}.$$

$$\left(\frac{\sqrt{m-n}}{\sqrt{m}+\sqrt{n}} = \frac{\sqrt{m}-\sqrt{n}}{\sqrt{m-n}}\right)\sqrt{m-n}(\sqrt{m}+\sqrt{n}).$$

$$m-n = m-n.$$

100. $$\frac{\sqrt[a+1]{\sqrt[a+2]{x}}}{\sqrt[a]{\sqrt[a+1]{x}}} = \frac{\sqrt[a+1]{x^2}}{\sqrt[a]{x}\cdot\sqrt[a+2]{x}}.$$

$$\frac{\dfrac{1}{x^{a^2+3a+2}}}{\dfrac{1}{x^{a^2+a}}} = \frac{\dfrac{2}{x^{a+1}}}{\dfrac{2a+2}{x^{a^2+2a}}}.$$

$$\frac{1}{x^{a^2+3a+2}} + \frac{2a+2}{a^2+2a} = \frac{2}{x^{a+1}} + \frac{1}{a^2+a}.$$

$$\frac{2a^2+5a+2}{x^{a(a+1)(a+2)}} = \frac{2a+1}{x^{a^2+a}}.$$

$$\frac{2a+1}{x^{a^2+a}} = \frac{2a+1}{x^{a^2+a}}.$$

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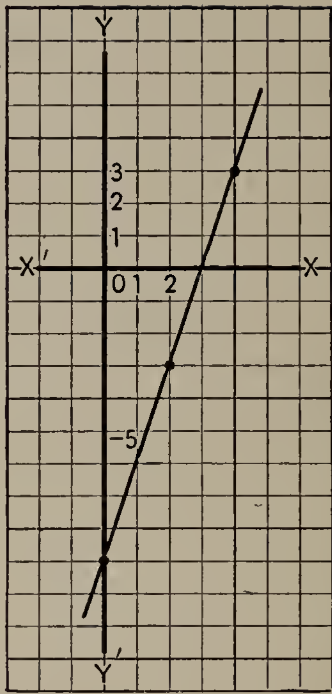
1. $f(x) = 3x - 9$.

x	0	2	4
$f(x)$	-9	-3	3



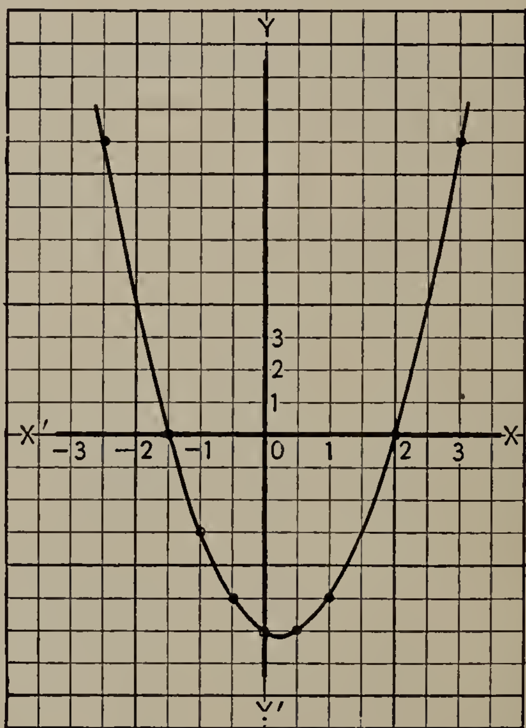
2. Yes. At that point the function equals zero.

3. The graph of a linear function of x is a straight line which cuts the x-axis.

4. $f(x) = 2x^2 - x - 6$.

x	3	2	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	-2	-3
$f(x)$	9	0	-5	-6	-6	-5	-3	$+4$	15

5. Yes. At those points the function equals zero.



6. A parabola.

7. Transpose terms so that the second member of the equation is zero. Graph the function in the first member. Read from the graph the x-values of the points where the curve crosses the x-axis.

8. It raises every point on the graph, and consequently the entire graph, by the amount added to the constant term. The same effect is produced by lowering the x-axis.

9. One root is decreased and the other increased in such a manner that the two approach the same value.

10. One real root only is given when the curve touches but does not cross the x-axis.

11. The graphical solution of an equation in x fails when the curve does not touch or cut the x-axis.

12. Yes. 2.3, $-.2$, and -2.1.

13. -2.6, 1.3. There is now but one point of intersection and one point of tangency.

14. $-2.7+$. There is now but one point of intersection. The other roots are not real.

15. 2.6. The other roots are not real.

16. $2.7+$. The real root is increasing.

17. (*a*) No. (*b*) No.

(*c*) $x + \frac{1}{x} + 5$ is rational in x but not integral.

$x - \sqrt{x} + 3$ is integral in x but not rational.

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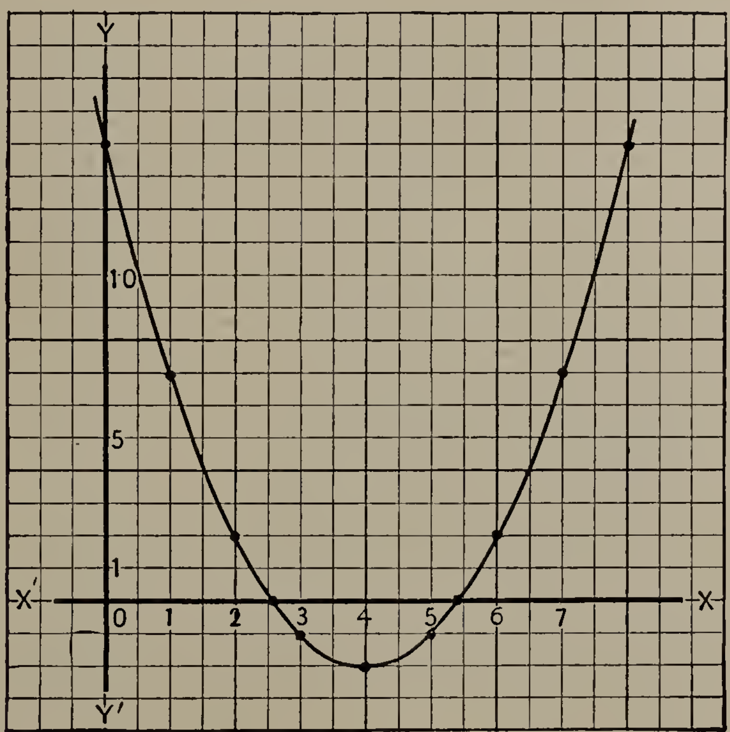
1. $f(x) = x^2 - 8x + 14$.

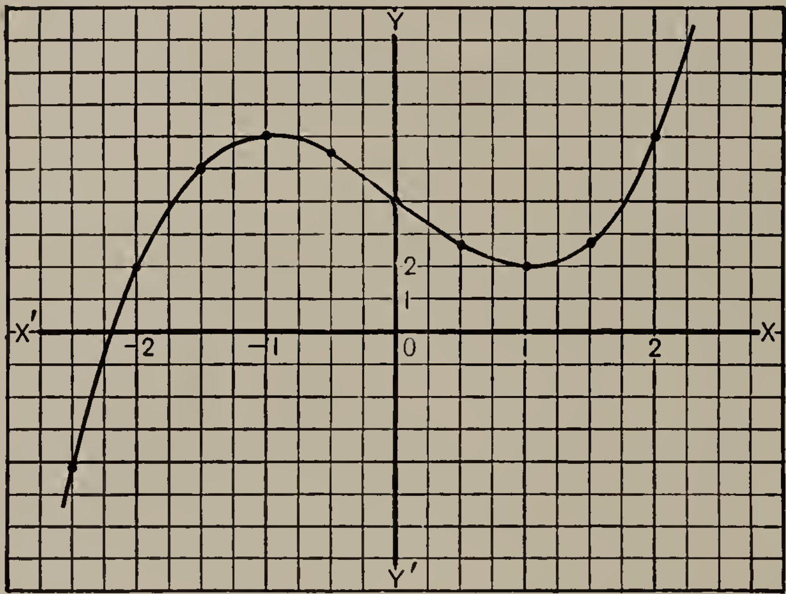
x	0	1	2	3	4	5	6	7	8
$f(x)$	14	7	2	-1	-2	-1	2	7	14



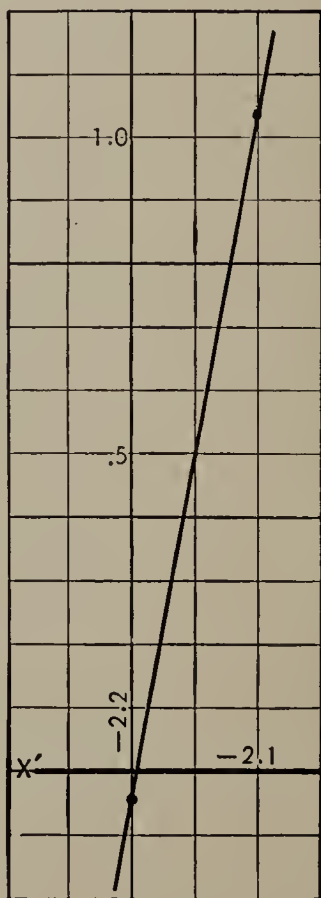
From the graph $x = 2.6$ and 5.4.

2. $f(x) = x^3 - 3x + 4$.

x	$-\frac{5}{2}$	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	-4.1	2	5.1	6	5.3	4	2.6	2	2.8	6



From the graph $x = -2.2$ approximately.

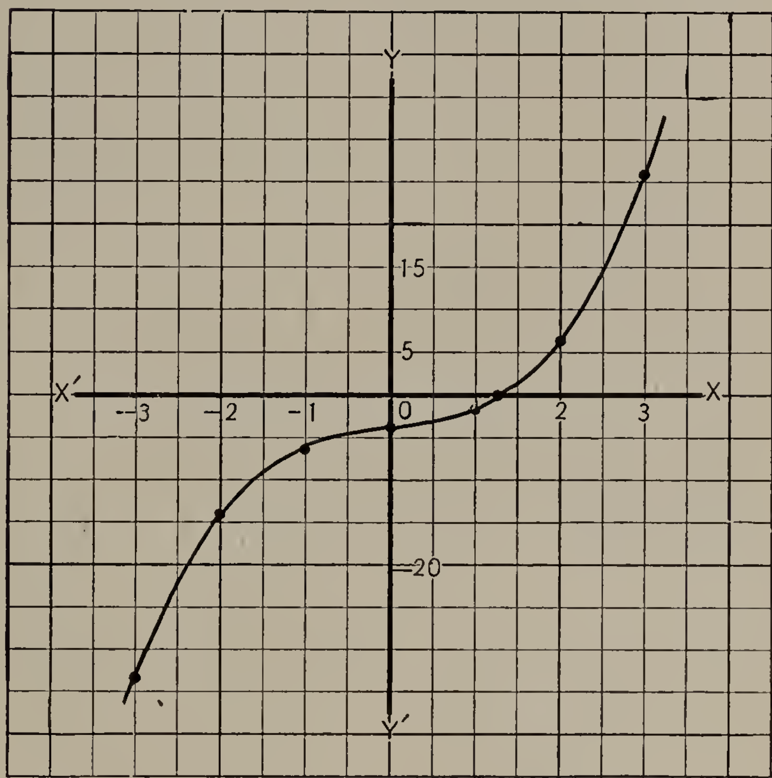


x	-2.1	-2.2
$f(x)$	1.04	$-.048$

From the graph $x = -2.19$.

3. $f(x) = x^3 + x - 4.$

x	3	2	1	0	-1	-2	-3
$f(x)$	26	6	-2	-4	-6	-14	-34



From the graph $x = 1.3$ approximately.

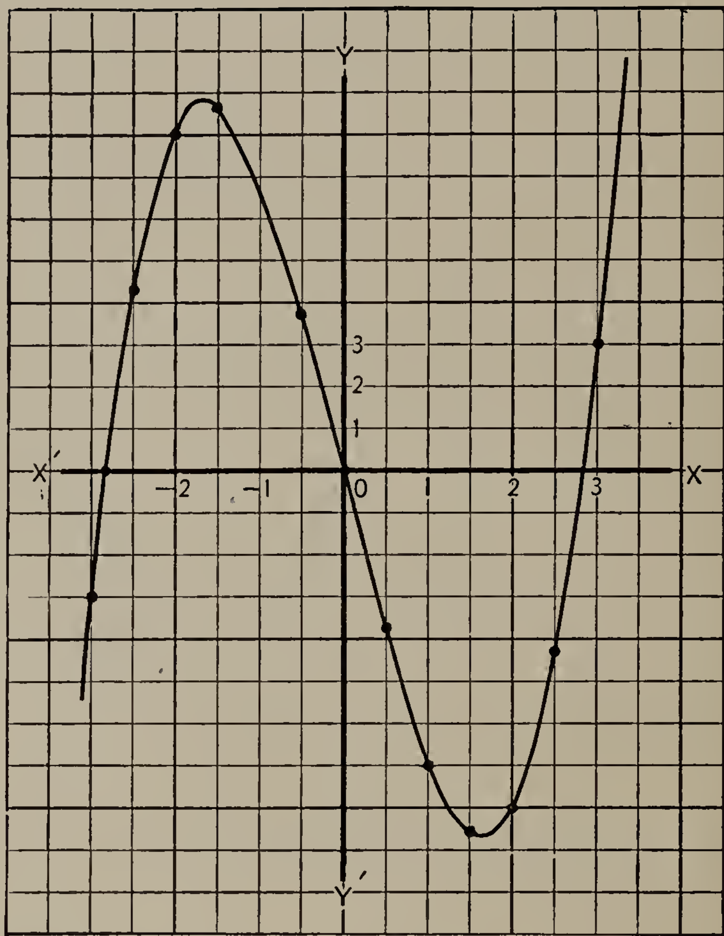
4. $f(x) = x^3 - 2x^2 - 5x + 6.$

x	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	1	2	$\frac{5}{2}$	3	$\frac{7}{2}$
$f(x)$	0	5.6	8	7.8	6	0	-4	-3.3	0	6.8

From the graph $x = -2$, 1, and 3.

5. $f(x) = x^3 - 8x$.

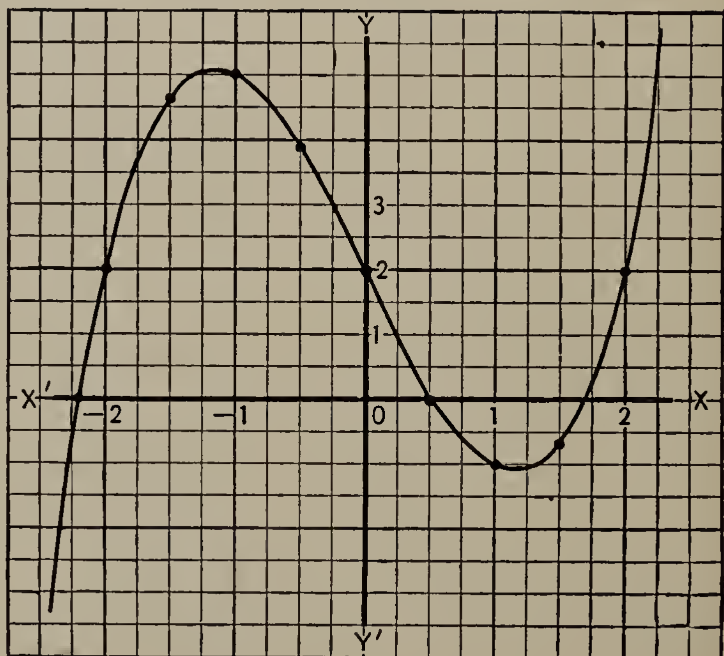
x	3	$\frac{5}{2}$	2	$\frac{3}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{3}{2}$	-2	$-\frac{5}{2}$	-3
$f(x)$	3	-4.3	-8	-8.6	-7	-3.8	0	3.8	7	8.6	8	4.3	-3



From the graph $x = 2.8$, 0, and -2.8.

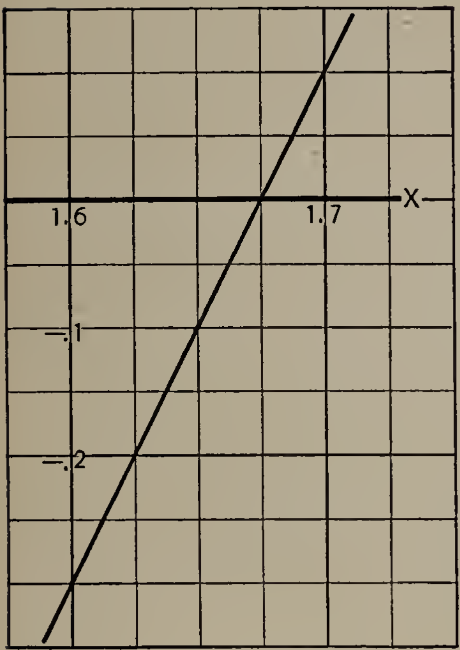
6. $f(x) = x^3 - 4x + 2$.

x	2	$\frac{3}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{3}{2}$	-2	$-\frac{5}{2}$
$f(x)$	2	$-\frac{5}{8}$	-1	$\frac{1}{8}$	2	$3\frac{7}{8}$	5	$4\frac{5}{8}$	2	$-3\frac{5}{8}$



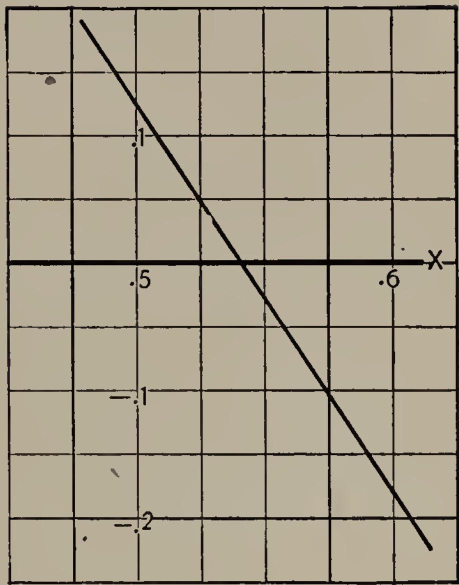
From the graph $x = 1.7$, $.5$, and -2.2.

x	1.6	1.7
$f(x)$	–.304	.113



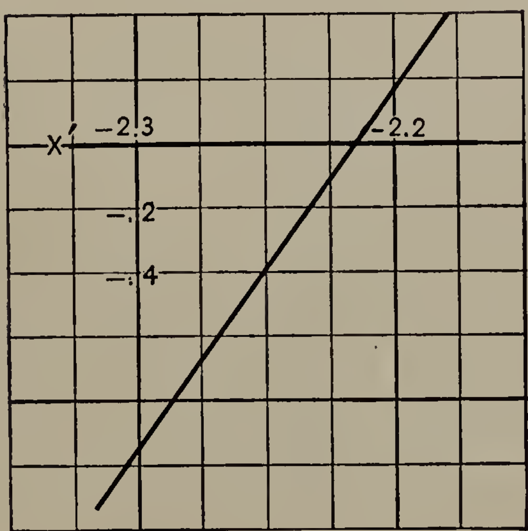
From the graph,
$x = 1.67$.

x	.5	.6
$f(x)$	.125	–.184



From the graph,
$x = .54$.

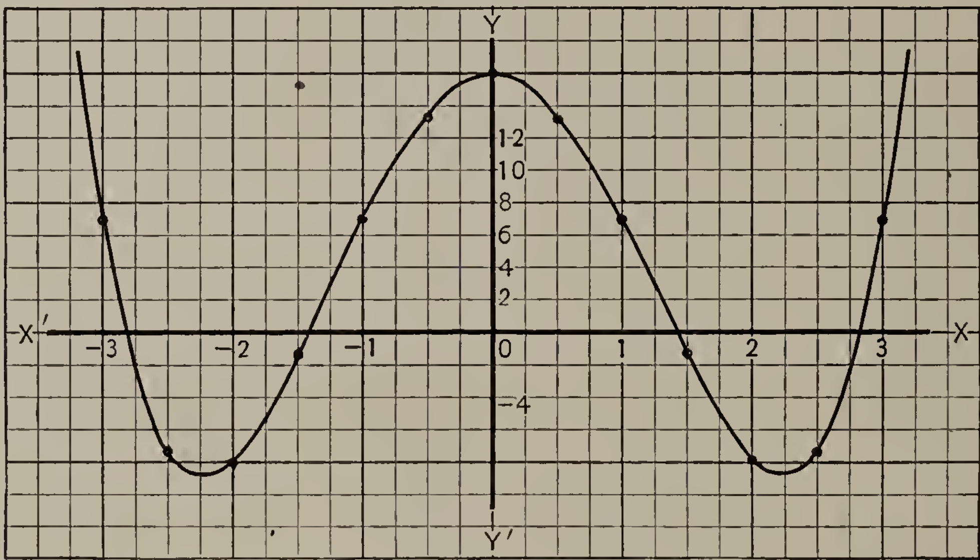
x	– 2.2	–2.3
$f(x)$	.15	–.96



From the graph,
$x = -2.22$.

7. $f(x) = x^4 - 10x^2 + 16$.

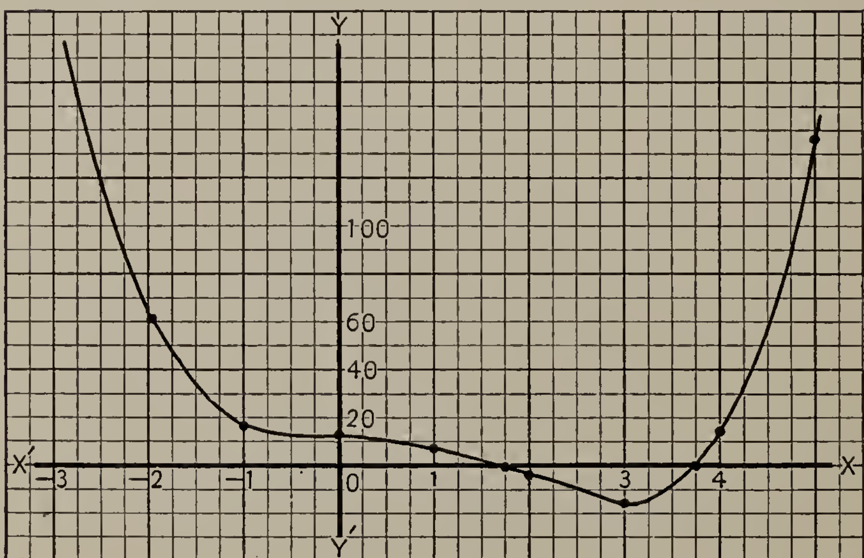
x	3	2.5	2	1.5	1	.5	0	$-.5$	-1	-1.5	-2	-2.5	-3
$f(x)$	7	-7.4	-8	-1.4	7	13.5	16	13.5	7	-1.4	-8	-7.4	7



From the graph $x = 2.8$, 1.4, -1.4, and -2.8.

8. $f(x) = x^4 - 4x^3 + 12$.

x	5	4	3	2	1	0	-1	-2
$f(x)$	137	12	-15	-4	9	12	17	60



From the graph $x = 3.77$ and 1.74.

9. (*a*) $x = -2.4$.

(*b*) By producing the graph upwards we have $x = 2.7$ approximately.

(*c*) $x = 2.2$, $-.5$, and -1.7 approximately.

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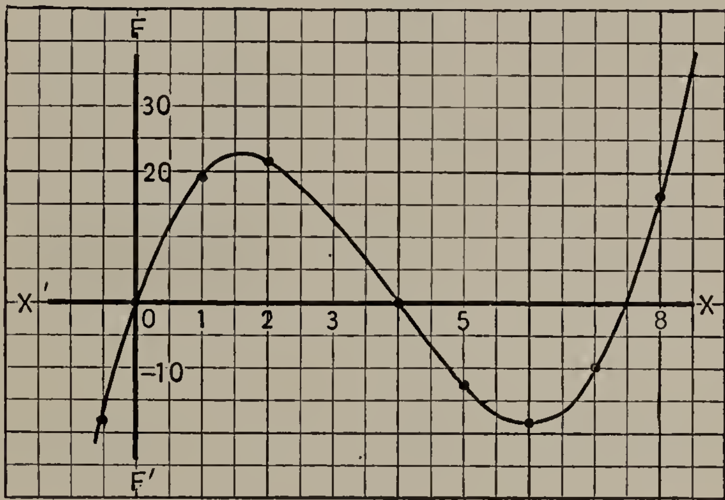
1. $\dfrac{V}{4}$ or $\dfrac{f(x)}{4} = x^3 - 11\frac{1}{2}x^2 + 30x.$

x	8	7	6	5	4	3	2	1	0	$-\frac{1}{2}$	$\frac{5}{3}$
$\dfrac{f(x)}{4}$	16	−10.5	−18	−12.5	0	13.5	22	19.5	0	−18	22.6

From the graph the function is greatest at $x = \frac{5}{3}$. Hence the side of the required square is $\frac{5}{3}$ inches.



2. (*a*) When $x = \frac{5}{3}$, $f(x) = 90$.

(*b*) $f(x) = 90$ when $x = 8.1$ also.

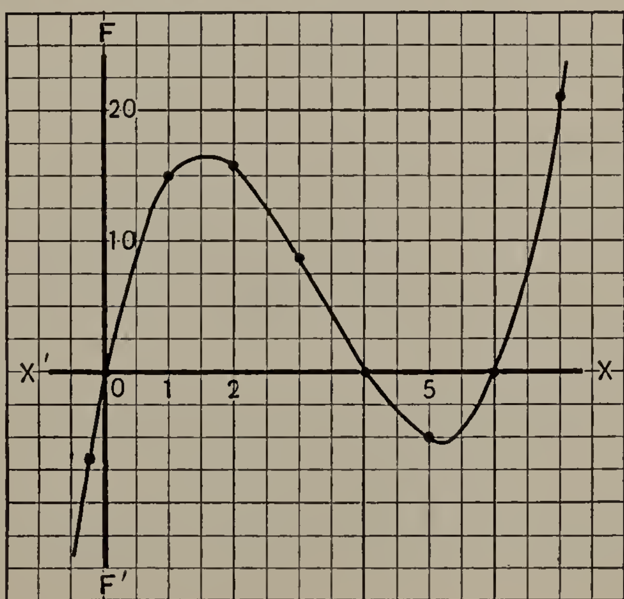
(*c*) Any value of x greater than 8.1 makes $f(x)$ greater than 90.

(*d*) The board has dimensions too small to allow for turning up a side whose edge is greater than 4.

3. $v = 4x^3 - 40x^2 + 96x.$

$\dfrac{f(x)}{4} = x^3 - 10x^2 + 24x.$

x	0	1	2	3	4	5	6	7	$-\frac{1}{4}$
$\dfrac{f(x)}{4}$	0	15	16	9	0	−5	0	21	−6.6

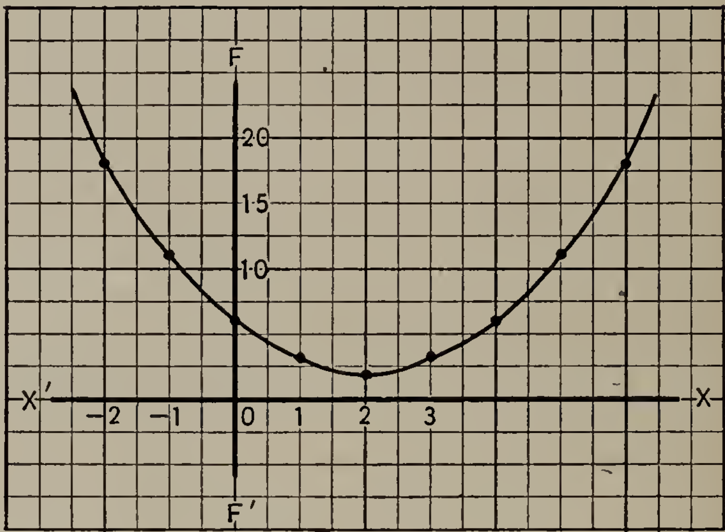


$\dfrac{f(x)}{4}$ and hence V is greatest for $x = 1.6$.

When $x = 1.6$, $V = 67$.

4. $f(x) = x^2 - 4x + 6.$

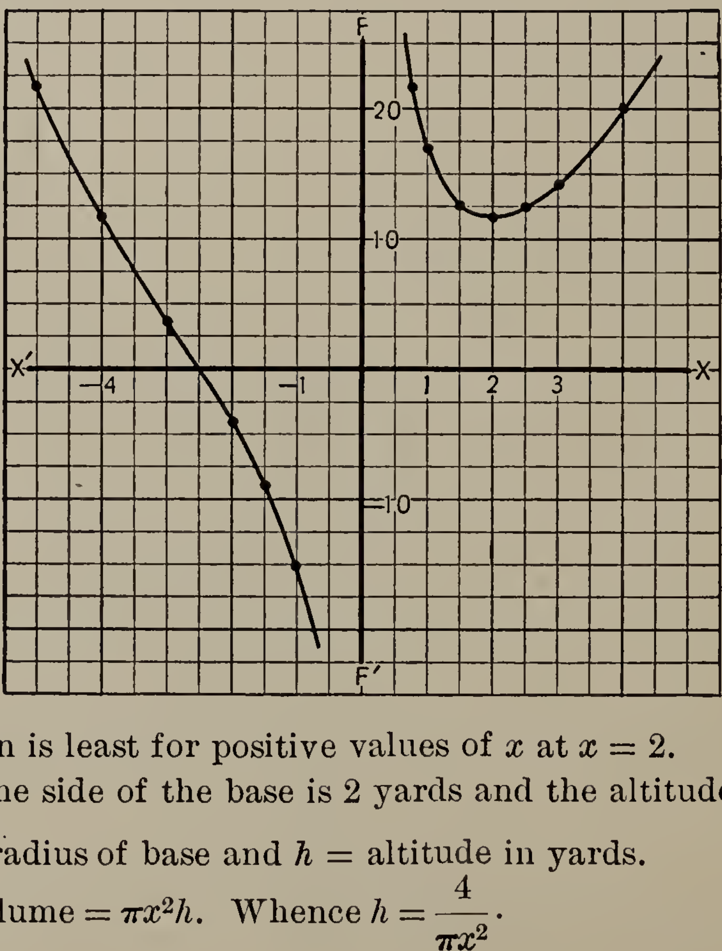
x	6	5	4	3	2	1	0	-1	-2
$f(x)$	18	11	6	3	2	3	6	11	18



The function is least for $x = 2$.

5. $f(x) = x^2 + \dfrac{16}{x}.$

x	-5	-4	-3	-2	$-\frac{3}{2}$	-1	$\frac{8}{4}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	4
$f(x)$	21.6	12	3.6	-4	$-.84$	-15	21.8	17	12.9	12	12.6	14.3	20



The function is least for positive values of x at $x = 2$.
Therefore the side of the base is 2 yards and the altitude is 1 yard.

6. Let x = radius of base and h = altitude in yards.
Then the volume $= \pi x^2 h$. Whence $h = \dfrac{4}{\pi x^2}$.

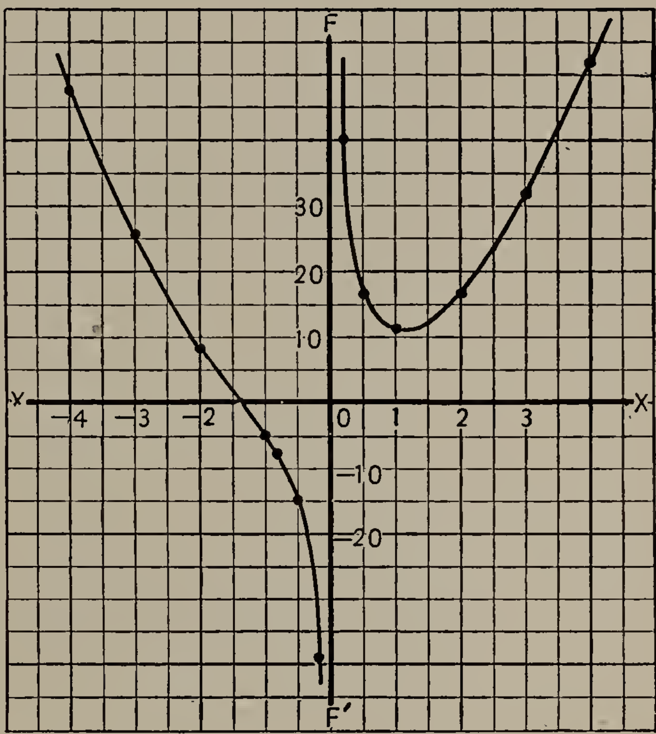
The inside surface $= \pi x^2 + 2\,\pi x h$. Substituting for h, $f(x) = \pi x^2 + \dfrac{8}{x}$.

x	-4	-3	-2	-1	$-.8$	$-.5$	$-.2$	$.2$	$.5$	$.8$	1	2	3	4
$f(x)$	48	25.6	8.5	-4.8	-7.8	-15.2	-39.8	40.1	16.7	12	11.1	16.5	30.9	52

It is evident that negative values for x have no meaning in the problem and need not have been considered.

From the graph the function is least for $x = 1$.

Hence $x = 1$ and $h = \dfrac{4}{\pi} = 1.26$

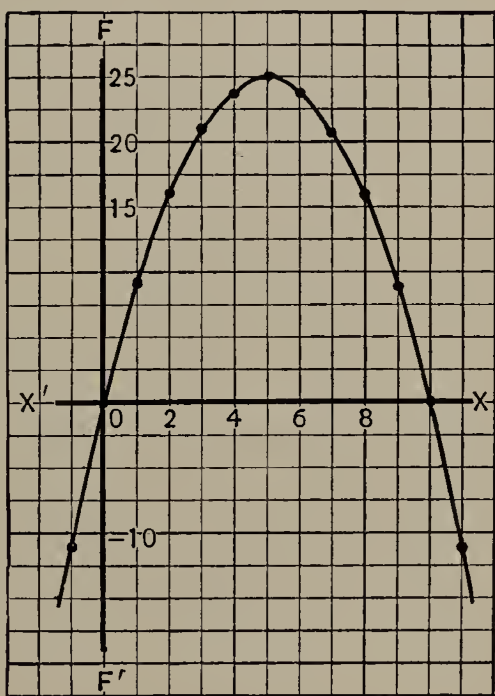


7. Let $x =$ the width and $10 - x =$ the length in rods.

Then $x(10 - x) =$ the area $= A$.

Hence $A = f(x) = 10\,x - x^2$.

x	-1	0	1	2	3	4	5	6	7	8	9	10	11
$f(x)$	-11	0	9	16	21	24	25	24	21	16	9	0	-11

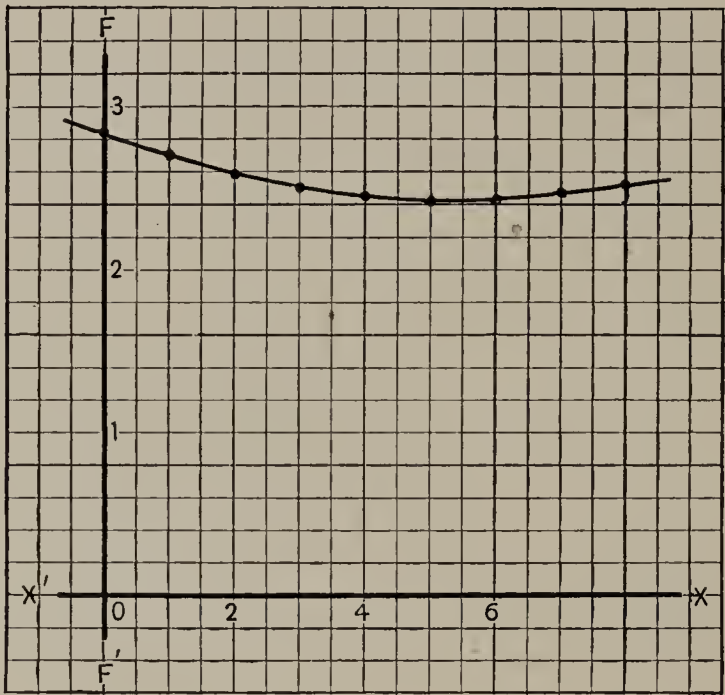


The function, and therefore the area A, is greatest for $x = 5$.

Hence length $=$ width $= 5$.

8. $f(x) = \frac{8-x}{6} + \frac{\sqrt{x^2+36}}{4} =$ time in hours.

x	0	1	2	3	4	5	6	7	8
$f(x)$	2.83	2.68	2.58	2.51	2.47	2.45	2.45	2.47	2.50



$f(x)$ is least for $x = 5.36$. Here $f(x) =$ time $= 2.44$ hours.
Then $KA = 5.36$ miles.

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1. $x^2 - 4x - 32 = 0$.

$$x^2 - 4x + 4 = 32 + 4 = 36.$$
$$x - 2 = \pm 6.$$
$$x = 8, \ -4.$$

2. $2x^2 + 5x + 3 = 0$.

$$x^2 + \tfrac{5}{2}x + \tfrac{25}{16} = -\tfrac{3}{2} + \tfrac{25}{16} = \tfrac{1}{16}.$$
$$x + \tfrac{5}{4} = \pm \tfrac{1}{4}.$$
$$x = -1, \ -\tfrac{3}{2}.$$

3. $\frac{x^2+3}{4} = \frac{5x^2-24}{7}$.

$$7x^2 + 21 = 20x^2 - 96.$$
$$13x^2 = 117.$$
$$x^2 = 9.$$
$$x = \pm 3.$$

4. $x^2 - 5x + 2 = 0$.

$$x^2 - 5x + \tfrac{25}{4} = -2 + \tfrac{25}{4} = \tfrac{17}{4}.$$
$$x - \frac{5}{2} = \pm \frac{\sqrt{17}}{2} = \pm \frac{4.12310}{2}.$$
$$x = 4.5615, \ .4384.$$

5. $7x^2 - 12x - 3 = 0.$

$$x^2 - \tfrac{12}{7}x + \tfrac{36}{49} = \tfrac{3}{7} + \tfrac{36}{49} = \tfrac{57}{49}.$$
$$x - \tfrac{6}{7} = \pm \tfrac{1}{7}\sqrt{57}.$$
$$x = \frac{6}{7} \pm \frac{7.54983}{7} = 1.9356, -.2214.$$

6. $\dfrac{3x^2}{2} - 3x\sqrt{2} = 9.$

$$x^2 - 2x\sqrt{2} + 2 = 6 + 2 = 8.$$
$$x - \sqrt{2} = \pm 2\sqrt{2}.$$
$$x = 3\sqrt{2}, \ -\sqrt{2}.$$

7. $(2x - 5)^2 - (x - 6)^2 = 80.$

$$4x^2 - 20x + 25 - x^2 + 12x - 36 = 80.$$
$$3x^2 - 8x = 91.$$
$$x^2 - \tfrac{8}{3}x + \tfrac{16}{9} = \tfrac{91}{3} + \tfrac{16}{9} = \tfrac{289}{9}.$$
$$x - \tfrac{4}{3} = \pm \tfrac{17}{3};\ x = 7, \ -\tfrac{13}{3}.$$

8. $\dfrac{1}{x} + \dfrac{1}{x+2} = \dfrac{3}{4}.$

$$4x + 8 + 4x = 3x^2 + 6x.$$
$$3x^2 - 2x - 8 = 0.$$
$$x^2 - \tfrac{2}{3}x + \tfrac{1}{9} = \tfrac{8}{3} + \tfrac{1}{9} = \tfrac{25}{9}.$$
$$x - \tfrac{1}{3} = \pm \tfrac{5}{3}.$$
$$x = 2, \ -\tfrac{4}{3}.$$

9. $\dfrac{1}{w+7} - \dfrac{1}{w-3} = \dfrac{2}{5}.$

$$5w - 15 - 5w - 35 = 2w^2 + 8w - 42.$$
$$2w^2 + 8w = -8.$$
$$w^2 + 4w + 4 = -4 + 4 = 0.$$
$$w + 2 = 0.$$
$$w = -2.$$

10. $\dfrac{m}{m+1} - \dfrac{2m}{m+2} + \dfrac{9}{2} = 0.$

$$2m^2 + 4m - 4m^2 - 4m + 9m^2 + 27m + 18 = 0.$$
$$7m^2 + 27m = -18.$$
$$m^2 + \tfrac{27}{7}m + (\tfrac{27}{14})^2 = -\tfrac{18}{7} + \tfrac{729}{196}.$$
$$m + \tfrac{27}{14} = \pm \tfrac{15}{14}.$$
$$m = -3, \ -\tfrac{6}{7}.$$

11. $\dfrac{10s^2-1-s^3}{6s-s^2-9}-\dfrac{s^2-9}{s+3}=0.$

$$\frac{s^3-10s^2+1}{s^2-6s+9}=s-3.$$
$$s^3-10s^2+1=s^3-9s^2+27s-27.$$
$$-s^2-27s+28=0.$$
$$s^2+27s+(\tfrac{27}{2})^2=28+\tfrac{729}{4}.$$
$$s+\tfrac{27}{2}=\pm\tfrac{29}{2}.$$
$$s=1,\ -28.$$

12. $\dfrac{4}{x-4}-\dfrac{x+3}{x^2-5x+4}-2=0.$

$$4x-4-x-3-2x^2+10x-8=0.$$
$$-2x^2+13x-15=0.$$
$$x^2-\frac{13}{2}x+\left(\frac{-13}{4}\right)^2=-\frac{15}{2}+\frac{169}{16}=\frac{49}{16}.$$
$$x-\tfrac{13}{4}=\pm\tfrac{7}{4}.$$
$$x=5,\ \tfrac{3}{2}.$$

13. $\dfrac{2(x^3+8)}{x+2}-\dfrac{x^3-1}{x-1}-\dfrac{19}{4}=0.$

$$2x^2-4x+8-x^2-x-1-\tfrac{19}{4}=0.$$
$$x^2-5x+\left(\frac{-5}{2}\right)^2=-\frac{9}{4}+\frac{25}{4}=4.$$
$$x-\tfrac{5}{2}=\pm 2.$$
$$x=4\tfrac{1}{2},\ \tfrac{1}{2}.$$

14. $\dfrac{r+2\sqrt{5}}{\sqrt{15}}-\dfrac{2}{r}=\dfrac{1}{\sqrt{5}}.$

$$r^2+2r\sqrt{5}-2\sqrt{15}=r\sqrt{3}.$$
$$r^2+(2\sqrt{5}-\sqrt{3})r+\left(\frac{2\sqrt{5}-\sqrt{3}}{2}\right)^2=2\sqrt{15}+\frac{23-4\sqrt{15}}{4}=\frac{23+4\sqrt{15}}{4}.$$
$$r+\frac{2\sqrt{5}-\sqrt{3}}{2}=\pm\sqrt{\frac{23+4\sqrt{15}}{4}}=\pm\frac{2\sqrt{5}+\sqrt{3}}{2}.$$
$$r=-\frac{2\sqrt{5}-\sqrt{3}}{2}\pm\frac{2\sqrt{5}+\sqrt{3}}{2}$$
$$=\sqrt{3},\ -2\sqrt{5}.$$

16. $x^3-10x^{\frac{3}{2}}-11=0.$

$$x^3-10x^{\frac{3}{2}}+25=11+25=36.$$
$$x^{\frac{3}{2}}-5=\pm 6.$$
$$x^{\frac{3}{2}}=11,\ -1.$$
$$x^3=121,\ 1.$$
$$x=\sqrt[3]{121},\ 1. \qquad \text{1 is not a root.}$$

Every number has three cube roots. The cube roots of 1 are 1 and $-\frac{1}{2}\pm\frac{1}{2}\sqrt{-3}$; and the cube roots of 121 are $\sqrt[3]{121}$ and $\sqrt[3]{121}\left(-\frac{1}{2}\pm\frac{1}{2}\sqrt{-3}\right)$ (see solution of Exercises 7-10, page 473). The student has not yet had sufficient practice in imaginaries to obtain such answers. Hence he cannot at this time solve completely equations like those of Exercises 16, 18, or 20. Therefore the imaginary answers to these will be omitted.

17. $x^4 - 26x^2 + 25 = 0.$

$$x^4 - 26x^2 + 169 = -25 + 169 = 144.$$
$$x^2 - 13 = \pm 12.$$
$$x^2 = 25, 1.$$
$$x = \pm 5, \pm 1.$$

18. $x^6 - 7x^3 - 8 = 0.$

$$x^6 - 7x^3 + \left(\frac{-7}{2}\right)^2 = 8 + \frac{49}{4} = \frac{81}{4}.$$
$$x^3 - \tfrac{7}{2} = \pm \tfrac{9}{2}.$$
$$x^3 = 8, -1.$$
$$x = 2, -1.$$

19. $x + x^{\frac{1}{2}} - 6 = 0.$

$$x + x^{\frac{1}{2}} + \tfrac{1}{4} = 6 + \tfrac{1}{4}.$$
$$x^{\frac{1}{2}} + \tfrac{1}{2} = \pm \tfrac{5}{2}.$$
$$x^{\frac{1}{2}} = 2, -3.$$
$$x = 4, 9. \quad 9 \text{ is not a root.}$$

20. $4x^6 - 7x^3 = 15.$

$$x^6 - \frac{7}{4}x^3 + \left(\frac{-7}{8}\right)^2 = \frac{15}{4} + \frac{49}{64} = \frac{289}{64}.$$
$$x^3 - \tfrac{7}{8} = \pm \tfrac{17}{8}.$$
$$x^3 = 3, -\tfrac{5}{4}.$$
$$x = \sqrt[3]{3}, \tfrac{1}{2}\sqrt[3]{-10}.$$

21. $4x^2 + \dfrac{4}{x^2} - \dfrac{97}{9} = 0.$

$$36x^4 + 36 - 97x^2 = 0.$$
$$x^4 - \frac{97}{36}x^2 + \left(\frac{-97}{72}\right)^2 = -1 + \frac{9409}{5184} = \frac{4225}{5184}.$$
$$x^2 - \tfrac{97}{72} = \pm \tfrac{65}{72}.$$
$$x = \pm \tfrac{3}{2}, \pm \tfrac{2}{3}.$$

22. $2x - 3x^{\frac{1}{2}} = 2.$

$$x - \tfrac{3}{2}x^{\frac{1}{2}} + (-\tfrac{3}{4})^2 = 1 + \tfrac{9}{16} = \tfrac{25}{16}.$$
$$x^{\frac{1}{2}} - \tfrac{3}{4} = \pm \tfrac{5}{4}.$$
$$x^{\frac{1}{2}} = 2, -\tfrac{1}{2}.$$
$$x = 4, \tfrac{1}{4}. \quad \tfrac{1}{4} \text{ is not a root.}$$

23. $3x^4 - 11x^2 + 6 = 0.$

$$x^4 - \tfrac{11}{3}x^2 + (-\tfrac{11}{6})^2 = -2 + \tfrac{121}{36} = \tfrac{49}{36}.$$
$$x^2 - \tfrac{11}{6} = \pm \tfrac{7}{6}.$$
$$x^2 = 3, \tfrac{2}{3}.$$
$$x = \pm\sqrt{3}, \pm \tfrac{1}{3}\sqrt{6}.$$

24. $9x^4 - 22x^2 + 8 = 0.$

$$x^4 - \tfrac{22}{9}x^2 + (-\tfrac{11}{9})^2 = -\tfrac{8}{9} + \tfrac{121}{81} = \tfrac{49}{81}.$$
$$x^2 - \tfrac{11}{9} = \pm \tfrac{7}{9}.$$
$$x^2 = 2, \tfrac{4}{9}.$$
$$x = \pm\sqrt{2}, \pm \tfrac{2}{3}.$$

25. $3x^{\frac{3}{2}} + 5x^{\frac{3}{4}} + 2 = 0.$

$$x^{\frac{3}{2}} + \tfrac{5}{3}x^{\frac{3}{4}} + (\tfrac{5}{6})^2 = -\tfrac{2}{3} + \tfrac{25}{36} = \tfrac{1}{36}.$$
$$x^{\frac{3}{4}} + \tfrac{5}{6} = \pm \tfrac{1}{6}.$$
$$x^{\frac{3}{4}} = -\tfrac{2}{3}, -1.$$
$$x^3 = \tfrac{16}{81}, 1.$$
$$x = \sqrt[3]{\tfrac{16}{81}}, 1. \quad \text{Neither is a root.}$$

26. $2x^{\frac{2}{3}} - 9\sqrt[3]{x} + 4 = 0.$

$$x^{\frac{2}{3}} - \tfrac{9}{2}x^{\frac{1}{3}} + (-\tfrac{9}{4})^2 = -2 + \tfrac{81}{16} = \tfrac{49}{16}.$$
$$x^{\frac{1}{3}} - \tfrac{9}{4} = \pm \tfrac{7}{4}.$$
$$x^{\frac{1}{3}} = 4, \tfrac{1}{2}.$$
$$x = 64, \tfrac{1}{8}.$$

27. $3x - 11x^{\frac{1}{2}} - 20 = 0.$

$$x - \tfrac{11}{3}x^{\frac{1}{2}} + (-\tfrac{11}{6})^2 = \tfrac{20}{3} + \tfrac{121}{36} = \tfrac{361}{36}.$$
$$x^{\frac{1}{2}} - \tfrac{11}{6} = \pm \tfrac{19}{6}.$$
$$x^{\frac{1}{2}} = 5, -\tfrac{4}{3}.$$
$$x = 25, \tfrac{16}{9}. \quad \tfrac{16}{9} \text{ is not a root.}$$

28. $6x^4 - 13x^2 + 6 = 0.$

$$x^4 - \tfrac{13}{6}x^2 + \left(\frac{-13}{12}\right)^2 = -1 + \frac{169}{144} = \frac{25}{144}.$$
$$x^2 - \tfrac{13}{12} = \pm \tfrac{5}{12}.$$
$$x^2 = \tfrac{3}{2}, \tfrac{2}{3}.$$
$$x = \pm \tfrac{1}{2}\sqrt{6}, \pm \tfrac{1}{3}\sqrt{6}.$$

29. $x^{-2} + 16x^{-1} - 17 = 0.$

$$x^{-2} + 16x^{-1} + 64 = 17 + 64 = 81.$$
$$x^{-1} + 8 = \pm 9.$$
$$x^{-1} = 1, -17.$$
$$x = 1, -\tfrac{1}{17}.$$

30. $y^{-4} - 10y^{-2} + 9 = 0.$

$$y^{-4} - 10y^{-2} + 25 = -9 + 25 = 16.$$
$$y^{-2} - 5 = \pm 4.$$
$$y^{-2} = 9, 1.$$
$$y^{-1} = \pm 3, \pm 1.$$
$$y = \pm \tfrac{1}{3}, \pm 1.$$

31. $x^{-1} - 13\,x^{-\frac{1}{2}} = -36.$

$$x^{-1} - 13\,x^{-\frac{1}{2}} + \left(\frac{-13}{2}\right)^2 = -36 + \frac{169}{4} = \frac{25}{4}.$$

$$x^{-\frac{1}{2}} - \tfrac{13}{2} = \pm \tfrac{5}{2}.$$

$$x^{-\frac{1}{2}} = 9,\ 4.$$

$$x^{-1} = 81,\ 16.$$

$$x = \tfrac{1}{81},\ \tfrac{1}{16}.$$

32. $x^{2m} + 4 - 5\,x^m = 0.$

$$x^{2m} = 5\,x^m = -4.$$

$$x^{2m} - 5\,x^m + \left(\frac{-5}{2}\right)^2 = -4 + \frac{25}{4} = \frac{9}{4}.$$

$$x^m - \tfrac{5}{2} = \pm \tfrac{3}{2}.$$

$$x^m = 4,\ 1.$$

$$x = \sqrt[m]{4},\ 1.$$

34. $3\,(x^2 + 3\,x)^2 - 7\,(x^2 + 3\,x) - 20 = 0.$

$$(x^2 + 3\,x)^2 - \frac{7}{3}\,(x^2 + 3\,x) + \left(\frac{-7}{6}\right)^2 = \frac{20}{3} + \frac{49}{36} = \frac{289}{36}.$$

$$x^2 + 3\,x - \tfrac{7}{6} = \pm \tfrac{17}{6}.$$

$$x^2 + 3\,x = 4,\ -\tfrac{5}{3}.$$

$$x^2 + 3\,x + \tfrac{9}{4} = 4 + \tfrac{9}{4} = \tfrac{25}{4}.$$

$$x + \tfrac{3}{2} = \pm \tfrac{5}{2}.$$

$$x = 1,\ -4.$$

$$x^2 + 3\,x + (\tfrac{3}{2})^2 = -\tfrac{5}{3} + \tfrac{9}{4} = \tfrac{7}{12}.$$

$$x + \tfrac{3}{2} = \pm \sqrt{\tfrac{7}{12}}.$$

$$x = -\tfrac{3}{2} \pm \tfrac{1}{6}\sqrt{21}.$$

35. $\left(x - \dfrac{1}{x}\right)^2 + 4\left(x - \dfrac{1}{x}\right) = 8\tfrac{1}{4}.$

$$\left(x - \frac{1}{x}\right)^2 + 4\left(x - \frac{1}{x}\right) + 4 = 8\tfrac{1}{4} + 4 = 12\tfrac{1}{4}.$$

$$x - \frac{1}{x} + 2 = \pm \frac{7}{2}.$$

$$x - \frac{1}{x} = \frac{3}{2},\ -\frac{11}{2}.$$

$$x^2 - \frac{3\,x}{2} + \frac{9}{16} = 1 + \frac{9}{16} = \frac{25}{16}.$$

$$x - \tfrac{3}{4} = \pm \tfrac{5}{4}.$$

$$x = 2,\ -\tfrac{1}{2}.$$

$$x^2 + \tfrac{11}{2}\,x + \tfrac{121}{16} = 1 + \tfrac{121}{16} = \tfrac{137}{16}.$$

$$x + \tfrac{11}{4} = \pm \tfrac{1}{4}\sqrt{137}.$$

$$x = \frac{-11 \pm \sqrt{137}}{4}.$$

36. $(4y+5)+2(4y+5)^{\frac{1}{2}}=15.$

$$(4y+5)+2(4y+5)^{\frac{1}{2}}+1=15+1=16.$$
$$(4y+5)^{\frac{1}{2}}=3,\ -5.$$
$$4y+5=9,\ 25.$$
$$y=1,\ 5. \quad \text{5 is not a root.}$$

37. $x^2+5x+3\sqrt{x^2+5x}-54=0.$

$$x^2+5x+3\sqrt{x^2+5x}+\tfrac{9}{4}=54+\tfrac{9}{4}=\tfrac{225}{4}.$$
$$\sqrt{x^2+5x}+\tfrac{3}{2}=\pm\tfrac{15}{2}.$$
$$\sqrt{x^2+5x}=6,\ -9.$$
$$x^2+5x=36,\ 81.$$
$$x^2+5x-36=0.$$
$$(x-4)(x+9)=0.\ \therefore x=4,\ -9.$$
$$x^2+5x+\tfrac{25}{4}=81+\tfrac{25}{4}=\tfrac{349}{4}.$$
$$x+\tfrac{5}{2}=\pm\tfrac{1}{2}\sqrt{349}.$$
$$x=\frac{-5\pm\sqrt{349}}{2}\ \text{(not roots).}$$

38. $x^2-2x-5\sqrt{x^2-2x-4}+2=0.$

$$x^2-2x-4-5\sqrt{x^2-2x-4}+\tfrac{25}{4}=-6+\tfrac{25}{4}=\tfrac{1}{4}.$$
$$\sqrt{x^2-2x-4}-\tfrac{5}{2}=\pm\tfrac{1}{2}.$$
$$\sqrt{x^2-2x-4}=3,\ 2.$$
$$x^2-2x-4=9,\ 4.$$
$$x^2-2x-13=0,\ x=1\pm\sqrt{14}.$$
$$x^2-2x-8=0.$$
$$(x-4)(x+2)=0,\ x=4,\ -2.$$

39. $2y(2y+1)+3\sqrt{8y^2+12y+5}=25-4y.$

$$8y^2+4y+6\sqrt{8y^2+12y+5}=50-8y.$$
$$8y^2+12y+5+6\sqrt{8y^2+12y+5}+9=55+9=64.$$
$$\sqrt{8y^2+12y+5}+3=\pm 8.$$
$$\sqrt{8y^2+12y+5}=5,\ -11.$$
$$8y^2+12y+5=25,\ 121.$$
$$8y^2+12y-20=0,\ 2y^2+3y-5=0.\quad\therefore y=-\tfrac{5}{2},\ 1.$$
$$8y^2+12y-116=0,\ 2y^2+3y-29=0.\quad\therefore y=\frac{-3\pm\sqrt{241}}{4}.$$

40. $2x^2+ax-6a^2=0.$

$$x^2+\frac{a}{2}x+\frac{a^2}{16}=3a^2+\frac{a^2}{16}=\frac{49a^2}{16}.$$
$$x+\frac{a}{4}=\pm\frac{7a}{4}.$$
$$x=-2a,\ \frac{3a}{2}.$$

41. $2x^3 - 17bx^2 + 8b^2x = 0.$

$$x(x - 8b)(2x - b) = 0.$$

$$\therefore\ x = 0,\ 8b,\ \frac{b}{2}.$$

42. $7a^2x^2 - 4ax - 11 = 0.$

$$(7ax - 11)(ax + 1) = 0.$$

$$x = \frac{11}{7a},\ -\frac{1}{a}.$$

43. $ax^2 + bx + c = 0.$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2} = \frac{b^2 - 4ac}{4a^2}.$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}.$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

44. $12kx - 4k^2 - 5x^2 = 0.$

$$5x^2 - 12kx + 4k^2 = 0.$$

$$x = \frac{12k \pm \sqrt{144k^2 - 80k^2}}{10} = \frac{12k \pm 8k}{10} = 2k,\ \frac{2k}{5}.$$

45. $c^2(x - d)^2 - d^2(c - x)^2 = 0.$

$$c^2x^2 - 2c^2dx + c^2d^2 - c^2d^2 + 2d^2cx - d^2x^2 = 0.$$

$$(c^2 - d^2)x^2 - (2c^2d - 2cd^2)x = 0.$$

$$x[(c^2 - d^2)x - (2c^2d - 2cd^2)] = 0.$$

$$x = 0.$$

$$x = \frac{2c^2d - 2cd^2}{c^2 - d^2} = \frac{2cd}{c + d}.$$

46. $mx^2 - x(m^2 + 1) + m = 0.$

$$x = \frac{m^2 + 1 \pm \sqrt{m^4 + 2m^2 + 1 - 4m^2}}{2m} = \frac{m^2 + 1 \pm (m^2 - 1)}{2m} = m,\ \frac{1}{m}.$$

47. $(x - r)^2 + (S - x)^2 = r^2 + S^2.$

$$x^2 - 2rx + r^2 + S^2 - 2Sx + x^2 = r^2 + S^2.$$

$$2x^2 - (2r + 2S)x = 0.$$

$$x(2x - 2r - 2S) = 0.$$

$$x = 0,\ r + S.$$

48. $\dfrac{m}{m - x} - \dfrac{m - x}{m} = 2.$

$$m^2 - m^2 + 2mx - x^2 = 2m^2 - 2mx.$$

$$x^2 - 4mx + 2m^2 = 0.$$

$$x = \frac{4m \pm \sqrt{16m^2 - 8m^2}}{2} = 2m \pm m\sqrt{2}.$$

$$x = 2m + m\sqrt{2},\ 2m - m\sqrt{2}.$$

49. $\dfrac{x-c}{c}-\dfrac{c}{x-c}=\dfrac{3}{2}.$

$$2x^2-4cx+2c^2-2c^2=3cx-3c^2.$$
$$2x^2-7cx+3c^2=0.$$
$$x=\frac{7c\pm\sqrt{49c^2-24c^2}}{4}=\frac{7c\pm 5c}{4}=3c,\ \frac{c}{2}.$$

50. $\dfrac{x^2-3mx}{m-n}+2m=\dfrac{nx}{n-m}.$

$$x^2-3mx+nx+2m^2-2mn=0.$$
$$x=\frac{3m-n\pm\sqrt{9m^2-6mn+n^2+8mn-8m^2}}{2}$$
$$=\frac{3m-n\pm(m+n)}{2}=2m,\ m-n.$$

51. $\dfrac{x^2+2ax}{a-b}-\dfrac{x}{2b}=\dfrac{a}{b}.$

$$2bx^2+4abx-ax+bx=2a^2-2ab.$$
$$2b(x^2)+x(4ab-a+b)-2a^2+2ab=0.$$
$$x=\frac{a-b-4ab\pm\sqrt{a^2+16a^2b^2+b^2-2ab-8a^2b+8ab^2+16a^2b-16ab^2}}{4b}.$$
$$x=\frac{a-b-4ab\pm(b-a-4ab)}{4b}=-2a,\ \frac{a-b}{2b}.$$

52. $\dfrac{2s+x}{x+s}-\dfrac{5s-x}{x-s}-\dfrac{s(x+s)}{s^2-x^2}=0.$

$$x^2+sx-2s^2+x^2-4sx-5s^2+sx+s^2=0.$$
$$2x^2-2sx-6s^2=0,\ \text{or}\ x^2-sx-3s^2=0.$$
$$x=\frac{s\pm\sqrt{s^2+12s^2}}{2}=\frac{s\pm s\sqrt{13}}{2}.$$

53. $x=-\dfrac{5-3c}{4-2c}\pm\dfrac{7-3c}{4-2c}=-3,\ \dfrac{1}{2-c}.$

54. $ax^2+3x^2+ax-5x-a+1=a-1.$

$$(a+3)x^2+(a-5)x-2(a-1)=0.$$
$$x=\frac{5-a\pm\sqrt{a^2-10a+25+8a^2+16a-24}}{2a+6}$$
$$=\frac{5-a\pm(3a+1)}{2a+6}=1,\ \frac{-2a+2}{a+3}.$$

55. $cx^2 - ax^2 + cx + ax - c = c + a - ax.$

$$(c-a)x^2 + (c+2a)x - 2c - a = 0.$$

$$x = \frac{-c-2a \pm \sqrt{c^2+4ca+4a^2+8c^2-4ca-4a^2}}{2c-2a}$$

$$= \frac{-c-2a \pm 3c}{2c-2a} = 1, \frac{-2c-a}{c-a}.$$

56. $x^2 + 2s + s^2 = 2sx + 2x.$

$$x^2 - 2sx - 2x = -2s - s^2.$$

$$x^2 - 2x(s+1) + (s+1)^2 = -2s - s^2 + s^2 + 2s + 1.$$

$$x - s - 1 = \pm 1.$$

$$x = s + 2, s.$$

57. $x^2 - 2x + 1 = ax - ax^2.$

$$(1+a)x^2 - (2+a)x + 1 = 0.$$

$$x = \frac{2+a \pm \sqrt{4+4a+a^2-4-4a}}{2+2a} = \frac{2+a \pm a}{2+2a} = 1, \frac{1}{1+a}.$$

58. $x^2 + 2x + 1 = hx^2 + hx.$

$$(1-h)x^2 + (2-h)x + 1 = 0.$$

$$x = \frac{h-2 \pm \sqrt{4-4h+h^2-4+4h}}{2-2h} = \frac{h-2 \pm h}{2-2h} = -1, \frac{1}{h-1}.$$

59. $cx^2 + 3x = 2x^2 + 2cx - 2.$

$$(c-2)x^2 + (3-2c)x + 2 = 0.$$

$$x = \frac{2c-3 \pm \sqrt{9-12c+4c^2-8c+16}}{2c-4} = \frac{2c-3 \pm (5-2c)}{2c-4}.$$

$$x = \frac{1}{c-2}, 2.$$

60. $\dfrac{a}{x+a} + \dfrac{b}{x+b} - \dfrac{2c}{x+c} = 0.$

$$ax^2 + abx + acx + abc + bx^2 + abx + cbx$$

$$+ abc - 2cx^2 - 2acx - 2cbx - 2abc = 0.$$

$$(a+b-2c)x^2 + (2ab - ac - cb)x = 0.$$

$$x[(a+b-2c)x + (2ab - ac - bc)] = 0.$$

$$x = 0, \frac{ac+bc-2ab}{a+b-2c}.$$

61. $\dfrac{a+x}{b+x} + \dfrac{b+x}{a+x} = \dfrac{5}{2}.$

$$2a^2 + 4ax + 2x^2 + 2b^2 + 4bx + 2x^2 = 5ab + 5ax + 5bx + 5x^2.$$

$$x^2 + (a+b)x - 2a^2 - 2b^2 + 5ab = 0.$$

$$x = \frac{-a-b \pm \sqrt{a^2+2ab+b^2+8a^2-20ab+8b^2}}{2}$$

$$= \frac{-a-b \pm (3a-3b)}{2}.$$

$$x = a - 2b, -2a + b.$$

62. $\dfrac{ax+b}{bx+a}=\dfrac{mx-n}{nx-m}.$

$$anx^2+bnx-amx-bm=mbx^2-nbx+amx-an.$$

$$(an-bm)x^2+(2bn-2am)x+an-bm=0.$$

$$x=\frac{2am-2bn\pm\sqrt{4b^2n^2-8abmn+4a^2m^2-4a^2n^2+8abmn-4b^2m^2}}{2an-2bm}.$$

$$x=\frac{am-bn\pm\sqrt{(a^2-b^2)(m^2-n^2)}}{an-bm}.$$

63. $\dfrac{ax^2+bx+c}{bx^2-mx+n}=\dfrac{c}{n}.$

$$anx^2+bnx+cn=bcx^2-cmx+cn.$$

$$(an-bc)x^2+(bn+cm)x=0.$$

$$x=0,\ -\frac{bn+cm}{an-bc}.$$

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1. $2x^2-7x+3=0.$

$$x=\frac{7\pm\sqrt{49-24}}{4}=3,\ \frac{1}{2}.$$

2. $3x^2-x-2=0.$

$$x=\frac{1\pm\sqrt{1+24}}{6}=1,\ -\frac{2}{3}.$$

3. $3x^2-11hx-20h^2=0.$

$$x=\frac{11h\pm\sqrt{121h^2+240h^2}}{6}=\frac{11h\pm19h}{6}=5h,\ \frac{-4h}{3}.$$

4. $5x^2+2cx-16c^2=0.$

$$x=\frac{-2c\pm\sqrt{4c^2+320c^2}}{10}=\frac{-2c\pm18c}{10}=-2c,\ \frac{8c}{5}.$$

5. $22x^2-3mx-7m^2=0.$

$$x=\frac{3m\pm\sqrt{9m^2+616m^2}}{44}=\frac{3m\pm25m}{44}=\frac{-m}{2},\ \frac{7m}{11}.$$

6. $6x^2-7rx+2r^2=0.$

$$x=\frac{7r\pm\sqrt{49r^2-48r^2}}{12}=\frac{7r\pm r}{12}=\frac{2r}{3},\ \frac{r}{2}.$$

7. $3x^2-6ax+2a^2=0.$

$$x=\frac{6a\pm\sqrt{36a^2-24a^2}}{6}=\frac{6a\pm2a\sqrt{3}}{6}=\frac{3a\pm a\sqrt{3}}{3}.$$

8. $3m^2+4mx-7x^2=0.$

$$x=\frac{-4m\pm\sqrt{16m^2+84m^2}}{-14}=\frac{-4m\pm10m}{-14}=m,\ \frac{-3m}{7}.$$

9. $4ax-10a^2x^2+3=0.$

$$x=\frac{-4a\pm\sqrt{16a^2+120a^2}}{-20a^2}=\frac{-4a\pm2a\sqrt{34}}{-20a^2}=\frac{2\pm\sqrt{34}}{10a}.$$

10. $12\,v^4 + 7\,v^2x - 10\,x^2 = 0.$

$$x = \frac{-7\,v^2 \pm \sqrt{49\,v^4 + 480\,v^4}}{-20} = \frac{-7\,v^2 \pm 23\,v^2}{-20} = \frac{3\,v^2}{2},\ \frac{-4\,v^2}{5}.$$

11. $$x = \frac{-(3-m) \pm \sqrt{9 - 6\,m + m^2 + 12\,m}}{2}$$

$$= \frac{-3 + m \pm (3+m)}{2} = m,\ -3.$$

12. $x^2 + nx = cx + cn.$

$$x^2 + (n-c)\,x - cn = 0.$$

$$x = \frac{c - n \pm \sqrt{n^2 - 2\,cn + c^2 + 4\,cn}}{2} = \frac{c - n \pm (n+c)}{2} = c,\ -n.$$

13. $3\,x^2 - 6\,cx + 2\,c = x.$

$$3\,x^2 - (6\,c + 1)\,x + 2\,c = 0.$$

$$x = \frac{6\,c + 1 \pm \sqrt{36\,c^2 + 12\,c + 1 - 24\,c}}{6}$$

$$= \frac{6\,c + 1 \pm (6\,c - 1)}{6} = 2\,c,\ \frac{1}{3}.$$

14. $mx^2 + mkx = kc + cx.$

$$mx^2 + (mk - c)\,x - kc = 0.$$

$$x = \frac{c - mk \pm \sqrt{m^2k^2 + c^2 - 2\,mkc + 4\,mkc}}{2\,m}$$

$$= \frac{c - mk \pm (mk + c)}{2\,m} = \frac{c}{m},\ -k.$$

15. $a^2x^2 - 2\,ax = b^2x^2 - 1.$

$$x^2(a^2 - b^2) - 2\,ax + 1 = 0.$$

$$x = \frac{2\,a \pm \sqrt{4\,a^2 - 4\,a^2 + 4\,b^2}}{2\,a^2 - 2\,b^2} = \frac{2\,a \pm 2\,b}{2\,a^2 - 2\,b^2} = \frac{1}{a-b},\ \frac{1}{a+b}.$$

16. $n^3x^2 + 2\,nx = 5\,n^2x + 10.$

$$n^3x^2 + (2\,n - 5\,n^2)\,x - 10 = 0.$$

$$x = \frac{5\,n^2 - 2\,n \pm \sqrt{25\,n^4 - 20\,n^3 + 4\,n^2 + 40\,n^3}}{2\,n^3}$$

$$= \frac{5\,n^2 - 2\,n \pm (5\,n^2 + 2\,n)}{2\,n^3} = \frac{5}{n},\ -\frac{2}{n^2}.$$

17. $hkx^2 - hk = h^2x + h^2x^2.$

$$(hk - h^2)\,x^2 - h^2x - hk = 0.$$

$$x = \frac{h^2 \pm \sqrt{h^4 + 4\,h^2k^2 - 4\,h^3k}}{2\,hk - 2\,h^2}$$

$$= \frac{h^2 \pm (h^2 - 2\,hk)}{2\,hk - 2\,h^2} = -1,\ \frac{k}{k-h}.$$

18. $cx^2 + cmx + 5 = cx + 5(x + m).$

$$cx^2 + (cm - c - 5)x + 5 - 5m = 0.$$

$$x = \frac{-cm + c + 5 \pm \sqrt{c^2m^2 + c^2 + 25 - 2c^2m - 10cm + 10c - 20c + 20cm}}{2c} = \frac{-cm + c + 5 \pm (cm - c + 5)}{2c} = \frac{5}{c},\ 1 - m.$$

19. $n^2x + 3nx + 2x = nx^2 + 2n + 3n^2.$

$$nx^2 - (n^2 + 3n + 2)x + 2n + 3n^2 = 0.$$

$$x = \frac{n^2 + 3n + 2 \pm \sqrt{n^4 + 9n^2 + 4 + 6n^3 + 4n^2 + 12n - 8n^2 - 12n^3}}{2n} = \frac{n^2 + 3n + 2 \pm \sqrt{n^4 - 6n^3 + 5n^2 + 12n + 4}}{2n}$$

$$= \frac{n^2 + 3n + 2 \pm (n^2 - 3n - 2)}{2n} = n,\ \frac{2}{n} + 3.$$

20. $m^2x^2 + 4mx + hmx + 3hx = 9x^2 + 12x - 4h.$

$$(m^2 - 9)x^2 + (4m + hm + 3h - 12) + 4h = 0.$$

$$x = \frac{-4m - hm - 3h + 12 \pm \sqrt{16m^2 + h^2m^2 + 9h^2 + 144 + 8hm^2 + 24mh - 96m + 6mh^2 - 24hm - 72h - 16hm^2 + 144h}}{2(m^2 - 9)}.$$

$$x = \frac{-4m - hm - 3h + 12 \pm (4m - hm - 3h - 12)}{2(m^2 - 9)}.$$

$$x = -\frac{4}{m + 3},\ -\frac{h}{m - 3}.$$

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1. Let $x =$ the first part.
Then $20 - x =$ the second part.
Then $20 - x = x^2.$
Solving, $x = 4$ or $- 5.$ $- 5$ is impossible.
Therefore 4 and 16 are the required parts.

2. Let $h =$ the hypotenuse in feet.
Then $h - 2 =$ one leg in feet.
Then $h^2 = (h - 2)^2 + 8^2.$
Whence $h = 17$, and $h - 2 = 15.$
$\therefore$ area $= 8 \cdot 15 \div 2 = 60.$

3. Let $t =$ the time of trip in hours.
Then $\frac{112}{t} =$ rate in miles per hour.
$$\therefore \frac{112}{t} + 6 = t.$$
Whence $t = 14$ or $- 8.$ $- 8$ is impossible.
$\frac{112}{14} = 8$, the rate.

4. Let $n =$ the less integer.
Then $n + 1 =$ the next greater integer.
$$\therefore \frac{1}{n} + \frac{1}{n+1} = \frac{5}{6}.$$
Whence $n = 2$ or $- \frac{3}{5}.$ $- \frac{3}{5}$ is impossible.
$n + 1 = 3.$

5. Let $x =$ altitude in feet.
Then $x + 4 =$ base in feet.
$$\therefore \frac{x^2 + 4x}{2} = 48.$$
Whence $x = 8$ or $- 12.$ $- 12$ is impossible.
$8 =$ altitude; $12 =$ base.

6. Let $x =$ shorter leg in feet.
Then $x + 7 =$ longer leg in feet.
$$\therefore \frac{x^2 + 7x}{2} = 30.$$
Whence $x = 5, - 12.$ $- 12$ is impossible.
Hypotenuse $= \sqrt{25 + 144} = 13.$
Therefore the sides are 5, 12, and 13.

7. Let $x =$ altitude in feet.
Then $7x + 2 =$ base in feet.
$$\therefore \frac{7x^2 + 2x}{2} = 360.$$
$$7x^2 + 2x = 720.$$
Whence $x = 10$ or $- 10\frac{2}{7}.$ $- 10\frac{2}{7}$ is impossible.
$\therefore 10 =$ altitude; $7x + 2 = 72 =$ base.

8. Let $x =$ altitude in feet.

Then $x + 2$ and $2x =$ bases.

$$\therefore \frac{3x^2 + 2x}{2} = 60.$$

Whence $x = 6, -\frac{20}{3}$. $-\frac{20}{3}$ is impossible.

$\therefore 6 =$ altitude; 8 and 12 are the bases.

9. Let $x =$ number of days B takes.

Then $x + 4 =$ number of days A takes.

$$\therefore \frac{1}{x} + \frac{1}{x+4} = \frac{9}{80}.$$

$$160x + 320 = 9x^2 + 36x.$$

$$9x^2 - 124x - 320 = 0.$$

Whence $x = 16$ or $-\frac{20}{9}$. $-\frac{20}{9}$ is impossible.

$\therefore 16 =$ number of days B takes,

and $20 =$ number of days A takes.

10. Let $x =$ the amount in inches the radius is shortened.

Then $(21 - x)^2 \cdot \frac{22}{7} = \frac{22}{7}(441) - 770.$

$$x^2 - 42x + 245 = 0.$$

Whence $x = 7$ or 35. 35 is impossible.

$\therefore 7 =$ the amount the radius is shortened.

11. Let $x =$ number of gallons of wine in the cask.

Then $$\frac{(x-5) - 5 \cdot \dfrac{x-5}{x}}{x} = \frac{64}{100},$$

or $$36x^2 - 1000x + 2500 = 0.$$

Whence $x = 25$ or $2\frac{7}{9}$. $2\frac{7}{9}$ is impossible.

12. Let $x =$ number of letters per line.

Then $x + 15 =$ number of lines.

$$\therefore x(x + 15) = (x - 10)(x + 30).$$

Whence $x = 60,$

and $x + 15 = 75.$

Therefore $60 =$ number of letters per line,

$75 =$ number of lines,

and $60 \cdot 75 = 4500$, letters on a page.

13. Let $x =$ rate of interest.

Then $(50x + 300) + \frac{x}{100}(5000 + 50x + 300) + 300 = 1236.$

$$\frac{x^2}{2} + 103x - 636 = 0.$$

Whence $x = 6$ or -212. -212 is impossible.

Therefore $6\% =$ rate of interest.

14. Let $x =$ number of seconds.

Then $(123 + 239x)^2 + (239 - 123x)^2 = \overline{850}^2$.

$$72{,}250x^2 + 72{,}250 = 722{,}500.$$

Whence $x = +3$ or -3.

Ans. 3 seconds before, and 3 seconds after the stated time.

15. Let $x =$ one dimension in inches.

Then $x + 1$ and $x + 2 =$ other dimensions.

$\therefore 2[x(x+1) + x(x+2) + (x+1)(x+2)] = 292,$

or $x^2 + 2x - 48 = 0.$

Whence $x = 6$ or -8. -8 is impossible.

The dimensions in inches are 6, 7, and 8.

16. Let $x =$ side of square in inches.

Then $x - 6 =$ length of the square box in inches.

$$\therefore 3(x-6)^2 = 300.$$

$$x^2 - 12x - 64 = 0.$$

Whence $x = 16$ or -4. -4 is impossible.

17. Let $BC = x$.

Then $144 = x(x + 20)$.

$$\therefore x^2 + 20x - 144 = 0.$$

Whence $x = \dfrac{-20 \pm 31.24}{2} = 5.62$ or -25.62.

-25.62 is impossible.

18. If BC in the figure on page 374 is very small with respect to CD, $\overline{AB}^2 = BC \cdot BD$ becomes $\overline{AB}^2 = BC(BC + CD)$ or approximately $BC \cdot CD$.

Referring to the figure, $\overline{80}^2 = BC \cdot 7920$.

Whence $BC = \frac{640}{792} = \frac{80}{99}$ miles, or $4266\frac{2}{3}$ feet.

19. Referring to the figure, let $AB =$ the distance in miles.

$$\overline{AB}^2 = \tfrac{6}{5280} \cdot 7920.$$

$$\overline{AB}^2 = 9.$$

$AB = 3$ or -3. -3 is impossible.

20. The distance between them consists of the sum of two tangents. Let AB represent one tangent in miles and $A'B'$ the other.

$AB = \pm\sqrt{\frac{250}{5280} \cdot 7920} = \pm 5\sqrt{15} = \pm 19.364$. -19.364 is impossible.

$A'B' = \pm\sqrt{\frac{150}{5280} \cdot 7920} = \pm 15$. -15 is impossible.

$$AB + A'B' = 19.364 + 15 = 34.364.$$

The distance $= 34.364$ miles.

21. $S = \dfrac{gt^2}{2} = \dfrac{32 \cdot 144}{2} = 2304$ feet.

22. Let $x =$ time in seconds for falling.

Then $13 - x =$ time in seconds for sound to return; and, from Exercise 21, $16x^2 =$ distance fallen in feet.

$$(13 - x)1120 = \text{distance the sound returns in feet.}$$
$$16x^2 = 14560 - 1120x.$$
$$x^2 + 70x - 910 = 0.$$
$$x = 11.206. \quad -81.206 \text{ is impossible.}$$
$$\text{The height} = 16x^2 = 2009.28 \text{ feet.}$$

23. Let x = the number of miles the head of army moves while the messenger travels from its rear to its head.

Then $$\frac{x}{28 + x} = \frac{28 - x}{x}.$$
$$x^2 = -x^2 + 784 \text{ or } 2x^2 = 784.$$

Whence $x^2 = 392$, or $x = \pm 14\sqrt{2}$. $-14\sqrt{2}$ is impossible.

Altogether he traveled $28 + 2x$ or $28 + 28\sqrt{2} = 67.5979$ miles.

Page 380

1. $\sqrt{x + 3} = 8.$
$$x + 3 = 64.$$
$$\therefore x = 61.$$

2. $\sqrt{2x - 6} + 4 = 7.$
$$\sqrt{2x - 6} = 3.$$
$$2x - 6 = 9.$$
$$\therefore x = 7\tfrac{1}{2}.$$

3. $3\sqrt{2x - 8} - 7 = 17.$
$$3\sqrt{2x - 8} = 24.$$
$$\sqrt{2x - 8} = 8.$$
$$2x - 8 = 64.$$
$$\therefore x = 36.$$

4. $\sqrt[3]{3x - 4} = 2.$
$$3x - 4 = 8.$$
$$\therefore x = 4.$$

5. $(7x + 15)^{\frac{1}{3}} + 18 = 17.$
$$\sqrt[3]{7x + 15} = -1.$$
$$7x + 15 = -1.$$
$$\therefore x = -\tfrac{16}{7}.$$

6. $2\sqrt[3]{3n - 25} + 3 = 7.$
$$2\sqrt[3]{3n - 25} = 4.$$
$$\sqrt[3]{3n - 25} = 2.$$
$$3n - 25 = 8.$$
$$\therefore n = 11.$$

7. $3x\sqrt{3x} = 18\sqrt{27x}.$
$$x\sqrt{3x} = 18\sqrt{3x}.$$
$$x^3 = 324x.$$
$$x(x^2 - 324) = 0.$$
$$\therefore x = 0, +18, -18.$$
-18 is extraneous.

8. $(9x)^{\frac{1}{2}} = x^{\frac{1}{2}} + 4.$
$$3\sqrt{x} = \sqrt{x} + 4.$$
$$2\sqrt{x} = 4.$$
$$\sqrt{x} = 2.$$
$$\therefore x = 4.$$

9. $\sqrt{5n^2 + 19} + n = 7.$
$$\sqrt{5n^2 + 19} = 7 - n.$$
$$5n^2 + 19 = n^2 - 14n + 49.$$
$$4n^2 + 14n - 30 = 0.$$
$$\therefore n = -5, \tfrac{3}{2}.$$

10. $(3x - 4)^{\frac{1}{3}} + (4x + 3)^{\frac{1}{3}} = 0.$
$$(3x - 4)^{\frac{1}{3}} = -(4x + 3)^{\frac{1}{3}}.$$
$$3x - 4 = -4x - 3.$$
$$\therefore x = \tfrac{1}{7}.$$

11. $4\sqrt{v^2 - v - 4} + 3 = 15.$
$$4\sqrt{v^2 - v - 4} = 12.$$
$$v^2 - v - 4 = 9.$$
$$v^2 - v - 13 = 0.$$
$$\therefore v = \frac{1 \pm \sqrt{53}}{2}.$$

12. $\sqrt{x + 4} = \sqrt[4]{x^2 - 5x + 6}.$
$$x + 4 = \sqrt[4]{(x^2 - 5x + 6)^2}.$$
$$x^2 + 8x + 16 = x^2 - 5x + 6.$$
$$13x = -10.$$
$$\therefore x = -\tfrac{10}{13}.$$

13. $\sqrt[3]{x+1}=\sqrt{x+1}$.

$\sqrt[8]{x^2+2x+1}=x+1.$

$x^2+2x+1=x^3+3x^2+3x+1.$

$x^3+2x^2+x=0.$

$x(x^2+2x+1)=0.$

$\therefore\ x=0,-1.$

14. $(s-2)^{\frac{1}{2}}=s^{\frac{1}{2}}+2^{\frac{1}{2}}$.

$s-2=s+2+2\sqrt{2s}.$

$-4=2\sqrt{2s}.$

$16=8s.$

$\therefore\ s=2.$ 2 is extraneous.

15. $3\sqrt{r+1}-2\sqrt{r+3}=\sqrt{2r+4}-\sqrt{r+3}+2\sqrt{r+1}$.

$$\sqrt{r+1}-\sqrt{r+3}=\sqrt{2r+4}.$$

$$r+1-2\sqrt{r^2+4r+3}+r+3=2r+4.$$

$$r^2+4r+3=0.$$

$\therefore\ r=-1,-3.$ -1 is extraneous.

16. $\sqrt{2x-3}+3\sqrt{x-5}=\sqrt{3x-8}+2\sqrt{x-5}$.

$$\sqrt{2x-3}+\sqrt{x-5}=\sqrt{3x-8}.$$

$$2x-3+2\sqrt{2x^2-13x+15}+x-5=x-8.$$

$$2x^2-13x+15=0.$$

$$\therefore\ x=5,\ \tfrac{3}{2}.$$

17. $\sqrt{4x-12}+\sqrt{5x-2}+\sqrt{9x-14}=0$.

$$\sqrt{4x-12}+\sqrt{5x-2}=-\sqrt{9x-14}.$$

$$4x-12+5x-2+2\sqrt{(4x-12)(5x-2)}=9x-14.$$

$$\sqrt{(4x-12)(5x-2)}=0.$$

$\therefore\ x=3,\ \tfrac{2}{5}.$ Both are extraneous.

18. $\sqrt{4x-12}-\sqrt{5x-2}-\sqrt{9x-14}=0$.

$$\sqrt{4x-12}-\sqrt{5x-2}=\sqrt{9x-14}.$$

$$4x-12+5x-2-2\sqrt{(4x-12)(5x-2)}=9x-14.$$

$$-\sqrt{(4x-12)(5x-2)}=0.$$

$\therefore\ x=3,\ \tfrac{2}{5}.$ 3 is extraneous.

19. $\sqrt{4x-12}+\sqrt{5x-2}-\sqrt{9x-14}=0$.

$$\sqrt{4x-12}+\sqrt{5x-2}=\sqrt{9x-14}.$$

$$4x-12+5x-2+2\sqrt{(4x-12)(5x-2)}=9x-14.$$

$$\sqrt{(4x-12)(5x-2)}=0.$$

$$x=3,\ \tfrac{2}{5}.$$

20. $\sqrt{4x-12}-\sqrt{5x-2}+\sqrt{9x-14}=0$.

$$\sqrt{4x-12}-\sqrt{5x-2}=-\sqrt{9x-14}.$$

$$4x-12+5x-2-2\sqrt{(4x-12)(5x-2)}=9x-14.$$

$$\sqrt{(4x-12)(5x-2)}=0.$$

$x=3,\ \tfrac{2}{5}.$ $\tfrac{2}{5}$ is extraneous.

21. $\dfrac{7\sqrt{n}+10}{\sqrt{4n}-2}=3.$

$$7\sqrt{n}+10=6\sqrt{n}-6.$$
$$\sqrt{n}=-16.$$
$$n=256. \quad 256 \text{ is extraneous.}$$

22. $\dfrac{2\sqrt{a}}{\sqrt{2x-a}}=\dfrac{\sqrt{2x+4a}}{3\sqrt{a}}.$

$$6a=\sqrt{4x^2+6ax-4a^2}.$$
$$36a^2=4x^2+6ax-4a^2.$$
$$2x^2+3ax-20a^2=0.$$
$$x=\frac{5a}{2}, \ -4a. \quad -4a \text{ is extraneous.}$$

23. $\dfrac{n^{\frac{1}{2}}-3}{n^{\frac{1}{2}}}-\dfrac{5-n^{\frac{1}{2}}}{4}=0.$

$$4\sqrt{n}-12=5\sqrt{n}-n.$$
$$n-\sqrt{n}-12=0.$$
$$(\sqrt{n}-4)(\sqrt{n}+3)=0.$$
$$\therefore\ n=16, \text{ or } n=9. \quad 9 \text{ is extraneous.}$$

24. $\dfrac{\sqrt{x+16}}{\sqrt{4-x}}+\dfrac{\sqrt{4-x}}{\sqrt{x+16}}=\dfrac{5}{2}.$

$$2x+32+8-2x=5\sqrt{-x^2-12x+64}.$$
$$-x^2-12x+64=64.$$
$$x=0, \ -12.$$

25. $5r-13r^{\frac{1}{2}}+6=0.$

$$(5r^{\frac{1}{2}}-3)(r^{\frac{1}{2}}-2)=0.$$
$$r=\tfrac{9}{25}, \ r=4.$$

26. $7x-3x^{\frac{1}{2}}-2x^{\frac{3}{2}}=0.$

$$x^{\frac{1}{2}}(2x-7x^{\frac{1}{2}}+3)=0.$$
$$x^{\frac{1}{2}}(2x^{\frac{1}{2}}-1)(x^{\frac{1}{2}}-3)=0.$$
$$x=0, \ \tfrac{1}{4}, \ 9.$$

27. $\dfrac{(r+5)^{\frac{1}{2}}}{(r+3)^{\frac{1}{2}}}-\dfrac{(r+3)^{\frac{1}{2}}}{(r+5)^{\frac{1}{2}}}=\dfrac{2}{\sqrt{3}}.$

$$(r+5)\sqrt{3}-(r+3)\sqrt{3}=2\sqrt{r^2+8r+15}.$$
$$r\sqrt{3}+5\sqrt{3}-r\sqrt{3}-3\sqrt{3}=2\sqrt{r^2+8r+15}.$$
$$\sqrt{3}=\sqrt{r^2+8r+15}.$$
$$r^2+8r+12=0.$$
$$r=-6, \ -2.$$

28. $\sqrt{7+4x+3\sqrt{2x^2+5x+7}}-3=0.$

$$7+4x+3\sqrt{2x^2+5x+7}=9.$$
$$3\sqrt{2x^2+5x+7}=2-4x.$$
$$18x^2+45x+63=4-16x+16x^2.$$
$$2x^2+61x+59=0.$$
$$x=-\tfrac{59}{2},\ -1.$$

29. $\sqrt{17+2\sqrt{3+s+\sqrt{s+7}}}-5=0.$

$$17+2\sqrt{3+s+\sqrt{s+7}}=25.$$
$$3+s+\sqrt{s+7}=16.$$
$$s+7=169-26s+s^2.$$
$$s^2-27s+162=0.$$
$$s=9,\ 18. \quad 18 \text{ is extraneous.}$$

30. $4x^2=10x+10-2\sqrt{4x^2-10x-2}.$

$$4x^2-10x-2+2\sqrt{4x^2-10x-2}+1=8+1.$$
$$\sqrt{4x^2-10x-2}+1=\pm 3.$$
$$\sqrt{4x^2-10x-2}=2,\ -4.$$
$$4x^2-10x-6=0.$$
$$x=3,\ -\tfrac{1}{2}.$$
$$\sqrt{4x^2-10x-2}=-4.$$
$$4x^2-10x-18=0.$$
$$\therefore x=\frac{5\pm\sqrt{97}}{4}. \quad \text{Both are extraneous.}$$

31. $3m^2=6\sqrt{3m^2-m-6}+m+22.$

$$3m^2-m-6-6\sqrt{3m^2-m-6}+9=16+9.$$
$$\sqrt{3m^2-m-6}-3=\pm 5.$$
$$\sqrt{3m^2-m-6}=8,\ -2.$$
$$3m^2-m-6=64,\ 4.$$
$$3m^2-m-70=0.$$
$$m=5,\ -4\tfrac{2}{3}.$$
$$3m^2-m-6=4.$$
$$3m^2-m-10=0.$$
$$m=2,\ -\tfrac{5}{3}. \quad \text{Both are extraneous.}$$

The teacher should note that the roots resulting in the solutions of Exercises 30 and 31 from the equations formed by setting the radical expression equal to the negative number are extraneous. This step, then, need not be performed.

32. $\sqrt{x+15}+\sqrt{x-24}-\sqrt{x-13}=\sqrt{x}.$

$$\sqrt{x+15}-\sqrt{x-13}=\sqrt{x}-\sqrt{x-24}.$$
$$x+15+x-13-2\sqrt{x^2+2x-195}=x+x-24-2\sqrt{x^2-24x}.$$
$$\sqrt{x^2+2x-195}=\sqrt{x^2-24x}+13.$$

$$x^2 + 2x - 195 = x^2 - 24x + 169 - 26\sqrt{x^2 - 24x}.$$
$$26x - 364 = -26\sqrt{x^2 - 24x}.$$
$$x - 14 = -\sqrt{x^2 - 24x}.$$
$$x^2 - 28x + 196 = x^2 - 24x.$$
$$-4x = -196.$$
$$x = 49.$$

33. $t = \pi\sqrt{\frac{l}{g}}.$

$$t^2 = \pi^2 \frac{l}{g}.$$
$$gt^2 = \pi^2 l.$$
$$\therefore\ l = \frac{gt^2}{\pi^2},$$

and $$g = \frac{\pi^2 l}{t^2}.$$

34. $s = \frac{gt^2}{2}.$

$$2s = gt^2.$$
$$t^2 = \frac{2s}{g}.$$
$$t = \sqrt{\frac{2s}{g}}.$$
$$t = \frac{\sqrt{2sg}}{g}.$$

35. $a = \frac{R}{2}\sqrt{2}$ and $K = 2R^2.$

$$2a = R\sqrt{2}.$$
$$R = a\sqrt{2}.$$
$$K = 2(a\sqrt{2})^2.$$
$$K = 4a^2.$$

36. $K = \frac{3R^2}{2}\sqrt{3}$ and $a = \frac{R}{2}\sqrt{3}.$

$$2a = R\sqrt{3}.$$
$$R = \frac{2a}{\sqrt{3}}.$$
$$K = \frac{3R^2}{2}\sqrt{3}.$$
$$K = \frac{3}{2}\left(\frac{2a}{\sqrt{3}}\right)^2\sqrt{3}$$
$$= 2a^2\sqrt{3}.$$

37. $K = 2r^2\sqrt{2}$ and $a = \frac{r}{2}\sqrt{2 + \sqrt{2}}.$

$$2a = r\sqrt{2 + \sqrt{2}}.$$
$$r = \frac{2a}{\sqrt{2 + \sqrt{2}}}.$$
$$K = 2r^2\sqrt{2}.$$
$$K = 2\left(\frac{2a}{\sqrt{2 + \sqrt{2}}}\right)^2\sqrt{2} = \frac{2 \cdot 4a^2}{2 + \sqrt{2}}\sqrt{2} = \frac{8a^2}{\sqrt{2} + 1} = 8a^2(\sqrt{2} - 1).$$

38. $K = 3r^2$ and $a = \frac{r}{2}\sqrt{2 + \sqrt{3}}.$

$$2a = r\sqrt{2 + \sqrt{3}}.$$
$$r = \frac{2a}{\sqrt{2 + \sqrt{3}}}.$$
$$K = 3r^2 = 3\left(\frac{2a}{\sqrt{2 + \sqrt{3}}}\right)^2 = \frac{3 \cdot 4a^2}{2 + \sqrt{3}} = \frac{12a^2}{2 + \sqrt{3}} = 12a^2(2 - \sqrt{3}).$$

39. Let $x =$ the side of the first square.

Then $\dfrac{4x - 72}{4} =$ the side of the second square.

$$x^2 - (x - 18)^2 = 900.$$
$$36x - 324 = 900.$$
$$36x = 1224.$$
$$x = 34,$$
and
$$x - 18 = 16.$$

40. $V = \sqrt{\frac{64}{5280} \cdot 4000}$ miles.

$$V = \sqrt{\tfrac{1600}{33}}.$$
$$V = \tfrac{40}{33}\sqrt{33}.$$
$$V = 7.$$

41. $.976\,v^2x - gx^2 = 0.$

$$.976 \cdot 10{,}000\,x - 32\,x^2 = 0.$$
$$9760\,x - 32\,x^2 = 0.$$
$$x\,(9760 - 32\,x) = 0.$$
$$x = 0,\ 305. \quad 0 \text{ is impossible.}$$

42. Substituting in the formula given in Problem 41,

$$.976\,v^2 - 32 \cdot 426.52 = 0.$$
$$.976\,v^2 - 13{,}648.6 = 0.$$
$$.976\,v^2 = 13{,}648.6.$$
$$v = \sqrt{\frac{13{,}648.6}{.976}} = 118.2.$$

43. Substituting in the formula of Problem 41:

(*a*) $$(.976\,v^2 - 32 \cdot 497.63)\,497.63 = 0,$$
$$.976\,v^2 = 15{,}924.76.$$
$$v^2 = 16{,}315.$$
$$v = 127.7.$$

(*b*) $$51\,(.976\,v^2 - 32 \cdot 51) = 0.$$
$$.976\,v^2 = 1632.$$
$$v^2 = 1672.$$
$$v = 40.8.$$

(*c*) $$184.3\,(.976\,v^2 - 32 \cdot 184.3) = 0.$$
$$.976\,v^2 = 5897.6.$$
$$v^2 = 6042.6.$$
$$v = 77.7.$$

(*d*) $$200\,(.976\,v^2 - 32 \cdot 200) = 0.$$
$$.976\,v^2 - 32 \cdot 200 = 0.$$
$$.976\,v^2 = 6400.$$
$$v^2 = 6557.3.$$
$$v = 80.9.$$

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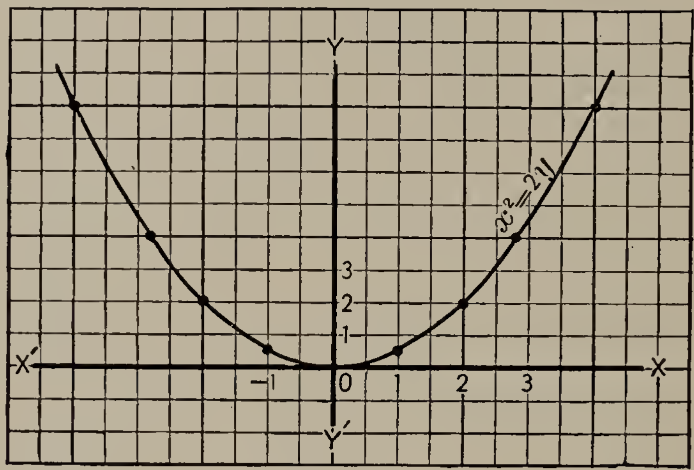
1. $x^2 = 2y. \quad \therefore x = \pm\sqrt{2y}.$

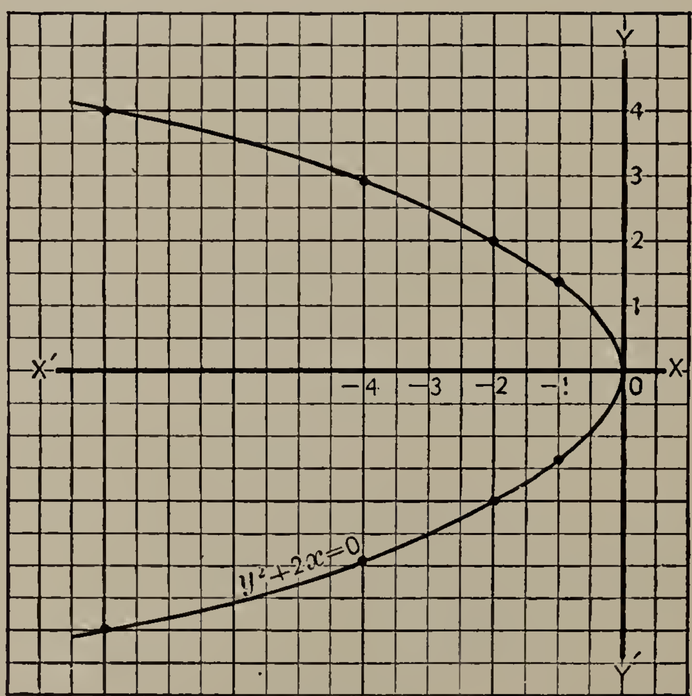
x	± 4	± 3.46	± 2.82	± 2	± 1.41	0	Imaginary
y	8	6	4	2	1	0	Negative values



Parabola

2. $y^2 + 2x = 0. \quad \therefore y = \pm\sqrt{-2x}.$

x	0	-1	-2	-4	-6	-8	Positive values
y	0	± 1.41	± 2	± 2.82	± 3.46	± 4	Imaginary

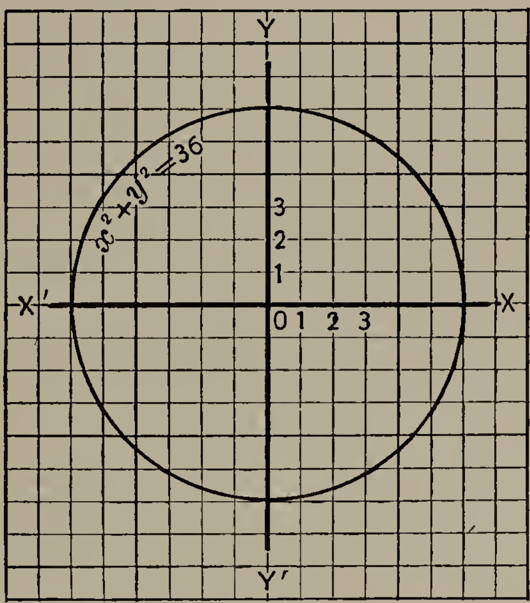


Parabola

3. $x^2 + y^2 = 36. \quad \therefore\ y = \pm\sqrt{36 - x^2}.$

x	± 6	± 5	± 4	± 3	± 2	± 1	0
y	0	± 3.31	± 4.47	± 5.19	± 5.65	± 5.91	± 6

or $r = 6$.

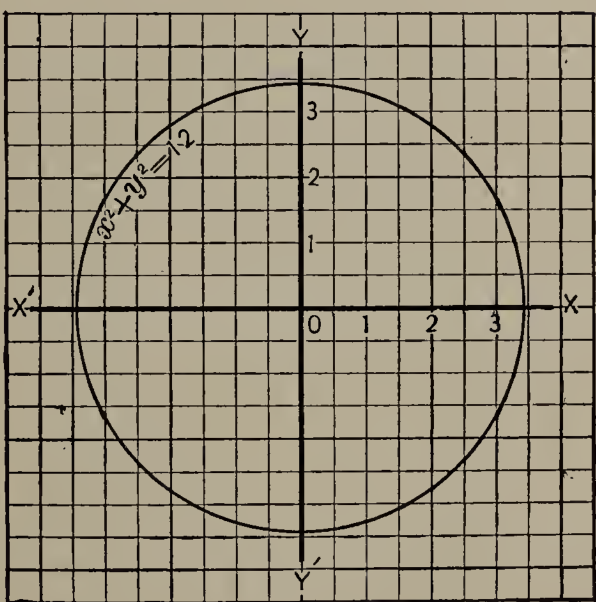


Circle

4. $x^2 + y^2 = 12. \quad \therefore\ y = \pm\sqrt{12 - x^2}.$

x	0	± 1	± 2	± 3	± 3.46
y	± 3.46	± 3.31	± 2.82	± 1.73	0

or $r = 3.46$.

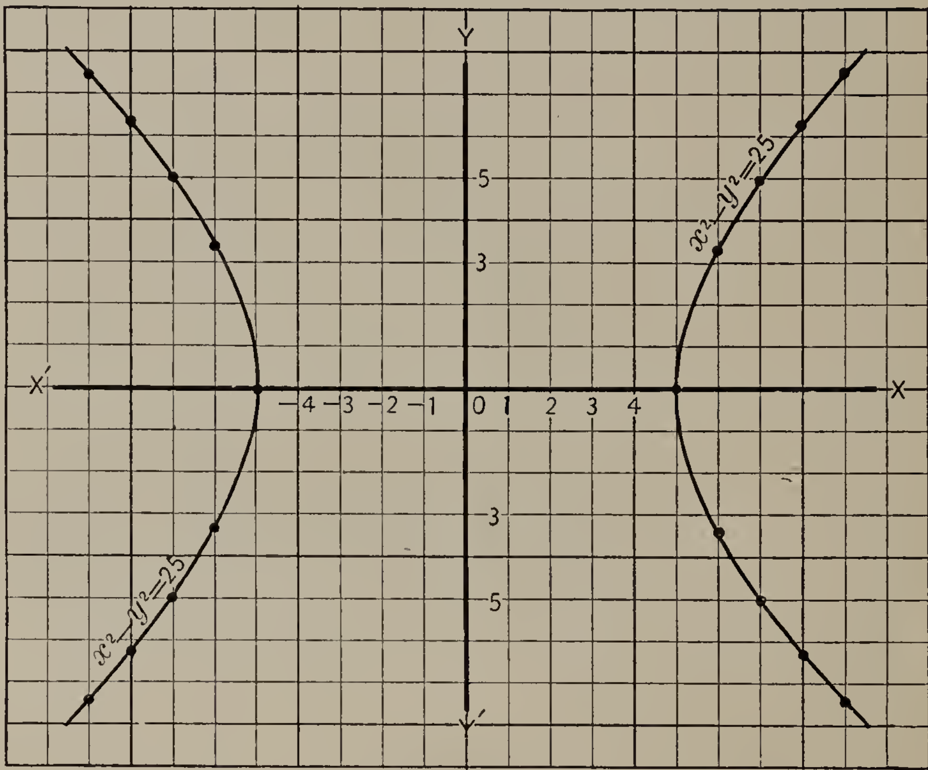


Circle

5. $x^2 - y^2 = 25. \quad \therefore\ y = \pm\sqrt{x^2 - 25}.$

x	Values between 5 and -5	$+5$	$+6$	$+7$	$+8$	$+9$	-5	-6	-7	-8	-9
y	Imaginary	0	± 3.31	± 4.89	± 6.24	± 7.48	0	± 3.31	± 4.89	± 6.24	± 7.48

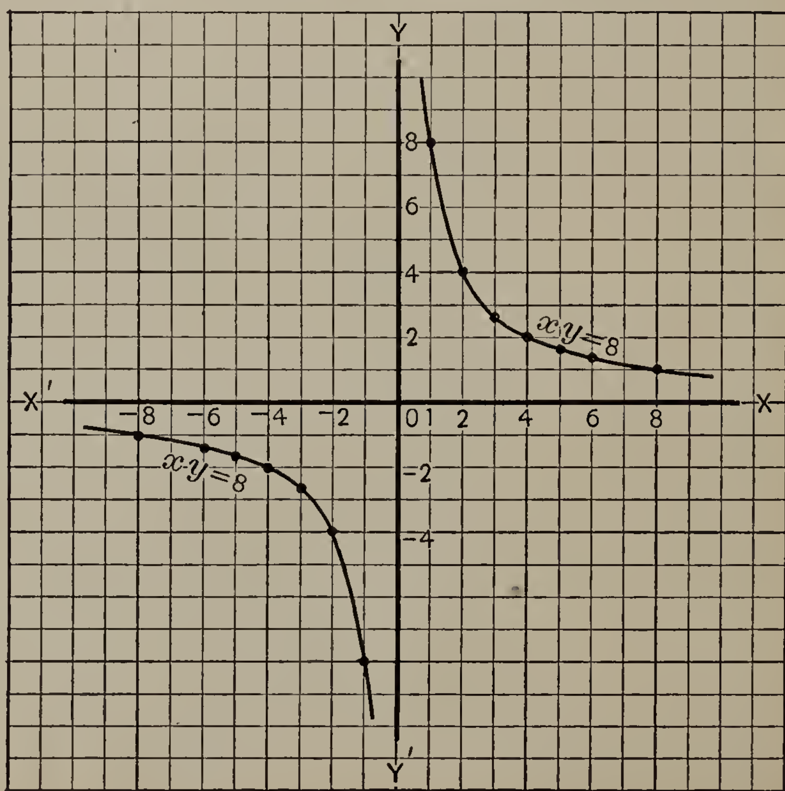
Hyperbola



6. $xy = 8. \quad \therefore\ y = \dfrac{8}{x}.$

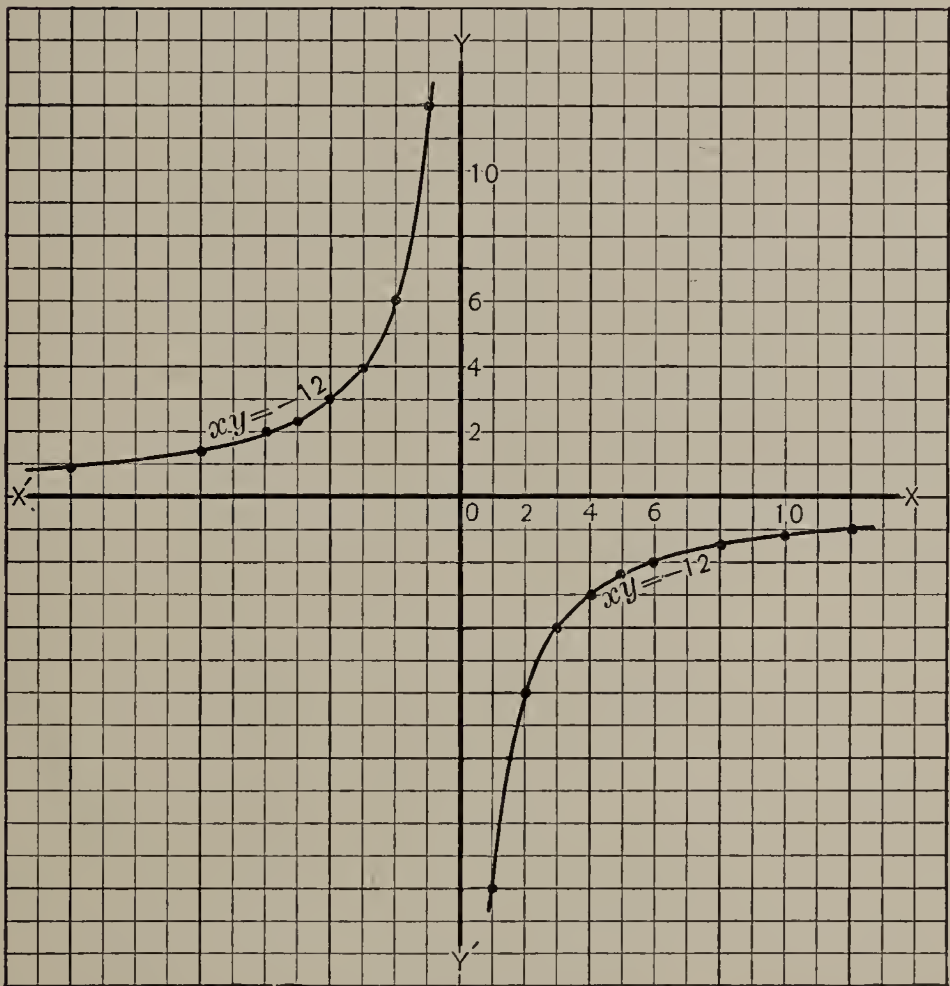
x	± 8	± 6	± 5	± 4	± 3	± 2	± 1	$\pm \frac{1}{2}$
y	± 1	$\pm \frac{4}{3}$	$\pm \frac{8}{5}$	± 2	$\pm \frac{8}{3}$	± 4	± 8	± 16

Hyperbola



7. $xy = -12. \quad \therefore y = \dfrac{-12}{x}.$

x	± 12	± 10	± 8	± 6	± 5	± 4	± 3	± 2	± 1
y	∓ 1	$\mp \frac{6}{5}$	$\mp \frac{3}{2}$	∓ 2	$\mp \frac{12}{5}$	∓ 3	∓ 4	∓ 6	∓ 12

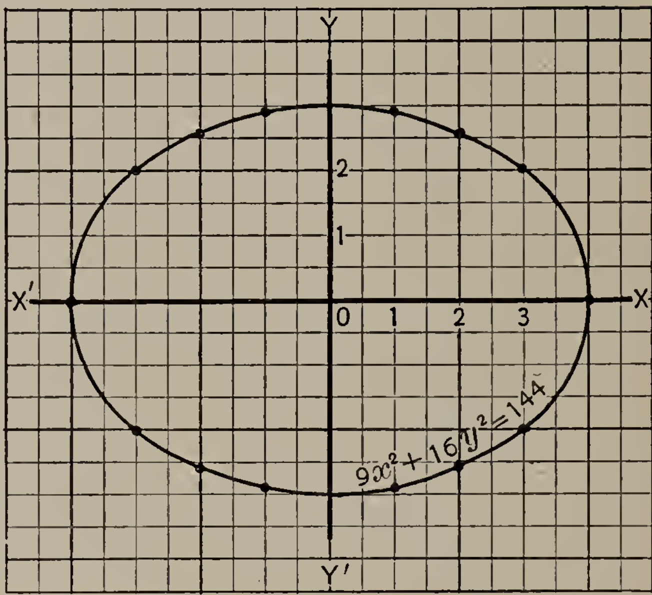


Hyperbola

8. $9x^2 + 16y^2 = 144. \quad \therefore\ y = \pm \frac{3}{4}\sqrt{16 - x^2}.$

x	4	3	2	1	0	-1	-2	-3	-4
y	0	± 1.98	± 2.59	± 2.90	± 3	± 2.90	± 2.59	± 1.98	0

Ellipse

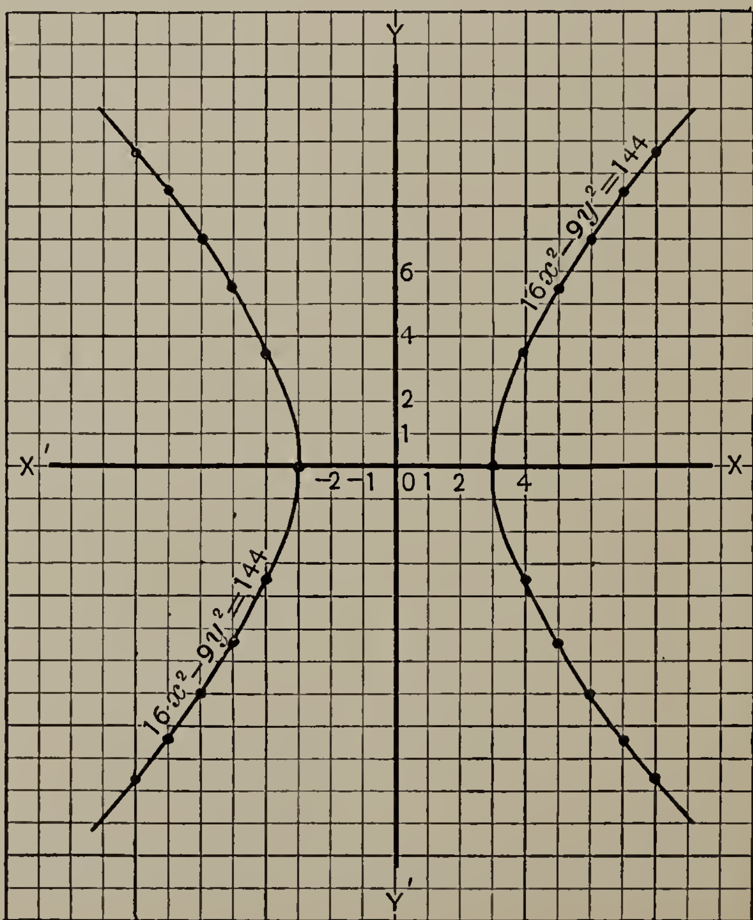


9. $16x^2 - 9y^2 = 144.$

$\therefore\ y = \pm \frac{4}{3}\sqrt{x^2 - 9}.$

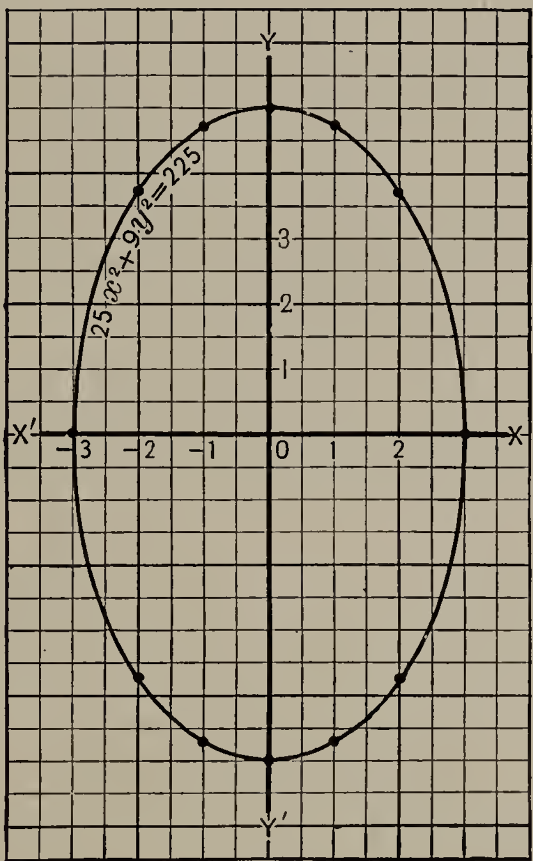
x	8	7	6	5	4	± 3	-4	-5	-6	-7	-8
y	± 9.88	± 8.43	± 6.92	± 5.33	± 3.52	0	± 3.52	± 5.33	± 6.92	± 8.43	± 9.88

Hyperbola



10. $25x^2 + 9y^2 = 225. \quad \therefore\ y = \pm\frac{5}{3}\sqrt{9 - x^2}.$

x	3	2	1	0	-1	-2	-3
y	0	± 3.72	± 4.71	± 5	± 4.71	± 3.72	0



Ellipse

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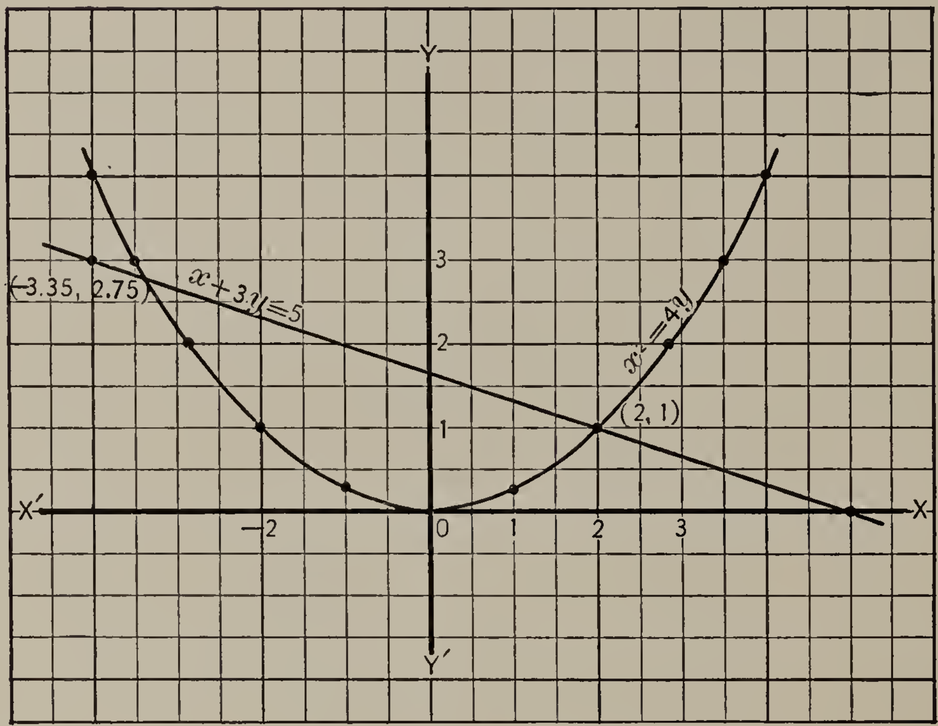
1. $x^2 = 4y$. $\therefore x = \pm 2\sqrt{y}$.

x	0	± 2	± 2.82	± 3.46	± 4
y	0	1	2	3	4



$x + 3y = 5$.

x	0	5
y	$\frac{5}{3}$	0

From the graphs

x	2	-3.35
y	1	2.75

2. $x^2 + y^2 = 25$.

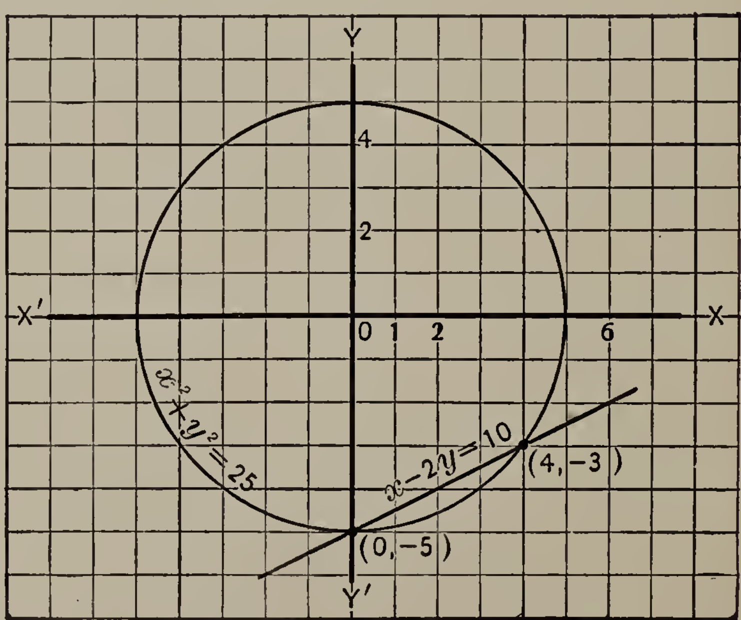
Circle, center (0,0) and $r = 5$.

$x - 2y = 10$.

x	0	6
y	-5	-2

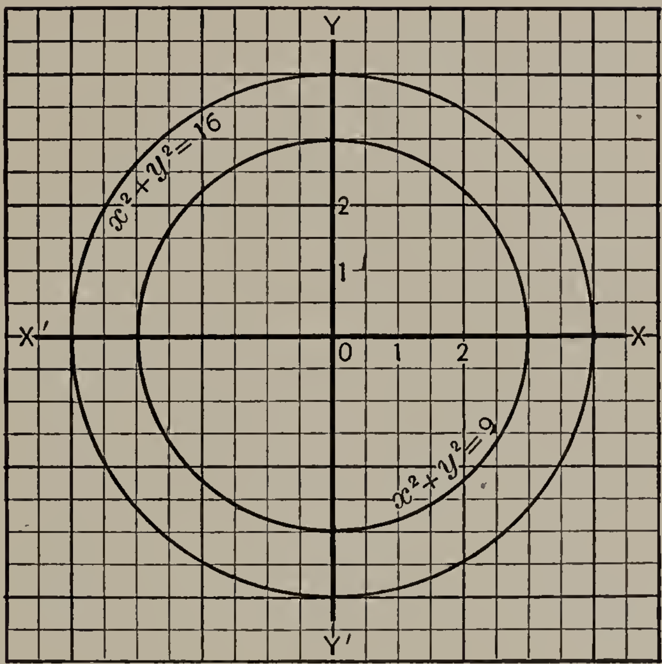
From the graphs

x	0	4
y	-5	-3



3. $x^2 + y^2 = 16.$

Circle, center at (0,0) and $r = 4$.



$x^2 + y^2 = 9.$

Circle, center at (0,0) and $r = 3$.

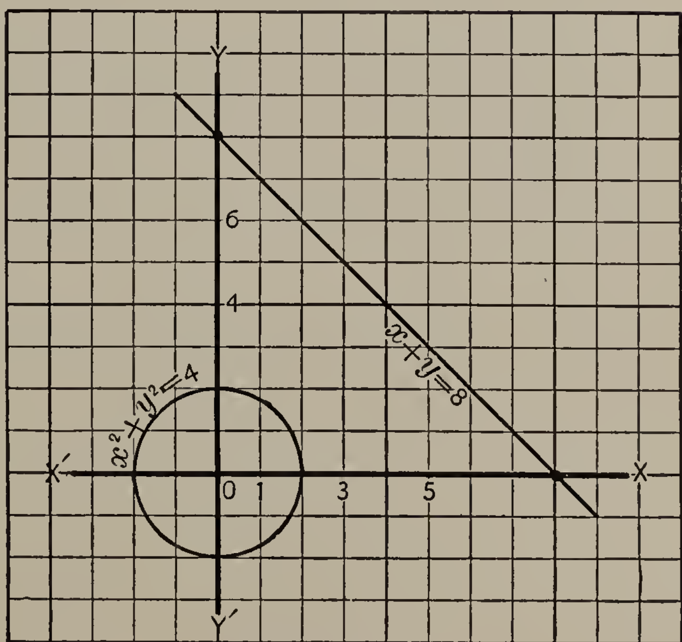
A graphical solution is impossible, for the graphs do not intersect.

4. $x^2 + y^2 = 4.$

Circle, center at (0,0) and $r = 2$.

$x + y = 8.$

x	0	8
y	8	0



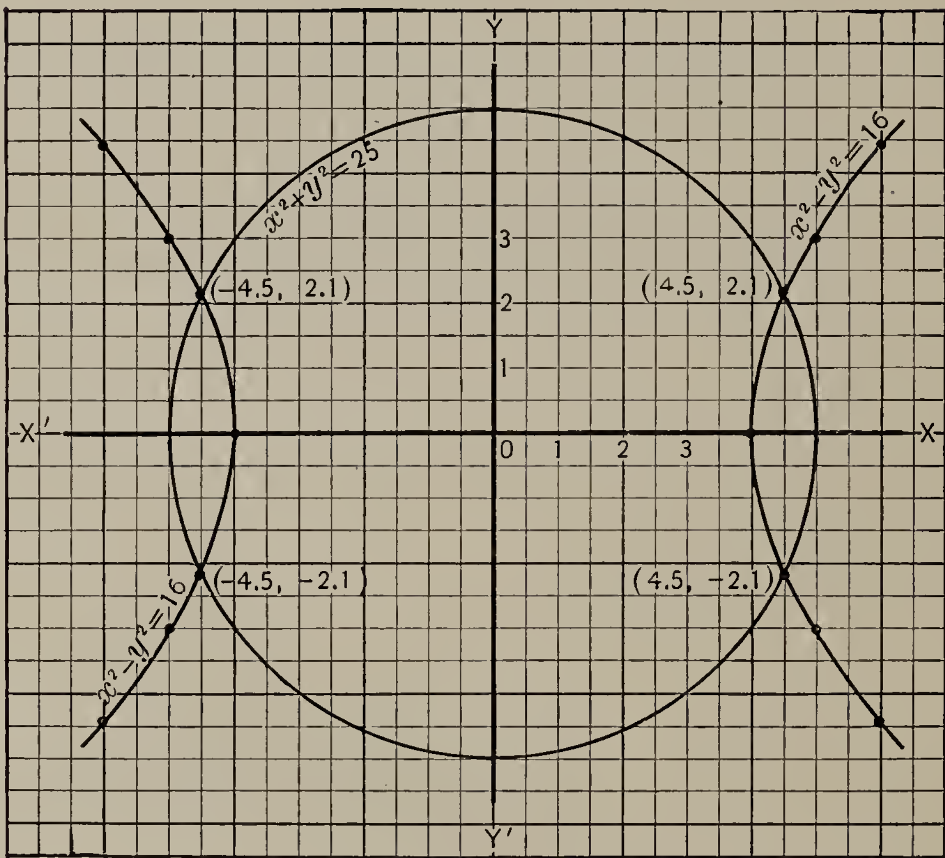
A graphical solution is impossible, for the graphs do not intersect.

5. $x^2 + y^2 = 25.$

Circle, center (0,0) and $r = 5$.

$x^2 - y^2 = 16. \quad \therefore y = \pm\sqrt{x^2 - 16}.$

x	6	5	4	-4	-5	-6
y	± 4.47	± 3	0	0	± 3	± 4.47



From the graphs

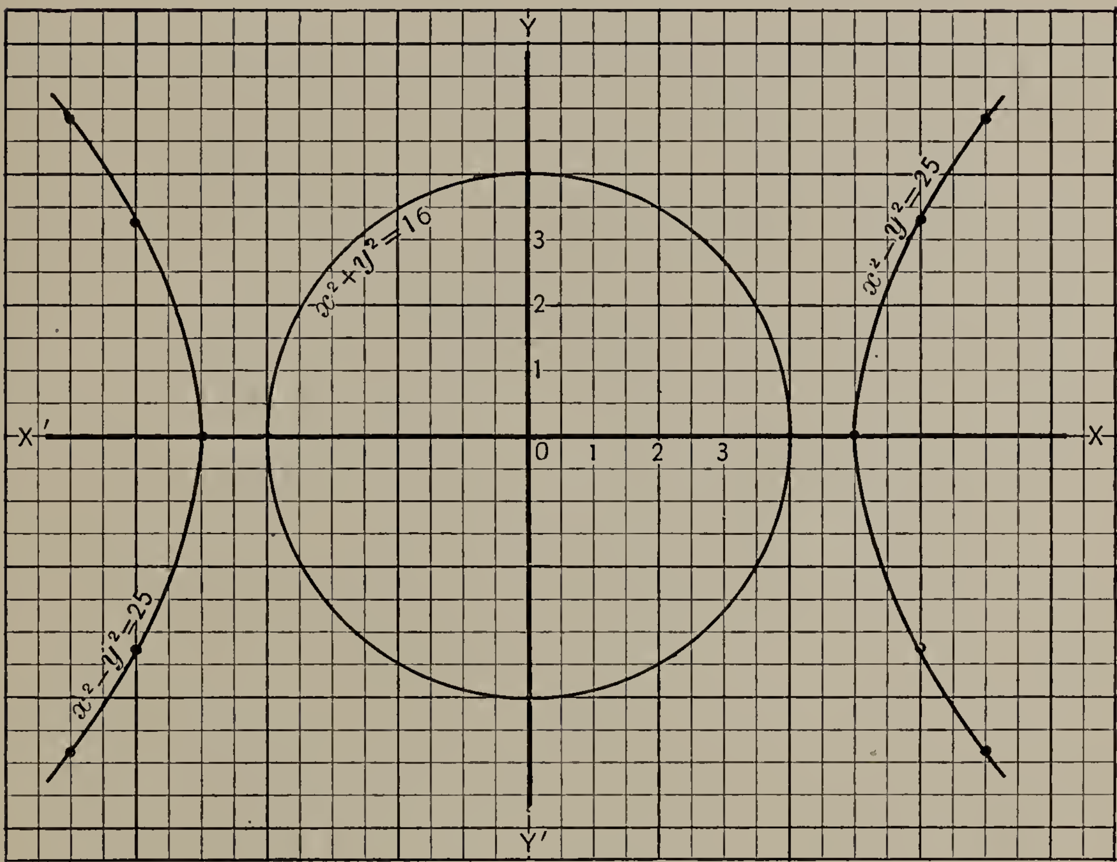
x	4.5	-4.5
y	± 2.1	± 2.1

6. $x^2 + y^2 = 16.$

Circle, center at (0,0) and $r = 4.$

$x^2 - y^2 = 25. \quad \therefore y = \pm\sqrt{x^2 - 25}.$

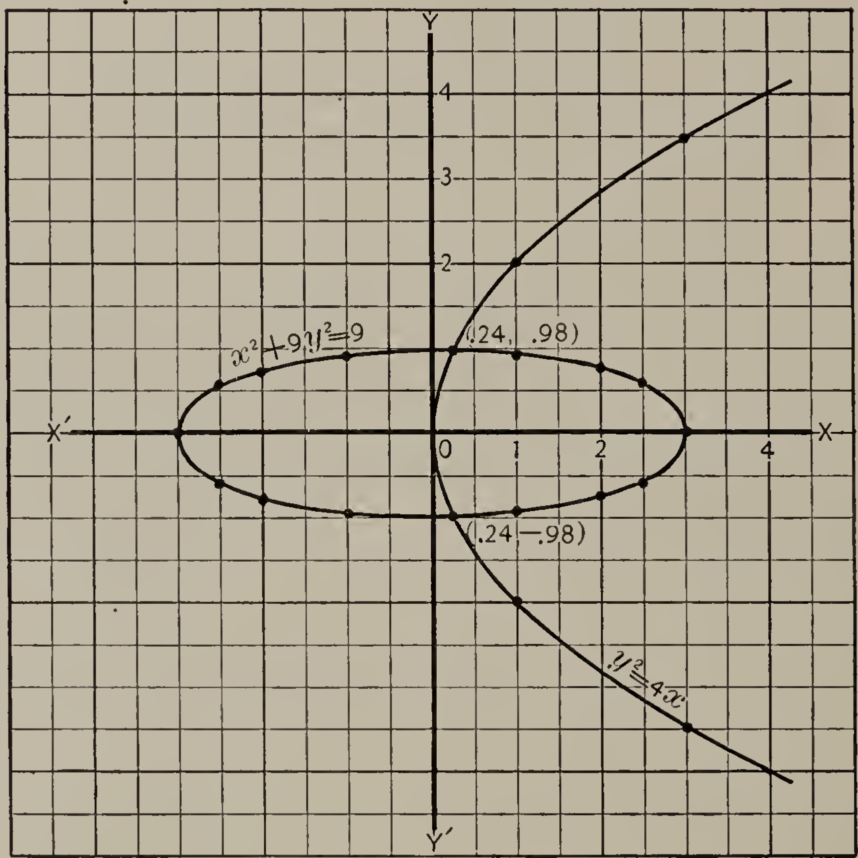
x	7	6	5	-5	-6	-7
y	± 4.89	± 3.31	0	0	± 3.31	± 4.89



The system has no graphical solution, for the curves do not intersect.

7. $y^2 = 4x \quad \therefore y = \pm 2\sqrt{x}.$

x	4	3	2	1	0
y	± 4	± 3.46	± 2.82	± 2	0



$x^2 + 9y^2 = 9. \quad \therefore y = \pm \frac{1}{3}\sqrt{9 - x^2}.$

x	3	2	1	0	-1	-2	-3	$\frac{5}{2}$	$-\frac{5}{2}$
y	0	$\pm .74$	$\pm .94$	± 1	± 9.4	$\pm .74$	0	$\pm .55$	$\pm .55$

From the graphs

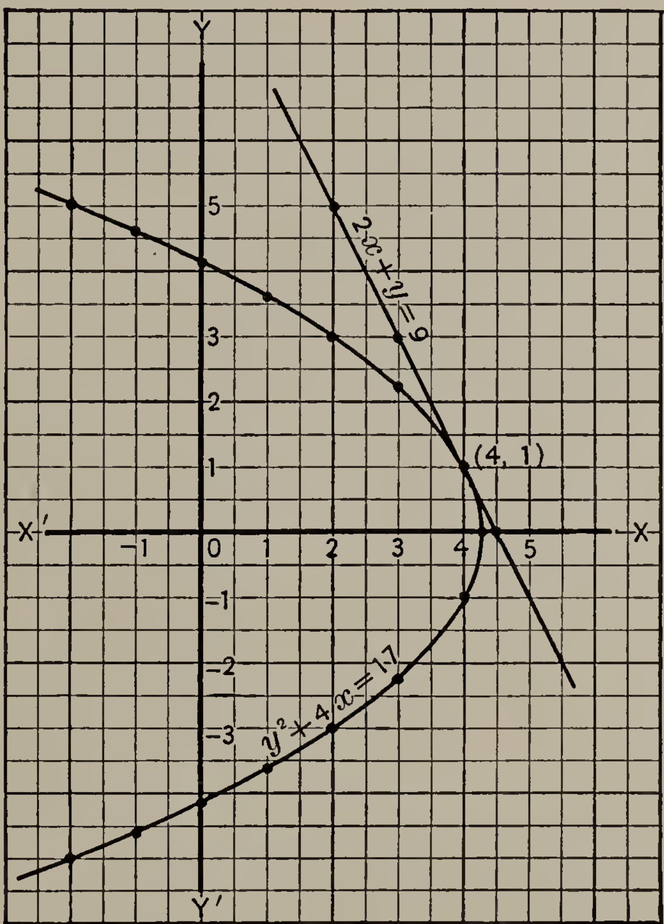
x	.24	.24
y	.98	$-.98$

8. $y^2 + 4x = 17. \quad \therefore y = \pm\sqrt{17 - 4x}.$

x	4	3	2	1	0	-1	-2	$4\frac{1}{4}$
y	± 1	± 2.23	± 3	± 3.60	± 4.12	± 4.58	± 5	0

$$2x + y = 9.$$

x	$\frac{9}{2}$	3	2
y	0	3	5



From the graphs

x	4
y	1

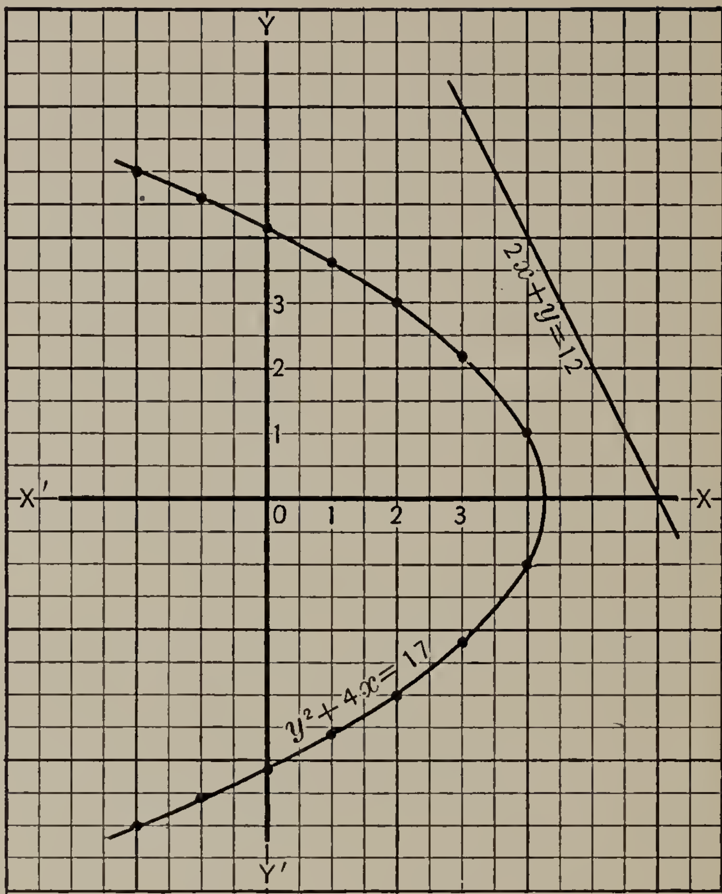
9. $y^2 + 4x = 17.$

For values of x and y, see Exercise 8.

$2x + y = 12.$

x	6	4	3
y	0	4	6

The system has no graphical solution, for the curves do not intersect.



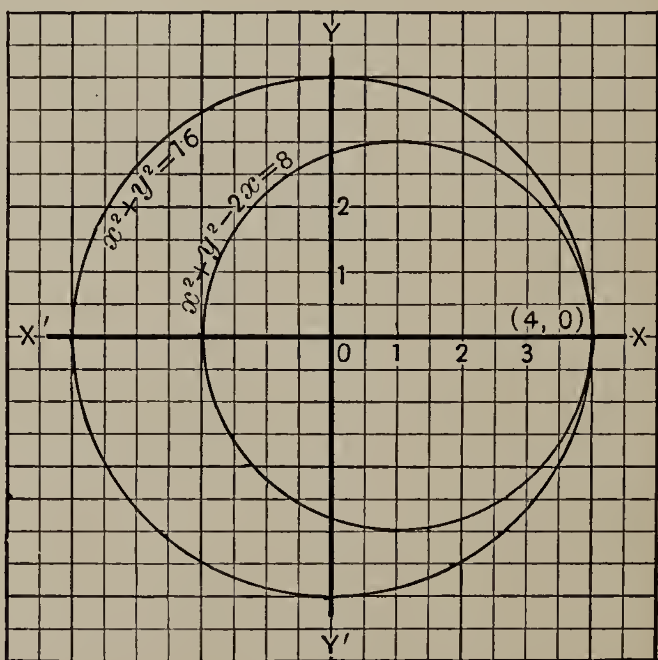
10. $x^2 + y^2 = 16.$

Circle, center at (0,0) and $r = 4$.

$x^2 + y^2 - 2x = 8. \quad \therefore y = \pm\sqrt{8 + 2x - x^2}.$

x	4	3	2	1	0	-1	-2
y	0	± 2.23	± 2.82	± 3	± 2.82	± 2.23	0

or circle, center at (0,1) and $r = 3$.



From the graphs

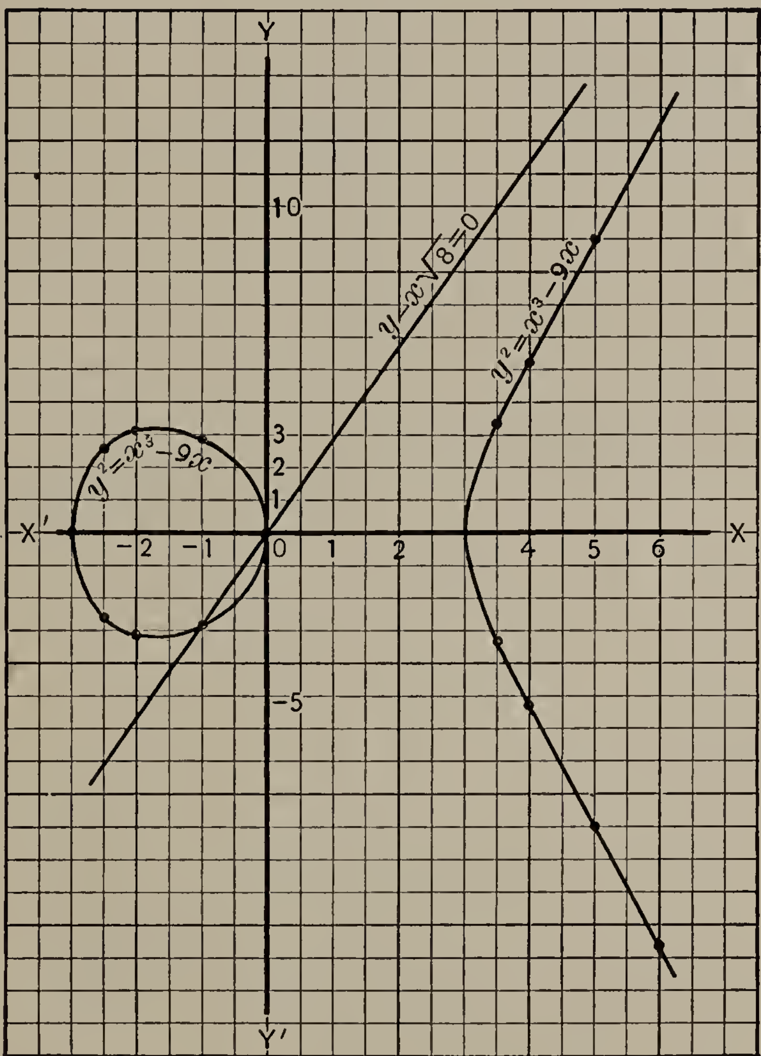
x	4
y	0

11. $y^2 = x^3 - 9x. \quad \therefore y = \pm\sqrt{x^3 - 9x}.$

x	0	3	3.5	4	5	6	-1	-2	-3
y	0	0	± 3.37	± 5.29	± 8.94	± 12.72	± 2.82	± 3.16	0

$$y - x\sqrt{8} = 0.$$

x	0	1	-1
y	0	2.82	-2.82



From the graphs

x	-1	0
y	-2.82	0

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1.
$$x + y = 6, \qquad (1)$$
$$x^2 + y^2 = 20. \qquad (2)$$
$$x = 6 - y. \qquad (3)$$
Substituting $6 - y$ for x in (2),
$$36 - 12y + y^2 + y^2 = 20. \qquad (4)$$
$$2y^2 - 12y + 16 = 0. \qquad (5)$$
$$y = \frac{12 \pm \sqrt{144 - 128}}{4}.$$
$$y = \frac{12 \pm 4}{4}.$$
$$y = 4,\ 2.$$
Substituting 4 for y in (3), $x = 6 - 4 = 2.$
Substituting 2 for y in (3), $x = 6 - 2 = 4.$
$$x = 2,\ 4.$$
$$y = 4,\ 2.$$

2.
$$4m + n = 28, \qquad (1)$$
$$2m^2 + 3mn = 98. \qquad (2)$$
$$n = 28 - 4m. \qquad (3)$$
From (3) and (2), $2m^2 + 84m - 12m^2 = 98. \qquad (4)$
$$-10m^2 + 84m - 98 = 0. \qquad (5)$$
$$m = \frac{-84 \pm \sqrt{7056 - 3920}}{-20}.$$
$$m = \frac{-84 \pm 56}{-20}.$$
$$m = 7,\ \tfrac{7}{5}.$$
Substituting 7 for m in (3), $n = 28 - 28.$
$$n = 0.$$
Substituting $\frac{7}{5}$ for m in (3), $n = 28 - \frac{28}{5}.$
$$n = \tfrac{112}{5}.$$
$$m = 7,\ \tfrac{7}{5}.$$
$$n = 0,\ \tfrac{112}{5}.$$

3.
$$m^2 + 2n^2 = 44, \qquad (1)$$
$$m - 2n\sqrt{5} = 0. \qquad (2)$$
$$m = 2n\sqrt{5}. \qquad (3)$$
From (3) and (1), $20n^2 + 2n^2 = 44.$
$$n^2 = 2.$$
$$n = \pm\sqrt{2}.$$
Substituting $\sqrt{2}$ for n in (3), $m = 2\sqrt{2}\sqrt{5} = 2\sqrt{10}.$

Substituting $-\sqrt{2}$ for n in (3), $m = 2(-\sqrt{2})\sqrt{5} = -2\sqrt{10}$.

$$m = 2\sqrt{10},\ -2\sqrt{10}.$$
$$n = \sqrt{2},\ -\sqrt{2}.$$

4.
$$4s + t = 6, \quad (1)$$
$$st = -10. \quad (2)$$
$$t = 6 - 4s. \quad (3)$$

From (2) and (3), $6s - 4s^2 = -10$. (4)

$$2s^2 - 3s - 5 = 0. \quad (5)$$

Whence $s = \frac{5}{2},\ -1$.

Substituting $\frac{5}{2}$ for s in (3), $t = 6 - 10,\ -4$.

Substituting -1 for s in (3), $t = 6 + 4,\ 10$.

$$s = \tfrac{5}{2},\ -1.$$
$$t = -4,\ 10.$$

5.
$$xy + 36 = 0, \quad (1)$$
$$4x - y = 30. \quad (2)$$
$$y = 4x - 30. \quad (3)$$

From (3) and (1), $4x^2 - 30x + 36 = 0$. (4)

$$(x - 6)(2x - 3) = 0. \quad (5)$$

Whence $x = \frac{3}{2},\ 6$.

Substituting $\frac{3}{2}$ for x in (3), $y = 6 - 30 = -24$.

Substituting 6 for x in (3), $y = 24 - 30 = -6$.

$$x = \tfrac{3}{2},\ 6.$$
$$y = -24,\ -6.$$

6.
$$x\sqrt{3} + 5y = -72, \quad (1)$$
$$xy = -15\sqrt{3}. \quad (2)$$
$$x = \frac{-15\sqrt{3}}{y}. \quad (3)$$

Substituting $\dfrac{-15\sqrt{3}}{y}$ for x in (1),

$$\frac{-45}{y} + 5y = -72. \quad (4)$$
$$5y^2 + 72y - 45 = 0. \quad (5)$$
$$(5y - 3)(y + 15) = 0. \quad (6)$$
$$y = -15.$$
$$y = \tfrac{3}{5}.$$

Substituting $\frac{3}{5}$ for y in (3), $x = \dfrac{-15\sqrt{3}}{\frac{3}{5}} = -25\sqrt{3}$.

Substituting -15 for y in (3), $x = \dfrac{-15\sqrt{3}}{-15} = \sqrt{3}$.

$$x = \sqrt{3},\ -25\sqrt{3}.$$
$$y = -15,\ \tfrac{3}{5}.$$

7.

$$3R_1 + 4R_2 = 5. \tag{1}$$

$$2R_1R_2 - 6R_1 = -3. \tag{2}$$

$$R_2 = \frac{5 - 3R_1}{4}. \tag{3}$$

Substituting $\frac{5 - 3R_1}{4}$ for R_2 in (2),

$$\frac{5R_1 - 3R_1^2}{2} - 6R_1 = -3. \tag{4}$$

$$-3R_1^2 - 7R_1 + 6 = 0. \tag{5}$$

$$R_1 = \frac{7 \pm \sqrt{49 + 72}}{-6} = \frac{7 \pm 11}{-6} = -3, \frac{2}{3}.$$

Substituting -3 for R_1 in (3),

$$R_2 = \frac{5 + 9}{4} = \frac{7}{2}.$$

Substituting $\frac{2}{3}$ for R_1 in (3), $R_2 = \frac{5 - 2}{4} = \frac{3}{4}.$

$$R_1 = -3, \tfrac{2}{3}.$$

$$R_2 = \tfrac{7}{2}, \tfrac{3}{4}.$$

8.

$$2xy + y^2 - 20 = 0, \tag{1}$$

$$xy + 40 = 0. \tag{2}$$

$2 \cdot (2)$, $$2xy + 80 = 0. \tag{3}$$

$(1) - (3)$, $$y^2 - 100 = 0. \tag{4}$$

$$y = \pm 10.$$

Substituting 10 for y in (2),

$$10x + 40 = 0. \tag{5}$$

$$x = -4.$$

Substituting -10 for y in (2),

$$-10x + 40 = 0. \tag{6}$$

$$x = 4.$$

$$x = 4, -4.$$

$$y = -10, 10.$$

9.

$$h^2 + k^2 + 2k = 40, \tag{1}$$

$$2 + h + k = 0. \tag{2}$$

$$h = -k - 2. \tag{3}$$

From (3) and (1),

$$k^2 + 4k + 4 + k^2 + 2k = 40. \tag{4}$$

$$2k^2 + 6k - 36 = 0. \tag{5}$$

$$(k - 3)(k + 6) = 0. \tag{6}$$

$$k = -6, 3.$$

Substituting -6 for k in (3), $h = 6 - 2 = 4.$

Substituting 3 for k in (3), $h = -3 - 2 = -5.$

$$h = 4, -5.$$

$$k = -6, 3.$$

10.
$$m^2 + 3\,mn + n^2 = 88, \quad (1)$$
$$2\,m = n. \quad (2)$$
$$m = \frac{n}{2}. \quad (3)$$
From (3) and (1), $\dfrac{n^2}{4} + \dfrac{3\,n^2}{2} + n^2 = 88.$ (4)

Whence $n = \pm\, 4\sqrt{2}.$

Substituting $4\sqrt{2}$ for n in (3), $m = 2\sqrt{2}.$

Substituting $-\,4\sqrt{2}$ for n in (3), $m = -\,2\sqrt{2}.$
$$m = 2\sqrt{2},\ -\,2\sqrt{2}.$$
$$n = 4\sqrt{2},\ -\,4\sqrt{2}.$$

11.
$$x^2 + y^2 + 4\,x + 6\,y = 40, \quad (1)$$
$$x - 10 = y. \quad (2)$$
$$x = y + 10. \quad (3)$$
From (3) and (1), $y^2 + 20\,y + 100 + y^2 + 4\,y + 40 + 6\,y = 40.$ (4)
$$y^2 + 15\,y + 50 = 0. \quad (5)$$
Whence $y = -\,10,\ -\,5.$

Substituting $-\,10$ for y in (3), $x = -\,10 + 10 = 0.$

Substituting $-\,5$ for y in (3), $x = -\,5 + 10 = 5.$
$$x = 0,\ 5.$$
$$y = -\,10,\ -\,5.$$

12.
$$y + x\sqrt{15} = 0. \quad (1)$$
$$y^2 + x^3 = 16x. \quad (2)$$
$$y = -\,x\sqrt{15}. \quad (3)$$
From (3) and (2), $15\,x^2 + x^3 = 16\,x.$ (4)
$$x\,(x + 16)\,(x - 1) = 0. \quad (5)$$
Whence $x = 0,\ -\,16,\ 1.$

Substituting 0 for x in (3), $y = -\,0\sqrt{15} = 0.$

Substituting $-\,16$ for x in (3), $y = 16\sqrt{15}.$

Substituting 1 for x in (3), $y = -\sqrt{15}.$
$$x = 0,\ -\,16,\ 1.$$
$$y = 0,\ 16\sqrt{15},\ -\sqrt{15}.$$

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1.
$$x^2 + xy = 3, \quad (1)$$
$$y^2 - xy = 10. \quad (2)$$
(1) · 10, $10\,x^2 + 10\,xy = 30.$ (3)

(2) · 3, $3\,y^2 - 3\,xy = 30.$ (4)

(3) – (4), $10\,x^2 + 13\,xy - 3\,y^2 = 0.$ (5)

$$(5x - y)(2x + 3y) = 0. \qquad (6)$$

$$\therefore\ x = \frac{y}{5},\ \frac{-3y}{2}. \qquad (7)$$

$$x = \frac{-3y}{2}. \qquad (8)$$

Substituting $\frac{y}{5}$ for x in (2), $\quad y^2 - \frac{y^2}{5} = 10.$

$$\therefore\ y = \pm \tfrac{5}{2}\sqrt{2}.$$

Substituting $\frac{5}{2}\sqrt{2}$ for y in (7), $\quad x = \dfrac{\frac{5}{2}\sqrt{2}}{5} = \frac{1}{2}\sqrt{2}.$

Substituting $-\frac{5}{2}\sqrt{2}$ for y in (7), $\quad x = \dfrac{-\frac{5}{2}\sqrt{2}}{5} = -\frac{1}{2}\sqrt{2}.$

Substituting $\frac{-3y}{2}$ for x in (2),

$$y^2 + \frac{3y^2}{2} = 10.$$

$$y = \pm 2.$$

Substituting 2 for y in (8), $\quad x = -3.$

Substituting -2 for y in (8), $\quad x = 3.$

$$x = \tfrac{1}{2}\sqrt{2},\ -\tfrac{1}{2}\sqrt{2},\ -3,\ 3.$$

$$y = \tfrac{5}{2}\sqrt{2},\ -\tfrac{5}{2}\sqrt{2},\ 2,\ -2.$$

2.

$$x^2 + y^2 = 10, \qquad (1)$$

$$3y^2 + xy = 6. \qquad (2)$$

$$6x^2 + 6y^2 = 60. \qquad (3)$$

$$30y^2 + 10xy = 60. \qquad (4)$$

$$6x^2 - 10xy - 24y^2 = 0. \qquad (5)$$

$$(3x + 4y)(x - 3y) = 0. \qquad (6)$$

$$x = -\frac{4y}{3}. \qquad (7)$$

$$x = 3y. \qquad (8)$$

From (7) and (2), $\quad 3y^2 - \frac{4y^2}{3} = 6.$

$$y = \pm \tfrac{3}{5}\sqrt{10}.$$

Substituting $\frac{3}{5}\sqrt{10}$ for y in (7), $\quad x = -\frac{4}{3} \cdot \frac{3}{5}\sqrt{10} = -\frac{4}{5}\sqrt{10}.$

Substituting $-\frac{3}{5}\sqrt{10}$ for y in (7), $\quad x = -\frac{4}{3},\ -\frac{3}{5}\sqrt{10} = \frac{4}{5}\sqrt{10}.$

From (8) and (2), $\quad 3y^2 + 3y^2 = 6.$

$$y = \pm 1.$$

Substituting 1 for y in (8), $\quad x = 3.$

Substituting -1 for y in (8), $\quad x = -3.$

$$x = \pm \tfrac{4}{5}\sqrt{10},\ \pm 3.$$

$$y = \mp \tfrac{3}{5}\sqrt{10},\ \pm 1.$$

3.
$$u^2 + 2\,uv = 0, \tag{1}$$
$$2\,v^2 + 3\,uv = -16. \tag{2}$$
$$u(u + 2\,v) = 0. \tag{3}$$
$$u = 0. \tag{4}$$
$$u = -2\,v. \tag{5}$$

From (4) and (2), $2\,v^2 + 0 = -16.$

$$v = \pm 2\sqrt{-2}.$$

From (5) and (2), $2\,v^2 - 6\,v^2 = -16.$

$$v = \pm 2.$$

Substituting 2 for v in (5), $u = -4.$

Substituting -2 for v in (5), $u = 4.$

$$u = 0,\ \pm 4.$$
$$v = \pm 2\sqrt{-2},\ \mp 2.$$

4.
$$s^2 - 3\,st = 4, \tag{1}$$
$$3\,t^2 + 3\,s^2 = 12. \tag{2}$$
$$3\,s^2 - 9\,st = 12. \tag{3}$$

(3) – (2), $-9\,st - 3\,t^2 = 0. \quad (4)$

From (4), $t = 0, \quad (5)$

and $t = -3\,s. \quad (6)$

From (6) and (1), $s^2 + 9\,s^2 = 4,\ s = \pm \frac{1}{5}\sqrt{10}.$

Substituting $\frac{1}{5}\sqrt{10}$ for s in (6), $t = -\frac{3}{5}\sqrt{10}.$

Substituting $-\frac{1}{5}\sqrt{10}$ for s in (6), $t = \frac{3}{5}\sqrt{10}.$

Substituting 0 for t in (1), $s^2 = 4.$

$$s = \pm 2.$$
$$s = \pm \tfrac{1}{5}\sqrt{10},\ \pm 2.$$
$$t = \mp \tfrac{3}{5}\sqrt{10},\ 0.$$

5.
$$x(x + 2\,y) = 16, \tag{1}$$
$$y(y - x) = 3. \tag{2}$$
$$x^2 + 2\,xy = 16. \tag{3}$$
$$y^2 - xy = 3. \tag{4}$$
$$3\,x^2 + 6\,xy = 48. \tag{5}$$
$$16\,y^2 - 16\,xy = 48. \tag{6}$$
$$3\,x^2 + 22\,xy - 16\,y^2 = 0. \tag{7}$$
$$(3\,x - 2\,y)(x + 8\,y) = 0. \tag{8}$$
$$x = \frac{2\,y}{3}. \tag{9}$$
$$x = -8\,y. \tag{10}$$

From (9) and (4), $y^2 - \dfrac{2\,y^2}{3} = 3.$

$$y = \pm 3.$$

Substituting 3 for y in (9), $x = 2.$
Substituting -3 for y in (9), $x = -2.$
From (10) and (4), $y^2 + 8y^2 = 3.$

$$y = \pm \tfrac{1}{3}\sqrt{3}.$$

Substituting $\frac{1}{3}\sqrt{3}$ for y in (10), $x = -\frac{8}{3}\sqrt{3}.$
Substituting $-\frac{1}{3}\sqrt{3}$ for y in (10), $x = \frac{8}{3}\sqrt{3}.$

$$x = \pm \tfrac{8}{3}\sqrt{3},\ \pm 2.$$
$$y = \mp \tfrac{1}{3}\sqrt{3},\ \pm 3.$$

6.
$$x^2 + xy + y^2 = 4, \qquad (1)$$
$$x^2 - 2xy = 12. \qquad (2)$$
$$3x^2 + 3xy + 3y^2 = 12. \qquad (3)$$
$$2x^2 + 5xy + 3y^2 = 0. \qquad (4)$$
$$(x + y)(2x + 3y) = 0.$$
$$x = -y. \qquad (5)$$
$$x = -\frac{3y}{2}. \qquad (6)$$

From (5) and (2), $y^2 + 2y^2 = 12,\ y = \pm 2.$
Substituting 2 for y in (5), $x = -2.$
Substituting -2 for y in (5), $x = 2.$
From (6) and (2), $\dfrac{9y^2}{4} + 3y^2 = 12.$

$$y^2 = \tfrac{48}{21}.$$
$$y = \pm \tfrac{4}{7}\sqrt{7}.$$

Substituting $\frac{4}{7}\sqrt{7}$ for y in (6), $x = -\frac{6}{7}\sqrt{7}.$
Substituting $-\frac{4}{7}\sqrt{7}$ for y in (6), $x = \frac{6}{7}\sqrt{7}.$

$$x = \pm 2,\ \pm \tfrac{6}{7}\sqrt{7}.$$
$$y = \mp 2,\ \pm \tfrac{4}{7}\sqrt{7}.$$

7.
$$x^2 + xy = 2y^2, \qquad (1)$$
$$2x^2 + x = 2 + y^2. \qquad (2)$$

From (1), $(x + 2y)(x - y) = 0.$ (3)
Then $x = y.$ (4)

$$x = -2y. \qquad (5)$$

From (4) and (2), $2y^2 + y = 2 + y^2.$ (6)
From (6), $y = -2, 1.$ (7)
From (7) and (4), $x = -2, 1.$ (8)
From (5) and (2), $8y^2 - 2y = 2 + y^2.$ (9)
From (9), $y = \dfrac{1 \pm \sqrt{15}}{7}.$ (10)
From (10) and (5), $x = \dfrac{-2 \mp 2\sqrt{15}}{7}.$ (11)

8.
$$x^2 + 2xy - y^2 = 32, \quad (1)$$
$$2x^2 - 3xy + y^2 = 0. \quad (2)$$

From (2), $(2x - y)(x - y) = 0.$ (3)

Whence $x = \frac{y}{2},$ (4)

and $x = y.$ (5)

From (5) and (1), $y^2 + 2y^2 - y^2 = 32.$

$y = \pm 4.$

Substituting 4 for y in (5), $x = 4.$

Substituting -4 for y in (5), $x = -4.$

From (4) and (1), $\frac{y^2}{4} + y^2 - y^2 = 32.$

$y = \pm 8\sqrt{2}.$

Substituting $8\sqrt{2}$ for y in (4), $x = \frac{8\sqrt{2}}{2} = 4\sqrt{2}.$

Substituting $-8\sqrt{2}$ for y in (4), $x = -4\sqrt{2}.$

$x = \pm 4, \ \pm 4\sqrt{2}.$

$y = \pm 4, \ \pm 8\sqrt{2}.$

9.
$$2x^2 - xy + 2y^2 = 12, \quad (1)$$
$$2x^2 + xy + 2y^2 = 8. \quad (2)$$

(1) · 2, $4x^2 - 2xy + 4y^2 = 24.$ (3)

(2) · 3, $6x^2 + 3xy + 6y^2 = 24.$ (4)

(3) – (4), $-2x^2 - 5xy - 2y^2 = 0.$ (5)

$(2x + y)(x + 2y) = 0.$ (6)

From (6), $x = -\frac{y}{2},$ (7)

and $x = -2y.$ (8)

From (7) and (1), $\frac{y^2}{2} + \frac{y^2}{2} + 2y^2 = 12.$

$y = \pm 2.$

Substituting 2 for y in (7), $x = -1.$

Substituting -2 for y in (7), $x = 1.$

From (8) and (1), $8y^2 + 2y^2 + 2y^2 = 12.$

$y = \pm 1.$

Substituting 1 for y in (8), $x = -2.$

Substituting -1 for y in (8), $x = 2.$

$x = \pm 1, \ \pm 2.$

$y = \mp 2, \ \mp 1.$

10.
$$x^2 - xy - 5y^2 = 15, \quad (1)$$
$$x^2 - 6y^2 = 1. \quad (2)$$

(2) · 15, $15x^2 - 90y^2 = 15.$ (3)

(1) – (3), $-14x^2 - xy + 85y^2 = 0.$ (4)

$(7x - 17y)(2x + 5y) = 0.$ (5)

$x = \dfrac{17y}{7}.$ (6)

$x = -\dfrac{5y}{2}.$ (7)

From (7) and (2), $\dfrac{25y^2}{4} - 6y^2 = 1.$ (8)

From (8), $y = \pm 2.$

Substituting 2 for y in (7), $x = -5.$

Substituting -2 for y in (7), $x = 5.$

From (6) and (2), $\frac{289}{49}y^2 - 6y^2 = 1.$

Whence $y = \pm \frac{7}{5}\sqrt{-5}.$

Substituting $\frac{7}{5}\sqrt{-5}$ for y in (6), $x = \frac{17}{5}\sqrt{-5}.$

Substituting $-\frac{7}{5}\sqrt{-5}$ for y in (6), $x = -\frac{17}{5}\sqrt{-5}.$

$x = \pm 5, \pm \frac{17}{5}\sqrt{-5},$

$y = \mp 2, \pm \frac{17}{5}\sqrt{-5}.$

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1. $x - y = 4,$ (1)

$xy = 5.$ (2)

Squaring (1), $x^2 - 2xy + y^2 = 16.$ (3)

(2) · 4, $4xy = 20.$ (4)

(3) + (4), $x^2 + 2xy + y^2 = 36.$ (5)

From (5), $x + y = \pm 6.$ (6)

From (6) and (1), $\begin{cases} x - y = 4, \\ x + y = 6. \end{cases}$ $\begin{cases} x - y = 4, \\ x + y = -6. \end{cases}$

Whence, $x = 5, -1,$

$y = 1, -5.$

2. $x + 2y = 8,$ (1)

$xy + 6 = 0.$ (2)

From (2), $xy = -6.$ (3)

Squaring (1), $x^2 + 4xy + 4y^2 = 64.$ (4)

(3) · 8, $8xy = -48.$ (5)

(4) – (5), $x^2 - 4xy + 4y^2 = 112.$ (6)

$x - 2y = \pm 4\sqrt{7}.$ (7)

From (1) and (7), $\begin{cases} x - 2y = 4\sqrt{7}, \\ x + 2y = 8. \end{cases}$ $\begin{cases} x - 2y = -4\sqrt{7}, \\ x + 2y = 8. \end{cases}$

Whence $x = 4 + 2\sqrt{7}.$ $x = 4 - 2\sqrt{7}.$

$y = 2 - \sqrt{7}.$ $y = 2 + \sqrt{7}.$

$x = 4 + 2\sqrt{7}, 4 - 2\sqrt{7}.$

$y = 2 - \sqrt{7}, 2 + \sqrt{7}.$

3.

$$x^2 + 4y^2 = 101, \quad (1)$$
$$xy + 5 = 0. \quad (2)$$
$$xy = -5. \quad (3)$$
$$4xy = -20. \quad (4)$$
$$x^2 - 4xy + 4y^2 = 121. \quad (5)$$
$$x - 2y = \pm 11. \quad (6)$$
$$x^2 + 4xy + 4y^2 = 81. \quad (7)$$
$$x + 2y = \pm 9. \quad (8)$$

From (6) and (8), $\begin{cases} x - 2y = 11, \\ x + 2y = 9. \end{cases}$ $\begin{cases} x - 2y = 11, \\ x + 2y = -9. \end{cases}$

$\begin{cases} x - 2y = -11, \\ x + 2y = 9. \end{cases}$ $\begin{cases} x - 2y = -11, \\ x + 2y = -9. \end{cases}$

Whence $x = \pm 10, \pm 1.$

$y = \mp \frac{1}{2}, \mp 5.$

4.

$$6x - y = 24, \quad (1)$$
$$36x^2 + y^2 = 288. \quad (2)$$
$$36x^2 - 12xy + y^2 = 576. \quad (3)$$
$$12xy = -288. \quad (4)$$
$$36x^2 + 12xy + y^2 = 0. \quad (5)$$
$$6x + y = 0.$$
$$6x - y = 24.$$
$$x = 2.$$
$$y = -12.$$

5.

$$4x^2 - 6xy + y^2 = 24, \quad (1)$$
$$xy = 20. \quad (2)$$
$$6xy = 120.$$
$$4x^2 = 144.$$
$$x = \pm 6.$$

Substituting 6 for x in (2),

$$y = \frac{10}{3}.$$

Substituting -6 for x in (2),

$$y = -\frac{10}{3}.$$
$$x = \pm 6.$$
$$y = \pm \frac{10}{3}.$$

6.

$$4x^2 + y^2 = 25, \quad (1)$$
$$4x^2 + 4xy + y^2 = 49. \quad (2)$$

From (2), $2x + y = \pm 7.$ (3)

(1) − (2), $-4xy = -24.$ (4)

(1) + (4), $4x^2 - 4xy + y^2 = 1.$ (5)

From (5), $2x - y = \pm 1.$ (6)

Combining (3) and (6),

$\begin{cases} 2x + y = 7. \\ 2x - y = 1. \end{cases}$ $\begin{cases} 2x + y = 7. \\ 2x - y = -1. \end{cases}$

$\begin{cases} 2x + y = -7. \\ 2x - y = 1. \end{cases}$ $\begin{cases} 2x + y = -7. \\ 2x - y = -1. \end{cases}$

Whence $x = \pm 2, \pm \frac{3}{2}.$

$y = \pm 3, \pm 4.$

7.

$$x^2 + 4y^2 = 15, \quad (1)$$
$$x + 2y = 3\sqrt{3}. \quad (2)$$

Squaring (2), $x^2 + 4xy + 4y^2 = 27.$ (3)

(1) − (3), $\quad -4xy=-12.$ (4)

(1) + (4), $\quad x^2-4xy+4y^2=3.$ (5)

From (5), $\quad x-2y=\pm\sqrt{3}.$ (6)

From (2) and (6), $\quad \begin{cases} x+2y=3\sqrt{3}, \\ x-2y=\sqrt{3}. \end{cases} \quad \begin{cases} x+2y=3\sqrt{3}, \\ x-2y=-\sqrt{3}. \end{cases}$

Whence $\quad x=2\sqrt{3},\ \sqrt{3},$ and $y=\frac{1}{2}\sqrt{3},\ \sqrt{3}.$

8. $\quad x^2-2xy=16.$ (1)

$\quad 2y^2-xy=-6.$ (2)

(1) + 2 (2), $\quad x^2-4xy+4y^2=4.$ (3)

$\quad x-2y=\pm 2.$ (4)

From (1), $\quad x(x-2y)=16.$ (5)

From (5) and (4), $\quad x(\pm 2)=16,\ x=\pm 8.$ (6)

From (1) and (6), $\quad 64-2(\pm 8)y=16.$ (7)

From (7), $\quad y=\pm 3.$

$$x=8,\ -8.$$
$$y=3,\ -3.$$

9. $\quad \dfrac{1}{x^2}+\dfrac{1}{y^2}=13.$ (1)

$\quad \dfrac{1}{xy}=6.$ (2)

(2) · 2, $\quad \dfrac{2}{xy}=12.$ (3)

(1) + (3), $\quad \dfrac{1}{x^2}+\dfrac{2}{xy}+\dfrac{1}{y^2}=25.$ (4)

From (4), $\quad \dfrac{1}{x}+\dfrac{1}{y}=\pm 5.$ (5)

(1) − (3), $\quad \dfrac{1}{x^2}-\dfrac{2}{xy}+\dfrac{1}{y^2}=1.$ (6)

From (6), $\quad \dfrac{1}{x}-\dfrac{1}{y}=\pm 1.$ (7)

Combining (5) and (7),

$$A\begin{cases} \dfrac{1}{x}+\dfrac{1}{y}=5, \\ \dfrac{1}{x}-\dfrac{1}{y}=1. \end{cases} \qquad B\begin{cases} \dfrac{1}{x}+\dfrac{1}{y}=5, \\ \dfrac{1}{x}-\dfrac{1}{y}=-1. \end{cases}$$

$$C\begin{cases} \dfrac{1}{x}+\dfrac{1}{y}=-5, \\ \dfrac{1}{x}-\dfrac{1}{y}=1. \end{cases} \qquad D\begin{cases} \dfrac{1}{x}+\dfrac{1}{y}=-5, \\ \dfrac{1}{x}-\dfrac{1}{y}=-1. \end{cases}$$

Whence $\quad x=\frac{1}{3},\ \frac{1}{2},\ -\frac{1}{2},\ -\frac{1}{3},$

and $\quad y=\frac{1}{2},\ \frac{1}{3},\ -\frac{1}{3},\ -\frac{1}{2}.$

10.
$$\frac{1}{x^2}-\frac{1}{xy}+\frac{1}{y^2}=7. \qquad (1)$$
$$\frac{1}{x}-\frac{1}{y}=1. \qquad (2)$$
Squaring (2),
$$\frac{1}{x^2}-\frac{2}{xy}+\frac{1}{y^2}=1. \qquad (3)$$
(1) – (3),
$$\frac{1}{xy}=6. \qquad (4)$$
(4) · 3,
$$\frac{3}{xy}=18. \qquad (5)$$
(1) + (5),
$$\frac{1}{x^2}+\frac{2}{xy}+\frac{1}{y^2}=25. \qquad (6)$$
From (6),
$$\frac{1}{x}+\frac{1}{y}=\pm 5. \qquad (7)$$
From (2) and (7),
$$A\begin{cases}\dfrac{1}{x}-\dfrac{1}{y}=1,\\[4pt] \dfrac{1}{x}+\dfrac{1}{y}=5.\end{cases} \qquad B\begin{cases}\dfrac{1}{x}-\dfrac{1}{y}=1,\\[4pt] \dfrac{1}{x}+\dfrac{1}{y}=-5.\end{cases}$$
Whence
$$x=\tfrac{1}{3},\ -\tfrac{1}{2},$$
and
$$y=\tfrac{1}{2},\ -\tfrac{1}{3}.$$

11.
$$3x-3y=7. \qquad (1)$$
$$9x^2+18xy+9y^2=1. \qquad (2)$$
From (2),
$$3x+3y=\pm 1. \qquad (3)$$
From (1) and (3),
$$A\begin{cases}3x-3y=7,\\ 3x+3y=1.\end{cases} \qquad B\begin{cases}3x-3y=7,\\ 3x+3y=-1.\end{cases}$$
Whence
$$x=\tfrac{4}{3},\ 1,$$
and
$$y=-1,\ -\tfrac{4}{3}.$$

12.
$$\frac{1}{x^3}+\frac{1}{y^3}=35. \qquad (1)$$
$$\frac{1}{x}+\frac{1}{y}=5. \qquad (2)$$
(1) ÷ (2),
$$\frac{1}{x^2}-\frac{1}{xy}+\frac{1}{y^2}=7. \qquad (3)$$
Squaring (2),
$$\frac{1}{x^2}+\frac{2}{xy}+\frac{1}{y^2}=25. \qquad (4)$$
(3) – (4),
$$-\frac{3}{xy}=-18. \qquad (5)$$
From (5),
$$-\frac{1}{xy}=-6. \qquad (6)$$

(6) + (3), $$\frac{1}{x^2}-\frac{2}{xy}+\frac{1}{y^2}=1. \quad (7)$$

From (7), $$\frac{1}{x}-\frac{1}{y}=\pm 1.$$

From (2) and (8), $$A\begin{cases}\frac{1}{x}+\frac{1}{y}=5,\\ \frac{1}{x}-\frac{1}{y}=1.\end{cases} \qquad B\begin{cases}\frac{1}{x}+\frac{1}{y}=5,\\ \frac{1}{x}-\frac{1}{y}=-1.\end{cases}$$

Whence $$x=\tfrac{1}{3},\ \tfrac{1}{2},$$
and $$y=\tfrac{1}{2},\ \tfrac{1}{3}.$$

13. $$x^2+4y^2=c. \quad (1)$$
$$4xy=d. \quad (2)$$
(1) + (2), $$x^2+4xy+4y^2=c+d. \quad (3)$$
From (3), $$x+2y=\pm\sqrt{c+d}. \quad (4)$$
(1) − (2), $$x^2-4xy+4y^2=c-d. \quad (5)$$
From (5), $$x-2y=\pm\sqrt{c-d}. \quad (6)$$
From (4) and (6),
$$A\begin{cases}x+2y=\sqrt{c+d},\\ x-2y=\sqrt{c-d}.\end{cases} \qquad B\begin{cases}x+2y=\sqrt{c+d},\\ x-2y=-\sqrt{c-d}.\end{cases}$$
$$C\begin{cases}x+2y=-\sqrt{c+d},\\ x-2y=\sqrt{c-d}.\end{cases} \qquad D\begin{cases}x+2y=-\sqrt{c+d},\\ x-2y=-\sqrt{c-d}.\end{cases}$$
Solving A, B, C, and D,
$$x=\frac{\sqrt{c+d}+\sqrt{c-d}}{2},\quad \frac{\sqrt{c+d}-\sqrt{c-d}}{2},$$
$$\frac{-\sqrt{c+d}+\sqrt{c-d}}{2},\quad \frac{-\sqrt{c+d}-\sqrt{c-d}}{2}.$$
$$y=\frac{\sqrt{c+d}-\sqrt{c-d}}{4},\quad \frac{\sqrt{c+d}+\sqrt{c-d}}{4},$$
$$\frac{-\sqrt{c+d}-\sqrt{c-d}}{4},\quad \frac{-\sqrt{c+d}+\sqrt{c-d}}{4}.$$

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1. $$4x^2-y^2=16. \quad (1)$$
$$2x+y=8. \quad (2)$$
(1) ÷ (2), $$2x-y=2. \quad (3)$$
(2) + (3), $$4x=10.$$
$$x=\tfrac{5}{2}.$$
$$5+y=8.$$
$$y=3.$$
$$x=\tfrac{5}{2}.$$
$$y=3.$$

2.
$$R^2h - 75 = 0. \quad (1)$$
$$Rh = 15. \quad (2)$$
$$R^2h = 75. \quad (3)$$
(3) ÷ (2), $R = 5.$ (4)

From (2) and (4),
$$h = 3.$$

3.
$$\frac{1}{x^2} - \frac{1}{y^2} = 15. \quad (1)$$
$$\frac{1}{x} + \frac{1}{y} = 3. \quad (2)$$
(1) ÷ (2), $\dfrac{1}{x} - \dfrac{1}{y} = 5.$ (3)

(2) − (3), $\dfrac{2}{x} = 8.$ (4)

Whence $x = \frac{1}{4}$,

and $y = -1.$

4.
$$9h^2k - 100 = 0. \quad (1)$$
$$3k^2h + 80 = 0. \quad (2)$$
$$9h^2k = 100. \quad (3)$$
$$3hk^2 = -80. \quad (4)$$
(3) ÷ (4), $\dfrac{3h}{k} = -\dfrac{5}{4}.$ (5)

From (5), $h = \dfrac{-5k}{12}.$ (6)

From (6) and (4),
$$3 \cdot \frac{-5k}{12} \cdot k^2 + 80 = 0. \quad (7)$$
Whence $k = 4.$ (8)

From (8) and (2),
$$3 \cdot 4^2 \cdot h + 80 = 0.$$
$$48h = -80.$$
$$h = -\tfrac{5}{3}.$$

5.
$$P(1 + r)^2 = 224.72. \quad (1)$$
$$P + Pr = 212. \quad (2)$$
(1) ÷ (2), $1 + r = 1.06.$ (3)
$$r = .06. \quad (4)$$
From (2) and (4),
$$1.06P = 212.$$
$$P = 200.$$

6.
$$9x^2y^2 + 6 = 15xy. \quad (1)$$
$$3xy + 4 = 6. \quad (2)$$
From (1),
$$(3xy - 2)(3xy - 3) = 0.$$
From (2), $3xy - 3 = 0.$

∴ any set of roots for
$$3xy - 3 = 0,$$
will satisfy the system, $\frac{1}{2}$ and 2, for example.

7.
$$x^4 = 9y^4 + 48. \quad (1)$$
$$x^2 = 3y^2 + 2. \quad (2)$$
$$x^4 - 9y^4 = 48. \quad (3)$$
$$x^2 - 3y^2 = 2. \quad (4)$$
(3) ÷ (4), $x^2 + 3y^2 = 24.$ (5)

(4) + (5), $2x^2 = 26.$
$$x = \pm\sqrt{13}.$$
(4) − (5), $-6y^2 = -22.$
$$y = \pm\tfrac{1}{3}\sqrt{33}.$$
$$x = \sqrt{13},\quad \sqrt{13},\quad -\sqrt{13},\quad -\sqrt{13}.$$
$$y = \tfrac{1}{3}\sqrt{33},\quad -\tfrac{1}{3}\sqrt{33},\quad -\tfrac{1}{3}\sqrt{33},\quad -\tfrac{1}{3}\sqrt{33}.$$

8.
$$\frac{gt^2}{2} = .16. \quad (1)$$
$$gt = 3.2. \quad (2)$$
(1) ÷ (2), $\dfrac{t}{2} = \dfrac{1}{20}.$ (3)
$$t = \frac{1}{10} = .1. \quad (4)$$
From (4) and (2),
$$g = 32.$$

9.
$$1 - x = y. \quad (1)$$
$$1 - x^3 = y^2. \quad (2)$$
$$(1 - x)(1 + x + x^2) = y \cdot y. \quad (3)$$
$$\therefore \begin{cases} 1 + x + x^2 = y. & (4) \\ 1 - x = y. & (5) \end{cases}$$
and by § 81, $\begin{cases} 1 - x = 0, & (6) \\ y = 0. & (7) \end{cases}$

From (4) and (5),

$$1 - x = 1 + x + x^2.$$
$$x^2 + 2x = 0.$$
$$x = 0, -2.$$
$$y = 1, 3.$$

From (6) and (7), $x = 1,\ y = 0.$

$$\therefore x = 0, -2, 1.$$
$$y = 1, 3, 0.$$

10. $x^2 - 2xy - 24y^2 = 32.$ (1)

$x - 6y = 2.$ (2)

(1) ÷ (2), $x + 4y = 16.$

$$10y = 14.$$
$$y = \tfrac{7}{5}.$$
$$x = \tfrac{52}{5}.$$

11. $4x^2 + 7 = 8xy + 5y^2.$ (1)

$4x^2 = 1 + y^2.$ (2)

$4x^2 - 8xy - 5y^2 = -7.$ (3)

$4x^2 - y^2 = 1.$ (4)

$(2x - 5y)(2x + y) = -7.$ (5)

$(2x + y)(2x - y) = 1.$ (6)

(5) ÷ (6), $\dfrac{2x - 5y}{2x - y} = -7.$ (7)

From (7), $x = \dfrac{3y}{4}.$ (8)

From (2) and (8), $\dfrac{9y^2}{4} = 1 + y^2.$ (9)

From (9), $y = \pm \tfrac{2}{5}\sqrt{5}.$

From (8) and (9), $x = \pm \tfrac{3}{10}\sqrt{5}.$

$$x = \tfrac{3}{10}\sqrt{5}, -\tfrac{3}{10}\sqrt{5}.$$
$$y = \tfrac{2}{5}\sqrt{5}, -\tfrac{2}{5}\sqrt{5}.$$

12. $x^3 + y^3 = 4x - 6y - 8,$ (1)

$x + y = 2x - 3y - 4.$ (2)

$(x + y)(x^2 - xy + y^2) = (2x - 3y - 4)2.$ (3)

$\therefore$ by § 81 we obtain the two systems (4), (5) and (6), (7).

From (2) and (3), $\begin{cases} x^2 - xy + y^2 = 2, & (4) \\ x - 4y = 4. & (5) \end{cases}$

From (2), $\begin{cases} x + y = 0, & (6) \\ 2x - 3y - 4 = 0. & (7) \end{cases}$

Substituting, from (5), $4y + 4$ for x in (4),

$16x^2 + 32y + 16 - 4y^2 - 4y + y^2 = 2.$ (8)

$13y^2 + 28y + 14 = 0.$ (9)

Whence $y = \dfrac{-14 \pm \sqrt{14}}{13}.$ (10)

From (10) and (5), $x = \dfrac{-4 \pm 4\sqrt{14}}{13}.$ (11)

Solving (6), (7), $x = \tfrac{4}{5}$, and $y = -\tfrac{4}{5}.$

13. $x^3 - y^3 = 6x.$ (1)

$x - y = 3x.$ (2)

$(x - y)(x^2 + xy + y^2) = 2 \cdot 3x.$ (3)

By § 81,

$\begin{cases} x^2 + xy + y^2 = 2, & (4) \\ x - y = 3x, & (5) \end{cases}$

and $\begin{cases} x - y = 0, & (6) \\ 3x = 0. & (7) \end{cases}$

From (4) and (5),

$$x^2 - 2x^2 + 4x^2 = 2.$$
$$3x^2 = 2.$$
$$x = \pm \tfrac{1}{3}\sqrt{6}.$$
$$y = \mp \tfrac{2}{3}\sqrt{6}.$$

From (6) and (7), $x = 0.$

$$y = 0.$$
$$x = \tfrac{1}{3}\sqrt{6}, -\tfrac{1}{3}\sqrt{6}, 0.$$
$$y = -\tfrac{2}{3}\sqrt{6}, \tfrac{2}{3}\sqrt{6}, 0.$$

14. $x^3 + y^3 = 28.$ (1)

$x + y = 4.$ (2)

(1) ÷ (2),

$x^2 - xy + y^2 = 7.$ (3)

From (2) and (4),

$16 - 8y + y^2 - 4y + y^2 + y^2 = 7.$

$3y^2 - 12y + 9 = 0.$

$(y-3)(y-1) = 0.$

$y = 3, 1.$

$x = 1, 3.$

15. $x^2 - xy + y^2 = 7.$ (1)

$x^3 + y^3 = 28.$ (2)

(1) ÷ (2), $x + y = 4.$ (3)

From (3) and (1),

$16 - 8y + y^2 - 4y + y^2 + y^2 = 7.$

$3y^2 - 12y + 9 = 0.$

$y^2 - 4y + 3 = 0.$

$(y-3)(y-1) = 0.$

$y = 3, 1.$

$x = 1, 3.$

16. $x + y = 4.$ (1)

$x^2 - xy + y^2 = 7.$ (2)

Substituting,

$16 - 8y + y^2 - 4y + y^2 + y^2 = 7.$

Collecting, $y^2 - 4y + 3 = 0.$

$y = 1, 3.$

$x = 3, 1.$

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1. $2x^2 + y^2 = 33,$ (1)

$x^2 + 2y^2 = 54.$ (2)

$4x^2 + 2y^2 = 66.$ (3)

$x^2 + 2y^2 = 54.$ (4)

$3x^2 = 12.$ (5)

$x^2 = 4.$

$x = \pm 2,$

$y = \pm 5.$

$A \begin{cases} x = 2, \\ y = \pm 5. \end{cases}$ $B \begin{cases} x = -2, \\ y = \pm 5. \end{cases}$

2. $3h^2 - 8k^2 = 40,$ (1)

$5h^2 + k^2 = 81.$ (2)

$40h^2 + 8k^2 = 648.$ (3)

(3) + (1), $43h^2 = 688.$

$h^2 = 16.$

$h = \pm 4.$

$k = \pm 1.$

$A \begin{cases} h = 4, \\ k = \pm 1. \end{cases}$ $B \begin{cases} h = -4, \\ k = \pm 1. \end{cases}$

3. $4R_1^2 + 3 = 9R_2^2,$ (1)

$12R_1^2 + R_2^2 = \frac{31}{9}.$ (2)

$4R_1^2 - 9R_2^2 = -3.$ (3)

$108R_1^2 + 9R_2^2 = 31.$ (4)

(3) + (4), $112R_1^2 = 28.$ (5)

$4R_1^2 = 1.$

$R_1 = \pm \frac{1}{2}.$

$R_2 = \pm \frac{2}{3}.$

$R_1 = \frac{1}{2}, -\frac{1}{2}.$

$R_2 = \pm \frac{2}{3}, \pm \frac{2}{3}.$

4. $xy + x = 18,$ (1)

$xy + y = 20.$ (2)

(1) – (2), $x - y = -2.$ (3)

$x = y - 2.$ (4)

From (4) and (2),

$y^2 - 2y + y = 20.$

$y^2 - y - 20 = 0.$

$y = -4, 5.$

Then $x = -6, 3.$

5. $x^2 = y,$

$xy = 8.$

$x^2 \cdot x = 8.$

$x^3 - 8 = 0.$

$(x-2)(x^2 + 2x + 4) = 0.$

$x = 2, -1 \pm \sqrt{-3}.$

$y = 4, -2 \mp 2\sqrt{-3}.$

$x = 2, -1 + \sqrt{-3}, -1 - \sqrt{-3}.$

$y = 4, -2 - 2\sqrt{-3}, -2 + \sqrt{-3}.$

6.
$$x - xy = 5, \quad (1)$$
$$2y + xy = 6. \quad (2)$$

(1) + (2), $$x + 2y = 11. \quad (3)$$
$$x = 11 - 2y. \quad (4)$$

From (4) and (1),
$$11 - 2y - 11y + 2y^2 = 5. \quad (5)$$
$$2y^2 - 13y + 6 = 0.$$
$$y = \frac{13 \pm \sqrt{169 - 48}}{4}$$
$$= \frac{13 \pm 11}{4} = 6, \frac{1}{2}.$$
$$x = -1, 10.$$
$$y = 6, \tfrac{1}{2}.$$

7.
$$x^3 - y^3 = 19, \quad (1)$$
$$x - y = 1. \quad (2)$$

(1) ÷ (2), $$x^2 + xy + y^2 = 19. \quad (3)$$

From (2) and (3),
$$y^2 + 2y + 1 + y^2 + y + y^2 = 19.$$
$$3y^2 + 3y - 18 = 0.$$

Whence, $$y = -3, +2.$$
$$x = y + 1 = -3 + 1 = -2.$$
$$x = y + 1 = 2 + 1 = 3.$$
$$x = -2, 3.$$
$$y = -3, 2.$$

8.
$$x^3 - y^3 = 19, \quad (1)$$
$$x^2 + xy + y^2 = 19. \quad (2)$$

(1) ÷ (2), $$x - y = 1. \quad (3)$$

From (1) and (3),
$$y^2 + 2y + 1 + y^2 + y + y^2 = 19. \quad (4)$$

Factoring, $$(y + 3)(y - 2) = 0.$$

Whence $$y = +2, -3.$$

Then $$x = 3, -2.$$

9.
$$x^2 + xy + y^2 = 19, \quad (1)$$
$$x - y = 1. \quad (2)$$

From (2) and (1),
$$y^2 + 2y + 1 + y^2 + y + y^2 = 19.$$
$$(y + 3)(y - 2) = 0.$$

Whence $$y = -3 \text{ or } +2.$$

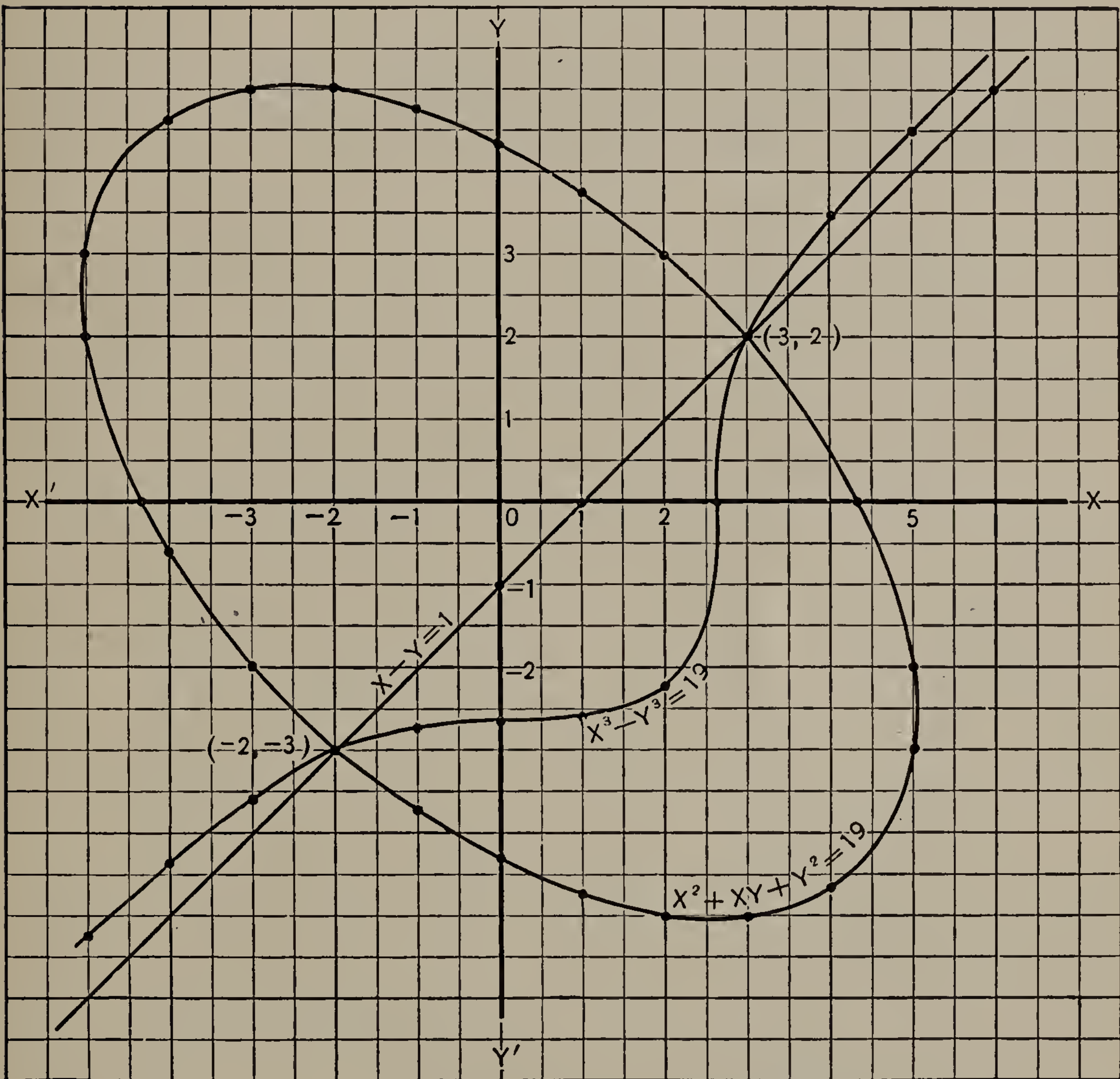
Then $$x = -2, +3.$$

10. $x^3 - y^3 = 19$. Then $y = \sqrt[3]{x^3 - 19}$. (1)

x	0	1	2	3	4	5	-1	-2	-3	-4	-5	2.62
y	-2.66	-2.62	-2.22	2	3.55	4.73	-2.71	-3	-3.58	-4.36	-5.24	0

$$x - y = 1. \qquad (2)$$

x	0	1	6
y	-1	0	5



$$x^2 + xy + y^2 = 19. \text{ Then } y = \frac{-x \pm \sqrt{76 - 3x^2}}{2}. \qquad (3)$$

x	5	4.35	4	3	2	1	0	-1	-2	-3	-4	-4.35	-5
y	-2	0	-4.63	-5	-5	3.77	± 4.35	4.77	5	$+5$	4.63	0	2
	-3		.63	2	$+3$	-4.77		-3.77	-3	-2	$-.63$		3

For (1), (2),

x	3	-2
y	2	-3

For (1), (3),

x	3	-2
y	2	-3

For (3), (2),

x	3	-2
y	2	-3

The systems (7), (8), and (9) are equivalent. The graph shows for all the systems but two points of intersection: 3, 2 and -2, -3.

11.
$$3s^2 - 2t^2 = 0, \quad (1)$$
$$5s^2 - 3t^2 = 1. \quad (2)$$
$$15s^2 - 10t^2 = 0. \quad (3)$$
$$15s^2 - 9t^2 = 3. \quad (4)$$
(3) − (4), $-t^2 = -3$. (5)
$$t = \pm\sqrt{3}. \quad (6)$$
$$9s^2 - 6t^2 = 0. \quad (7)$$
$$10s^2 - 6t^2 = 2. \quad (8)$$
(7) − (8), $-s^2 = -2,\ s^2 = 2.$
$$s = \pm\sqrt{2}.$$
$$\therefore\ t = \sqrt{3},\quad \sqrt{3},\ -\sqrt{3},\ -\sqrt{3}.$$
$$s = \sqrt{2},\ -\sqrt{2},\quad \sqrt{2},\ -\sqrt{2}.$$

12.
$$4n^2 + 7m^2 = 9, \quad (1)$$
$$2n^2 - \tfrac{9}{2} = m^2. \quad (2)$$
$$4n^2 + 7m^2 = 9. \quad (3)$$
$$4n^2 - 2m^2 = 9. \quad (4)$$
(3) − (4), $9m^2 = 0,\ m = 0.$ (5)

From (5) and (1),
$$4n^2 = 9.$$
Whence, $n = \pm\frac{3}{2}.$
$$\therefore\ m = 0,\ 0.$$
$$n = \tfrac{3}{2},\ -\tfrac{3}{2}.$$

13. $5W_1^2 - 6.8W_2^2 = 99.55.$ (1)
$$W_1^2 - W_2^2 = 20. \quad (2)$$
5 · (2), $5W_1^2 - 5W_2^2 = 100.$
$$-1.8W_2^2 = -.45.$$
$$W_2^2 = \tfrac{1}{4}.$$
$$W_2 = \pm\tfrac{1}{2}.$$
$$5W_1^2 - 6.8W_2^2 = 99.55.$$
$$6.8W_1^2 - 6.8W_2^2 = 136.0.$$
$$-1.8W_1^2 = -36.45.$$
$W_1^2 = \frac{81}{4}.$ Whence $W_1 = \pm\frac{9}{2}.$
$$\therefore\ W_2 = \tfrac{1}{2},\ \tfrac{1}{2},\ -\tfrac{1}{2},\ -\tfrac{1}{2}.$$
$$W_1 = \tfrac{9}{2},\ -\tfrac{9}{2},\ \tfrac{9}{2},\ -\tfrac{9}{2}.$$

14.
$$xy + 2y^2 = 2, \quad (1)$$
$$3xy + 5y^2 = 2. \quad (2)$$
(1) · 3, $3xy + 6y^2 = 6.$ (3)
$$-y^2 = -4. \quad (4)$$
$$y = \pm 2. \quad (5)$$
From (1) and (5), $x = \mp 3.$
$$\therefore\ x = 3,\ -3.$$
$$y = -2,\ 2.$$

15. $x^2 + 2xy + 2y^2 = 10.$ (1)
$$3x^2 - xy - y^2 = 51. \quad (2)$$
(2) · 2,
$$6x^2 - 2xy - 2y^2 = 102. \quad (3)$$
$$7x^2 = 112. \quad (4)$$
$$x = \pm 4. \quad (5)$$
From (1) and (5),
$$y = -3,\ -1,\quad 3,\quad 1.$$
$$x = 4,\quad 4,\ -4,\ -4.$$

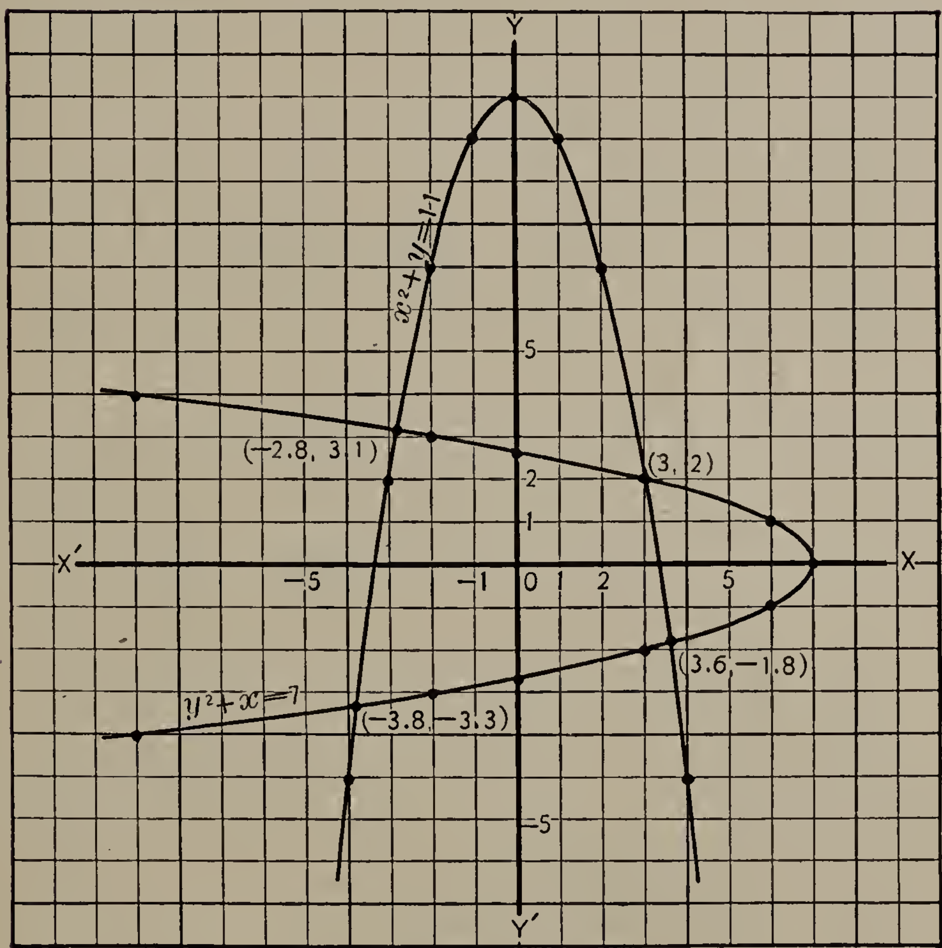
16.
$$y^2 + x = 7,$$
$$x^2 + y = 11.$$

$y^2 + x = 7$. Then $y = \pm\sqrt{7 - x}$.

x	7	6	3	-2	-9	0
y	0	± 1	± 2	± 3	± 4	± 2.64

$x^2 + y = 11$. Then $x = \pm\sqrt{11 - y}$.

x	0	± 1	± 2	± 3	± 4	± 5
y	11	10	7	2	-5	-14



The roots are :

x	-2.8	3	3.6	-3.8
y	3.1	2	-1.8	-3.3

17. $x^2 + xy + y^2 = 7,$ (1)

$x^2 + y^2 = 10.$ (2)

$xy = -3.$ (3)

(1) + (3),

$x^2 + 2xy + y^2 = 4.$ (4)

$x + y = \pm 2.$ (5)

$(3) \cdot -3, \quad -3xy = 9.$ (6)

(1) − (6), $x^2 - 2xy + y^2 = 16.$ (7)

$x - y = \pm 4.$ (8)

From (5) and (8),

$\begin{cases} x + y = 2. \\ x - y = \pm 4. \end{cases} \quad \begin{cases} x + y = -2. \\ x - y = \pm 4. \end{cases}$

$\therefore x = 3, -3, \quad 1, -1.$

$y = -1, \quad 1, -3, \quad 3.$

18. $x^2 + xy + x = 0,$

$x^2 + xy + 2x = 0.$

$x(x + y + 1) = 0.$

$x(x + y + 2) = 0.$

$\left.\begin{array}{l} x + y + 1 = 0. \\ x + y + 2 = 0. \end{array}\right\}$ contradictory.

$\therefore x = 0, y =$ any number.

19. $x^2 + xy + y = 0,$ (1)

$x^2 + xy + x = 0.$ (2)

$y = x.$ (3)

$x^2 + x^2 + x = 0.$ (4)

$x(2x + 1) = 0.$ (5)

Then $x = 0, -\frac{1}{2}.$

And $y = 0, -\frac{1}{2}.$

20. $\frac{1}{x^2} + \frac{1}{y^2} = 13,$ (1)

$\frac{1}{x} - \frac{1}{y} = 1.$ (2)

Squaring (2),

$\frac{1}{x^2} - \frac{1}{2xy} + \frac{1}{y^2} = 1.$ (3)

(1) − (3), $\frac{1}{2xy} = 12.$ (4)

(1) + (4), $\frac{1}{x^2} + \frac{1}{2xy} + \frac{1}{y^2} = 25.$ (5)

$\frac{1}{x} + \frac{1}{y} = \pm 5.$ (6)

From (6) and (2),

$\frac{2}{x} = 6, -4.$

$\therefore x = \frac{1}{3}, -\frac{1}{2}.$

$y = \frac{1}{2}, -\frac{1}{3}.$

21. $\frac{1}{x^3} - \frac{1}{y^3} = 7,$ (1)

$\frac{1}{x} - \frac{1}{y} = 1.$ (2)

$\frac{1}{x^2} + \frac{1}{xy} + \frac{1}{y^2} = 7.$ (3)

Squaring (2),

$\frac{1}{x^2} - \frac{2}{xy} + \frac{1}{y^2} = 1.$ (4)

(3) − (4), $\frac{3}{xy} = 6.$ (5)

$\frac{1}{xy} = 2.$ (6)

(3) + (6),

$\frac{1}{x^2} + \frac{2}{xy} + \frac{1}{y^2} = 9.$ (7)

$\frac{1}{x} + \frac{1}{y} = \pm 3.$ (8)

From (8) and (2), $x = \frac{1}{2}, -1.$

$y = 1, -\frac{1}{2}.$

22. $x^2 - 2xy + 2y^2 - y = 0,$ (1)

$2x^2 - 3xy - y^2 + 2y = 0.$ (2)

$(1) \cdot 2, \quad 2x^2 - 4xy + 4y^2 - 2y = 0.$ (3)

(2) + (3), $4x^2 - 7xy + 3y^2 = 0.$ (4)

From (4), $x = y,$ (5)

and $x = \frac{3y}{4}.$ (6)

From (1) and (5), $y^2 - 2y^2 + 2y^2 - y = 0.$ (7)

From (7), $y = 0, 1.$ (8)

From (8) and (5), $x = 0, 1.$ (9)

From (6) and (1), $\frac{9y^2}{16} - \frac{3y^2}{2} + 2y^2 - y = 0.$ (10)

From (10), $y = 0, \frac{16}{17}.$ (11)

From (11) and (6), $x = 0, \frac{12}{17}.$

$$\therefore x = 0, 1, \tfrac{12}{17}.$$
$$y = 0, 1, \tfrac{16}{17}.$$

23. $x^2 + z^2 = 34,$ (1)

$x^2 + y^2 = 25,$ (2)

$y^2 + z^2 = 41.$ (3)

(1) – (2), $-y^2 + z^2 = 9.$ (4)

(3) + (4), $2z^2 = 50.$ (5)

$z = \pm 5.$ (6)

From (6) and (3), $y^2 + 25 = 41.$ (7)

$y = \pm 4.$ (8)

From (6) and (1), $x^2 + 25 = 34.$

$x = \pm 3.$

Therefore:

$$\begin{array}{llllllll} x = 3, & 3, & 3, & 3, & -3, & -3, & -3, & -3, \\ y = 4, & -4, & 4, & -4, & 4, & -4, & 4, & -4, \\ z = 5, & 5, & -5, & -5, & 5, & 5, & -5, & -5. \end{array}$$

24. $3xy = x^2y^2 - 88,$ (1)

$x - y = 6.$ (2)

From (1), $(xy - 11)(xy + 8) = 0.$ (3)

From (3) and (2),

$(y^2 + 6y - 11)(y^2 + 6y + 8) = 0.$ (4)

$\therefore y = -2, -4,$ (5)

and $y = -3 \pm 2\sqrt{5}.$ (6)

From (3), (5), and (6), $x = 4, 2, 3 \pm 2\sqrt{5}.$

$x = 4, 2, 3 + 2\sqrt{5}, 3 - 2\sqrt{5}.$

$y = -2, -4, -3 + 2\sqrt{5}, -3 - 2\sqrt{5}.$

25. $x^3 = y^3 + 37,$ (1)

$x^2y = xy^2 + 12.$ (2)

$x^3 - y^3 = 37.$ (3)

$x^2y - xy^2 = 12.$ (4)

(3) ÷ (4), $\frac{x^2 + xy + y^2}{xy} = \frac{37}{12}.$ (5)

$12x^2 + 12xy + 12y^2 = 37xy.$ (6)

From (6), $$x = \frac{4y}{3}, \tag{7}$$

and $$x = \frac{3y}{4}. \tag{8}$$

From (2) and (8), $$\frac{9y^3}{16} = \frac{3y^3}{4} + 12. \tag{9}$$

$$y^3 + 64 = 0, \text{ or } (y+4)(y^2 - 4x + 16) = 0. \tag{10}$$

$$y = -4,\ 2 \pm 2\sqrt{-3}. \tag{11}$$

From (11) and (8), $$x = -3,\ \frac{3 \pm 3\sqrt{-3}}{2}.$$

From (2) and (7), $$\frac{16y^3}{9} = \frac{4y^3}{3} + 12. \tag{12}$$

From (12), $$y = 3,\ \frac{-3 \pm 3\sqrt{-3}}{2}. \tag{13}$$

From (13) and (7), $$x = 4,\ -2 \pm 2\sqrt{-3}.$$

$$x = 4,\quad -3,\quad -2 + 2\sqrt{-3},\quad -2 - 2\sqrt{-3}.$$

$$y = 3,\quad -4,\quad \frac{-3 + 3\sqrt{-3}}{2},\quad -3 - 3\sqrt{-3},$$

$$\frac{3 + 3\sqrt{-3}}{2},\qquad \frac{3 - 3\sqrt{-3}}{2},$$

$$2 + 2\sqrt{-3},\qquad 2 - 2\sqrt{-3}.$$

26. $$\frac{4}{x^2} - \frac{13}{xy} + \frac{9}{y^2} = 9, \tag{1}$$

$$\frac{1}{xy} - \frac{1}{y^2} = 3. \tag{2}$$

(2) · 3, $$\frac{3}{xy} - \frac{3}{y^2} = 9. \tag{3}$$

(1) – (3), $$\frac{4}{x^2} - \frac{16}{xy} + \frac{12}{y^2} = 0. \tag{4}$$

$$\left(\frac{1}{x} - \frac{3}{y}\right)\left(\frac{1}{x} - \frac{1}{y}\right) = 0. \tag{5}$$

$$x = \frac{y}{3}. \tag{6}$$

$$x = y. \tag{7}$$

From (6) and (2), $$\frac{3}{y^2} - \frac{1}{y^2} = 3. \tag{8}$$

$$y^2 = \tfrac{2}{3}. \tag{9}$$

$$y = \pm \tfrac{1}{3}\sqrt{6}.$$

From (9) and (6), $$x = \pm \tfrac{1}{9}\sqrt{6}.$$

No results can be obtained by using (7) and (1), or (7) and (2).

27. $$x^2 = 8x + 6y, \tag{1}$$

$$y^2 = 6x + 8y. \tag{2}$$

$$x^2 - y^2 = 2(x - y). \tag{3}$$

$$(x - y)(x + y - 2) = 0. \tag{4}$$

$$x = y, \tag{5}$$

$$x = 2 - y. \tag{6}$$

From (5) and (1), $$y^2 = 14y. \tag{7}$$

$$y = 0,\ 14.$$

From (6) and (1), $4 - 4y + y^2 = 16 - 8y + 6y.$

$$y^2 - 2y - 12 = 0.$$
$$y = 1 \pm \sqrt{13}.$$
$$x = 1 \mp \sqrt{13}.$$
$$x = 0,\ 14,\ 1 + \sqrt{13},\ 1 - \sqrt{13}.$$
$$y = 0,\ 14,\ 1 - \sqrt{13},\ 1 + \sqrt{13}.$$

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$$x^3 - y^3 = 2x + y - 4, \quad (1)$$
$$x + 2y = 4. \quad (2)$$

Transposing, $0 = x + 2y - 4. \quad (3)$

Adding $x - y$, $x - y = 2x + y - 4. \quad (4)$

From (1), $(x - y)(x^2 + xy + y^2) = (2x + y - 4)1. \quad (5)$

From (4) and (5), by § 81,

$$\begin{cases} x^2 + xy + y^2 = 1, & (6) \\ x + 2y = 4, & (7) \end{cases}$$

and

$$\begin{cases} x - y = 0, & (8) \\ 2x + y - 4 = 0. & (9) \end{cases}$$

From (6) and (7), $x = \pm 2\sqrt{-1},$

$$y = 2 \pm \sqrt{-1}.$$

From (8) and (9), $x = \frac{4}{3},$

$$y = \frac{4}{3}.$$

29.

$$xy = c, \quad (1)$$
$$x + y = a. \quad (2)$$
$$x = a - y. \quad (3)$$

From (1) and (3), $ay - y^2 = c. \quad (4)$

$$y = \frac{a \pm \sqrt{a^2 - 4c}}{2}. \quad (5)$$

From (3) and (5), $x = \dfrac{a \mp \sqrt{a^2 - 4c}}{2}.$

30.

$$x^{-2} - y^{-2} = 6, \quad (1)$$
$$x^{-1} + y^{-1} = 2. \quad (2)$$

(1) ÷ (2), $x^{-1} - y^{-1} = 3. \quad (3)$

(3) + (2), $2x^{-1} = 5. \quad (4)$

$$\frac{2}{x} = 5.$$
$$x = \frac{2}{5}.$$

(2) − (3), $2y^{-1} = -1.$

$$\frac{2}{y} = -1.$$
$$y = -2.$$
$$\therefore x = \frac{2}{5}.$$
$$y = -2.$$

31.

$$x - y = 16, \quad (1)$$
$$x^{\frac{1}{2}} - y^{\frac{1}{2}} = 2. \quad (2)$$

(1) ÷ (2), $x^{\frac{1}{2}} + y^{\frac{1}{2}} = 8. \quad (3)$

(2) + (3), $2x^{\frac{1}{2}} = 10.$

$$x = 25.$$

(3) − (2), $2y^{\frac{1}{2}} = 6.$

$$y^{\frac{1}{2}} = 3.$$
$$y = 9.$$
$$x = 25.$$
$$y = 9.$$

32.
$$\frac{1}{x-2}+\frac{1}{y-2}=\frac{3}{4}, \quad (1)$$
$$\frac{1}{x}-\frac{1}{y}=\frac{1}{12}. \quad (2)$$

From (2), $12y-12x=xy.$ (3)

From (1), $10x+10y=3xy+28.$ (4)

(3) · 3, $-36x+36y=3xy.$ (5)

(4) – (5), $46x-26y=28.$ (6)

$$x=\frac{13y+14}{23}. \quad (7)$$

From (7) and (3), $12y-\dfrac{156y+168}{23}=\dfrac{13y^2+14y}{23}.$ (8)

From (8), $y=6,\ \frac{28}{13}.$ (9)

From (9) and (7), $x=4,\ \frac{42}{23}.$

33. $x^2+6x^{-2}=36\frac{1}{6},$ (1)

$3xy-x^2=36.$ (2)

(1) · x^2, $x^4-36\frac{1}{6}x^2+6=0.$ (3)

From (3), $x^2=36,\ \frac{1}{6}.$ (4)

Whence $x=\pm 6,\ \pm\frac{1}{6}\sqrt{6}.$ (5)

From (2), $y=\dfrac{36+x^2}{3x}.$ (6)

From (5) and (6),

$y=\pm 4,\ \pm\frac{217}{18}\sqrt{6}.$ (7)

$x=\pm 6,\ \pm\frac{1}{6}\sqrt{6}.$

$y=\pm 4,\ \pm\frac{217}{18}\sqrt{6}.$

34. $x-y\sqrt{x}=24,$ (1)

$x^2+xy^2=320.$ (2)

Squaring (1),

$x^2-2xy\sqrt{x}+xy^2=576.$ (3)

(2) – (3), $2xy\sqrt{x}=-256.$ (4)

(3) – (4) · 2,

$x^2+2xy\sqrt{x}+xy^2=64.$ (5)

$x+y\sqrt{x}=\pm 8.$ (6)

$x-y\sqrt{x}=24.$ (7)

(6) + (7), $2x=32,\ 16.$

$x=16,\ 8.$

(6) – (7), $2y\sqrt{x}=-16,\ -32.$

$$y=\frac{-8}{\sqrt{x}},\ \frac{-16}{\sqrt{x}}$$
$$=\frac{-8}{4},\ \frac{-16}{2\sqrt{2}}=-2,\ -4\sqrt{2}.$$

$x=16,\ 8.$

$y=-2,\ -4\sqrt{2}.$

35.
$$\frac{x-1}{y-1}=3, \quad (1)$$
$$\frac{y^2+y+1}{x^2-x+1}=\frac{13}{43}. \quad (2)$$

From (1), $x=3y-2.$ (3)

From (2), $43y^2+43y+43=13x^2-13x+13.$ (4)

From (3) and (4), $43y^2+43y+30=117y^2-156y+52-39y+26.$ (5)

From (5), $37y^2-119y+24=0.$ (6)

From (6), $y=3,\ \frac{8}{37}.$ (7)

From (3) and (7), $x=7,\ \dfrac{-50}{37}.$

36. $$\frac{1}{x^2} + \frac{2}{xy} = 16, \quad (1)$$

$$\frac{3}{x^2} - \frac{4}{xy} + \frac{2}{y^2} = 6. \quad (2)$$

(1) · 3, $$\frac{3}{x^2} + \frac{6}{xy} = 48. \quad (3)$$

(2) · 8, $$\frac{24}{x^2} - \frac{32}{xy} + \frac{16}{y^2} = 48. \quad (4)$$

(4) − (3), $$\frac{21}{x^2} - \frac{38}{xy} + \frac{16}{y^2} = 0. \quad (5)$$

$$\left(\frac{7}{x} - \frac{8}{y}\right)\left(\frac{3}{x} - \frac{2}{y}\right) = 0. \quad (6)$$

From (6), $$x = \frac{3y}{2}, \quad (7)$$

and $$x = \frac{7y}{8}. \quad (8)$$

From (1) and (8), $$\frac{64}{49y^2} + \frac{16}{7y^2} = 16. \quad (9)$$

From (9), $$y = \pm \tfrac{1}{7}\sqrt{11}. \quad (10)$$

From (8) and (10), $$x = \pm \tfrac{1}{8}\sqrt{11}.$$

From (7) and (1), $$\frac{4}{9y^2} + \frac{4}{3y^2} = 16. \quad (11)$$

From (11), $$y = \pm \tfrac{1}{3}. \quad (12)$$

From (7) and (12), $$x = \pm \tfrac{1}{2}.$$

$$x = \pm \tfrac{1}{2}, \ \pm \tfrac{1}{8}\sqrt{11}.$$

$$y = \pm \tfrac{1}{3}, \ \pm \tfrac{1}{7}\sqrt{11}.$$

37. $$\frac{9x}{y} = 18, \quad (1)$$

$$xy = 18. \quad (2)$$

From (1), $$x = 2y. \quad (3)$$

From (2) and (3), $$2y^2 = 18.$$

$$y = \pm 3.$$

$$x = \pm 6.$$

38. $$4x + \frac{1}{y} = 46, \quad (1)$$

$$\frac{26x}{5} - \frac{1}{y} = 46. \quad (2)$$

(1) + (2), $$\frac{46x}{5} = 92. \quad (3)$$

$$x = 10. \quad (4)$$

From (1) and (4), $$40 + \frac{1}{y} = 46.$$
$$\frac{1}{y} = 6.$$
$$y = \tfrac{1}{6}.$$

39. $$x^2 + y^2 - (y - x) = 12, \quad (1)$$
$$x^2 - xy = 0. \quad (2)$$
$$x(x - y) = 0. \quad (3)$$
$$x = 0. \quad (4)$$
$$x = y. \quad (5)$$
From (1) and (4), $$y^2 - y - 12 = 0. \quad (6)$$
From (6), $$y = 4, -3.$$
From (5) and (1), $$y^2 + y^2 = 12.$$
$$y^2 = 6.$$
$$y = \pm\sqrt{6}.$$
$$x = 0, 0, \pm\sqrt{6}.$$
$$y = 4, -3, \pm\sqrt{6}.$$

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1. Let x equal one number and y equal the other.

Then $$x - y = 4,$$
and $$x^2 - y^2 = 88.$$
Dividing, $$x + y = 22.$$
$$x - y = 4.$$
$$x = 13, 13 - y = 4, y = 9.$$

2. Let x equal one number and y equal the other.

Then $$x + y = 21, \quad (1)$$
and $$x^2 + y^2 = 281. \quad (2)$$
Squaring (1), $$x^2 + 2xy + y^2 = 441. \quad (3)$$
(3) − (2), $$2xy = 160. \quad (4)$$
(2) − (4), $$x^2 - 2xy + y^2 = 121. \quad (5)$$
$$x - y = \pm 11. \quad (6)$$
From (1) and (6), $$\begin{cases} x = 16, 5. \\ y = 5, 16. \end{cases}$$

3. Let x equal one number and y equal the other.

Then $$xy = 192, \quad (1)$$
and $$\frac{x}{y} = \frac{3}{4}. \quad (2)$$
From (1) and (2), $$3y^2 = 4 \cdot 192. \quad (3)$$
From (3), $$y = \pm 16. \quad (4)$$
From (2) and (4), $$x = \pm 12.$$
$$\therefore x = 12, -12. \quad y = 16, -16.$$

4. Let x equal one leg in feet and y equal the other.

$$\frac{xy}{2} = 150. \qquad (1)$$

$$x^2 + y^2 = 625. \qquad (2)$$

(2) − (1) · 4, $$x^2 - 2xy + y^2 = 25. \qquad (3)$$

(2) + (1) · 4, $$x^2 + 2xy + y^2 = 1225. \qquad (4)$$

From (3), $$x - y = \pm 5.$$

From (4), $$x + y = \pm 35.$$

$$x = 20.$$

$$y = 15.$$

5. Let x and y equal the dimensions in rods.

$$x - y = 8.$$

$$xy = 1280.$$

$$8y + y^2 = 1280.$$

$$y^2 + 8y - 1280 = 0.$$

$$y = 32, -40.$$

∴ the dimensions are 32 by 40 rods.

6. Let x and y in feet equal the sides of the squares respectively.

$$x^2 - y^2 = 252.$$

$$4x - 4y = 24.$$

$$x - y = 6.$$

Dividing, $$x + y = 42.$$

$$x = 24.$$

$$y = 18.$$

∴ the sides are 24 feet and 18 feet.

7. Let x and y equal the dimensions in rods.

$$xy = \frac{18}{5} \cdot 160 = 576. \qquad (1)$$

$$x^2 + y^2 = 3600. \qquad (2)$$

(2) − 2 · (1), $$x^2 - 2xy + y^2 = 2448.$$

(2) + 2 · (1), $$x^2 + 2xy + y^2 = 4752.$$

$$x - y = \pm 49.47.$$

$$x + y = \pm 68.93.$$

$$x = 59.20.$$

$$y = 9.73.$$

$$2x + 2y = 137.86, \text{ perimeter.}$$

8. Let x and y equal the dimensions in feet.

$$2x + 2y = 112.$$

$$x + y = 56.$$

$$xy = 768.$$

$$56y - y^2 = 768.$$
$$y^2 - 56y + 768 = 0.$$
$$y = 32,\ 24.$$

$\therefore$ the dimensions are 32 by 24 feet.

9. Let x and y equal the numbers.

$$xy = 56.$$
$$x^2 + y^2 = 113.$$
$$x^2 - 2xy + y^2 = 1.$$
$$x^2 + 2xy + y^2 = 225.$$
$$x - y = \pm 1.$$
$$x + y = \pm 15.$$
$$x = 8,\ -8.$$
$$y = 7,\ -7.$$

10. Let $x =$ the numerator,

and $y =$ the denominator.

$$\frac{x}{y} = \frac{2}{3}. \qquad (1)$$
$$\frac{x^2 - 44}{y^2 - 44} = \frac{5}{14}. \qquad (2)$$
$$x = \frac{2y}{3}. \qquad (3)$$
$$14x^2 - 616 = 5y^2 - 220. \qquad (4)$$

From (3) and (4), $14 \cdot \frac{4y^2}{9} - 396 = 5y^2.$

$$11y^2 = 9 \cdot 396.$$
$$y^2 = 9 \cdot 36.$$
$$y = \pm 18.$$

Then $x = \pm 12.$

$$\therefore \text{ the fraction } = \tfrac{12}{18}.$$

11. Let $x =$ the base,

and $y =$ the altitude, each in inches.

$$x = y + 6.$$
$$\frac{xy}{2} = \frac{3}{2} \cdot 144.$$
$$y^2 + 6y = 432.$$
$$y = 18,\ -24.$$

-24 does not satisfy the conditions.

$\therefore$ base $= 24$ inches; altitude $= 18$ inches.

12. Let x and $y =$ the edges respectively, each in inches.

$$\therefore\ x^3 - y^3 = 1413. \qquad (1)$$
$$x - y = 3. \qquad (2)$$

$$x = y + 3. \qquad (3)$$

(1) ÷ (2), $\quad x^2 + xy + y^2 = 471. \qquad (4)$

From (3) and (4),

$$y^2 + 6y + 9 + y^2 + 3y + y^2 = 471.$$

$$y^2 + 3y - 154 = 0.$$

$$y = -14, +11. \quad -14 \text{ is not possible.}$$

∴ 14 inches and 11 inches are the sides.

13. Let $\quad x$ and y = the radii, each in inches.

$$x + y = 25. \qquad (1)$$

$$\pi x^2 - \pi y^2 = 125\pi. \qquad (2)$$

(2) ÷ (1), $\quad x - y = 5.$

Then $\quad x = 15,$

and $\quad y = 10.$

∴ 15 inches and 10 inches are the radii.

14. Let $\quad x$ and y = the dimensions.

$$2x + 2y = 5C. \qquad (1)$$

$$x = \frac{5C}{2} - y. \qquad (2)$$

$$xy = C^2. \qquad (3)$$

From (2) and (3), $y^2 - \dfrac{5Cy}{2} + C^2 = 0. \qquad (4)$

$$2y^2 - 5Cy + 2C^2 = 0. \qquad (5)$$

$$y = 2C, \frac{C}{2}. \qquad (6)$$

From (6) and (2), $\quad x = \dfrac{C}{2}, 2C.$

$$\therefore 2C = \text{the length}, \frac{C}{2} = \text{the width}.$$

15. Let $\quad x$ and y = the legs.

$$\frac{xy}{2} = 8a^2 - 8b^2. \qquad (1)$$

$$x^2 + y^2 = 32a^2 + 32b^2. \qquad (2)$$

From (1), $\quad 2xy = 32a^2 - 32b^2. \qquad (3)$

(2) − (3), $\quad (x - y)^2 = 64b^2.$

(2) + (3), $\quad (x + y)^2 = 64a^2.$

$$x - y = \pm 8b.$$

$$x + y = \pm 8a.$$

Then $\quad x = 4a + 4b,$

and $\quad y = 4a - 4b.$

∴ the legs are $4a + 4b$, and $4a - 4b$.

16. Let $x =$ one leg and $y =$ the other leg, each in feet.

$$y + x + \sqrt{x^2 + y^2} = 56. \quad (1)$$

$$\frac{xy}{2} = 84. \quad (2)$$

From (1), $x^2 + 2xy + y^2 - 112x - 112y + 3136 = x^2 + y^2. \quad (3)$

From (2), $2xy - 336 = 0. \quad (4)$

(3) − (4), $-112x - 112y + 3472 = 0. \quad (5)$

$$x + y = 31. \quad (6)$$

From (6) and (2), $y^2 - 31y + 168 = 0.$

$$y = 7,\ 24.$$

Then $x = 24,\ 7$

$\therefore$ the legs are 7 and 24 and the hypotenuse is 25.

17. Let $x =$ the units digit,

and $y =$ the tens digit.

$$(x + y)(10y + x) = 324. \quad (1)$$

$$10y + x + 3y + 3x = 10x + y. \quad (2)$$

From (1), $x^2 + 11xy + 10y^2 = 324. \quad (3)$

From (2), $12y = 6x. \quad (4)$

From (3) and (4),

$$x^2 + \frac{11x^2}{2} + \frac{5x^2}{2} = 324.$$

$$9x^2 = 324.$$

$$x = \pm 6.$$

Then $y = \pm 3.$

$\therefore$ the number $= 36.$

18. Let $x =$ the sum in dollars,

and $y =$ the rate per cent.

$$\frac{y}{100}x = \$42. \quad (1)$$

$$\frac{(x + 200)(y - 1)}{100} = \$48. \quad (2)$$

$$xy = 4200. \quad (3)$$

$$xy + 200y - x - 200 = 4800. \quad (4)$$

From (3) and (4), $x = 200y - 800. \quad (5)$

From (5) and (3), $200y^2 - 800y = 4200.$

$$y^2 - 4y - 21 = 0.$$

$$y = 7,\ -3.$$

$$x = \$600.$$

$\therefore$ principal $= \$600$; rate $= 7\%$.

19. Let $x =$ the rate in miles of man going north,
and $y =$ the rate in miles of man going east.

Then
$$\left(\frac{4x}{3}\right)^2 + \left(3 + \frac{4y}{3}\right)^2 = 289. \quad (1)$$
$$\left(\frac{14x}{3}\right)^2 + \left(3 + \frac{14y}{3}\right)^2 = 2809. \quad (2)$$
Solving (1) and (2), $x = 6$, $y = 9$.

20. Let $x =$ the circumference of fore wheel of carriage in feet,
and $y =$ the circumference of rear wheel of carriage in feet.

$$\frac{5280}{x+1} - \frac{5280}{y+3} = 88. \quad (1) \qquad \frac{60}{x+1} - \frac{60}{y+3} = 1. \quad (3)$$
$$\frac{5280}{x} - \frac{5280}{y} = 40. \quad (2) \qquad \frac{132}{x} - \frac{132}{y} = 1. \quad (4)$$
Solving (3) and (4),
$$x = 11.$$
$$y = 12.$$
$$x + 1 = 12.$$
$$y + 3 = 15.$$

21.

Let K be the point where A and B meet.

First. A goes from P to K, B from Q to K.

(A) $r = x$ (miles per hour).
$t = y$ (hours).
$d = PK = xy$.

(B) $r = \dfrac{xy - 40}{y}$.
$t = y$.
$d = QK = xy - 40$.

Second. A goes from K to Q, B from K to P.

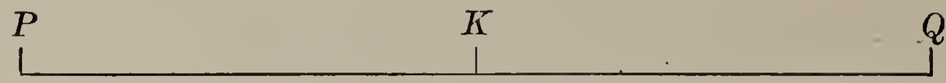
(A) $r = x$.
$t = \dfrac{xy - 40}{x} = \left(\dfrac{d}{r}\right)$.
$d = xy - 40$.

(B) $r = \dfrac{xy - 40}{y}$.
$t = \dfrac{xy^2}{xy - 40} = \left(\dfrac{d}{r}\right)$.
$d = xy$.

Therefore
$$\frac{xy - 40}{x} = 2, \quad (1)$$
and
$$\frac{xy^2}{xy - 40} = 8. \quad (2)$$
(1) · (2),
$$y^2 = 16. \quad (3)$$
$$y = \pm 4. \quad (4)$$
From (4) and (1), $x = 20$, A's rate.

Then
$$\frac{xy - 40}{y} = 10, \text{ B's rate,}$$
and
$$PQ = 2xy - 40 = 120 \text{ miles.}$$

22.



First. A goes from P to K, and B goes from Q to K.

(A) $r = x$ (in miles per hour).
$t = y$ (in hours).
$d = xy$.

(B) $r = \dfrac{300 - xy}{y}$.
$t = y$.
$d = 300 - xy$.

Second. A goes from K to Q, and B goes from K to P.

(A) $r = x$.
$t = \dfrac{300 - xy}{x}$.
$d = 300 - xy$.

(B) $r = \dfrac{300 - xy}{y}$.
$t = \dfrac{xy^2}{300 - xy}$.
$d = xy$.

Therefore $$\frac{300 - xy}{x} = 6\tfrac{2}{3}. \quad (1)$$

$$\frac{xy^2}{300 - xy} = 15. \quad (2)$$

(1) · (2), $$y^2 = 100. \quad (3)$$

$$y = \pm 10. \quad -10 \text{ is rejected.} \quad (4)$$

From (4) and (1), $$x = 18, \text{ or A's rate.}$$

$$\therefore \frac{300 - xy}{y} = 12, \text{ or B's rate.}$$

23. Let $b =$ the breadth.
Then $2b^2 = 1250$.
Whence $b = 25$,
and $2b = 50$, the length.
Now let $x =$ the width of the path.
Then $(50 - 2x)(25 - 2x) = 625$.

$$4x^2 - 150x + 625 = 0.$$

$$x = \frac{150 \pm \sqrt{22{,}500 - 10{,}000}}{8} = \frac{75 \pm 25\sqrt{5}}{4}.$$

$\dfrac{75 + 25\sqrt{5}}{4}$ is rejected.

$$\text{Width of path} = \frac{75 - 25\sqrt{5}}{4} = 4.77 + \text{ feet.}$$

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1.
$$x^2 + y^2 = 36.$$
$$(10 - x)^2 + y^2 = 64.$$
$$100 - 20x + x^2 + y^2 = 64.$$
$$-20x + 100 = 28.$$

$$20x = 72.$$
$$x = 3\tfrac{3}{5}.$$
$$y^2 = \tfrac{900}{25} - \tfrac{324}{25} = \tfrac{576}{25} = 1.$$
$$y = \tfrac{24}{5} = 4\tfrac{4}{5}, \text{ altitude.}$$

2.
$$x^2 + y^2 = 64.$$
$$(17 - x)^2 + y^2 = 225.$$
$$x^2 + y^2 = 64.$$
$$x^2 + y^2 - 34x + 289 = 225.$$
$$34x - 289 = -161.$$
$$34x = 128.$$
$$x = 3\tfrac{13}{17}.$$
$$y = 7\tfrac{1}{17}.$$
$$\therefore \text{ altitude} = 7\tfrac{1}{17}\text{ ; area} = 60.$$

3.
$$x^2 + y^2 = 169.$$
$$(20 - x)^2 + y^2 = 441.$$
Solving, $x = 3\tfrac{1}{5},\ y = 12\tfrac{3}{5}.$
$$\therefore \text{ altitude} = 12\tfrac{3}{5}\text{ ; area} = 126.$$

4.
$$x^2 + y^2 = 225.$$
$$(7 - x)^2 + y^2 = 400.$$
Solving, $x = 9,\ y = 12.$
$$\therefore \text{ altitude} = 12\text{ ; area} = 42.$$

5.
$$x^2 + y^2 = 289.$$
$$(10 - x)^2 + y^2 = 441.$$
Solving, $x = -2\tfrac{3}{5},\ y = 16\tfrac{8}{10}.$

The minus sign before x indicates that the perpendicular falls outside the triangle.
$$\therefore \text{ altitude} = 16\tfrac{4}{5}\text{ ; area} = 84.$$

6.
$$x^2 + y^2 = 144.$$
$$(16 - x)^2 + y^2 = 324.$$
Solving, $x = 2\tfrac{3}{8},$

and $y = 11.76$, the altitude.

7.
$$x^2 + y^2 = 100.$$
$$(12 - x)^2 + y^2 = 100.$$
Solving, $x = 6,\ y = 8.$
$$\therefore \text{ altitude} = 8.$$

8.
$$x^2 + y^2 = 144.$$
$$(8 - x)^2 + y^2 = 289.$$
Solving, $x = -5\tfrac{1}{16},\ y = 10.88.$
$$\therefore \text{ altitude} = 10.88.$$

9.
$$x^2 + y^2 = 900.$$
$$(5 - x)^2 + y^2 = 841.$$
Solving, $x = -8\tfrac{2}{5},\ y = 28\tfrac{4}{5}$, altitude.
$$\text{Area } \frac{28\tfrac{4}{5} \cdot (20 + 15)}{2} = 504.$$

10.
$$x^2 + y^2 = 144.$$
$$(25 - x)^2 + y^2 = 289.$$
Solving, $x = 9\tfrac{3}{5},\ y = 7\tfrac{1}{5}$, altitude.
$$\text{Area} = \frac{7\tfrac{1}{5} \cdot (20 + 45)}{2} = 234.$$

11.
$$x^2 + y^2 = 729.$$
$$(19 - x)^2 + y^2 = 900.$$
Solving, $y = 26.53$, altitude.
$$\text{Area} = \frac{26.53}{2}(21 + 40) = 809.16.$$

12.
$$x^2 + (100 - 23)^2 = \overline{85}^2.$$
$$x^2 = 1296.$$
$$x = 36, \text{ and area} = \tfrac{1}{2} \cdot 36 \cdot (123) = 2214.$$

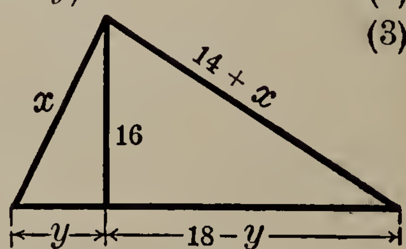
13.
$$x^2 + y^2 = \overline{123}^2.$$
$$(250 - 42 - x)^2 + y^2 = \overline{325}^2.$$
$$x^2 - 416x + y^2 = 105{,}625 - 43{,}264.$$
$$416x = 47{,}232.$$
$$x = 113\tfrac{71}{104}.$$
Substituting and solving, $y = 47\tfrac{4}{13}$, altitude.

Then $\text{area} = 47\tfrac{4}{13}\left(\dfrac{250 + 42}{2}\right) = 6906.9 +.$

14. Let x equal the third side, $x + 14$ the second, and y the projection of the third side on the first.

Then
$$x^2 + y^2 = 256. \qquad (1)$$
$$(14 + x)^2 = 256 + (18 - y)^2. \qquad (2)$$
From (1) and (2), $7x + 9y = 32.$ (3)

From (3) and (1), $x^2 + 14x - 680 = 0.$
$$\therefore x = -34, 20.$$
$$x + 14 = 34.$$
$\therefore$ the three sides are 18, 34, and 20.



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1. 6, 7, 8. **2.** 10, 12, 14. **3.** 5, 4, 3. **4.** $-9, -11, -13$. **5.** $7^2, 9^2, 11^2$. **6.** $\sqrt[3]{4}, \sqrt[3]{5}, \sqrt[3]{6}$. **7.** 32, 64, 128. **8.** $\frac{1}{16}, \frac{1}{32}, \frac{1}{64}$.

9. $\dfrac{1}{1\cdot2\cdot3\cdot4\cdot5}$, $\dfrac{1}{1\cdot2\cdot3\cdot4\cdot5\cdot6}$, $\dfrac{1}{1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7}$.

10. $1+3\sqrt{2}$, $1+4\sqrt{2}$, $1+5\sqrt{2}$.

Page 408 (First set)

1. Arithmetical progression ; $\frac{8}{3}$.
2. Arithmetical progression ; $6\frac{1}{2}$.
5. Arithmetical progression ; -4.
6. Arithmetical progression ; -10.
7. Arithmetical progression ; $-(2a+1)$.
9. Arithmetical progression ; $-\sqrt{3}$.

Page 408 (Second set)

1. $l = 1 + 11\cdot 4.$
$l = 45.$

2. $l = -18 + 22\cdot 3.$
$l = 48.$

3. $l = 13 + 14\cdot(-6).$
$l = -71.$

4. $l = a + 18\cdot 2a.$
$l = 37a.$

5. 7th term $= \frac{7}{3} + 6\cdot(-\frac{2}{3}) = -\frac{5}{3}.$
12th term $= \frac{7}{3} + 11\cdot(-\frac{2}{3})$
$= -\frac{15}{3} = -5.$

6. 5th term $= 1 + 4a.$
20th term $= 1 + 19a.$

7. $l = 7\sqrt{2} + 9\cdot(-2\sqrt{2}).$
$l = -11\sqrt{2}.$

8. $l = \dfrac{\sqrt{3}+5}{2} + 8\cdot\left(\dfrac{-\sqrt{3}-1}{2}\right).$
$l = \dfrac{-7\sqrt{3}-3}{2}.$

9. $l = a + (n-2)d.$

10. $l = a + (n-3)d.$

11. $l = \sqrt{5} - 1 + (n-4)\cdot(\sqrt{5}-1).$
$l = (\sqrt{5}-1)(n-3).$

12. $l = \dfrac{1}{n} + (n-1)\left(\dfrac{n-2}{n}\right).$
$l = \dfrac{n^2-3n+3}{n}.$

13. 3d term $= h + 2(k-h)$
$= 2k - h.$
nth term $= h + (n-1)(k-h).$

14. nth term $= h + (n-1)\dfrac{(k-h)}{2}.$

15. 10th term $= 16 + 9\cdot 32$
$= 304.$
nth term $= 16 + (n-1)\cdot 32$
$= 32n - 16.$

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1. $15 = 3 + 2d.$
$d = 6.$
$\therefore$ progression is:
3, 9, 15.

2. $4k = h + 2d.$
$d = \dfrac{4k-h}{2}.$
$\therefore$ progression is:
$h, \dfrac{4k+h}{2}, 4k.$

3. $17 = 2 + 3d.$
$d = 5.$
$\therefore$ progression is:
2, 7, 12, 17.

4. $$b = a + 3d.$$
$$d = \frac{b-a}{3}.$$

$\therefore$ progression is:

$$a, \frac{b+2a}{3}, \frac{2b+a}{3}, b.$$

5. $$16 = -4 + 4d.$$
$$d = 5.$$

$\therefore$ progression is:

$$-4, 1, 6, 11, 16.$$

6. $$n = m + 4d.$$
$$d = \frac{n-m}{4}.$$

$\therefore$ progression is:

$$m, \frac{3m+n}{4}, \frac{m+n}{2}, \frac{m+3n}{4}, n.$$

7. $$45 = 3 + 7d.$$
$$d = 6.$$

$\therefore$ progression is:

$$3, 9, 15, 21, 27, 33, 39, 45.$$

8. $$\frac{14}{3} = 3 + 10d.$$
$$d = \frac{1}{6}.$$

$\therefore$ progression is:

$$3, \frac{19}{6}, \frac{10}{3}, \frac{7}{2}, \frac{11}{3}, \frac{23}{6},$$
$$4, \frac{25}{6}, \frac{13}{3}, \frac{9}{2}, \frac{14}{3}.$$

9. $$9\sqrt{2} = -\sqrt{2} + 5d.$$
$$d = 2\sqrt{2}.$$

$\therefore$ progression is:

$$-\sqrt{2}, \sqrt{2}, 3\sqrt{2}, 5\sqrt{2},$$
$$7\sqrt{2}, 9\sqrt{2}.$$

10. $$13x + 9a = 7x - 3a + 6d.$$
$$6d = 6x + 12a.$$
$$d = x + 2a.$$

$\therefore$ progression is:

$$7x - 3a, 8x - a, 9x + a,$$
$$10x + 3a, 11x + 5a,$$
$$12x + 7a, 13x + 9a.$$

11. $$\frac{15\sqrt{5}}{2} = \frac{5}{2\sqrt{5}} + 7d.$$
$$d = \sqrt{5}.$$

$\therefore$ progression is:

$$\frac{5}{2\sqrt{5}}, \frac{15}{2\sqrt{5}}, \frac{5\sqrt{5}}{2},$$
$$\frac{7\sqrt{5}}{2}, \frac{9\sqrt{5}}{2}, \frac{11\sqrt{5}}{2},$$
$$\frac{13\sqrt{5}}{2}, \frac{15\sqrt{5}}{2}.$$

12. $$\sqrt{2}(1 - 2\sqrt{2}) = \frac{\sqrt{2}}{\sqrt{2}-1} + 3d.$$
$$\sqrt{2} - 4 = \sqrt{4} + \sqrt{2} + 3d.$$

$\therefore$ progression is:

$$\frac{\sqrt{2}}{\sqrt{2}-1}, \frac{2-\sqrt{2}}{\sqrt{2}-1}, \frac{4-3\sqrt{2}}{\sqrt{2}-1}, \sqrt{2}(1-2\sqrt{2}).$$

13. One half their sum.

14. Average velocity $= \dfrac{20+30}{2} = 25.$

$$\text{Time} = \frac{1 \text{ hour}}{25} = 2\tfrac{2}{5} \text{ minutes.}$$

15. (*a*) Average velocity $= \dfrac{64+96}{2} = 80.$

(*b*) Distance $= 80.$

16. Average velocity during first second $= \dfrac{0+32}{2} = 16.$

Distance the first second $= 16.$

Average velocity the second second $= \dfrac{32+64}{2} = 48.$

Distance the second second $= 48.$

17.
$$l = 2 + 24 \cdot 2.$$
$$l = 50.$$

Average $= \dfrac{2+50}{2} = \dfrac{4+48}{2}$, etc. $= 26.$

18.
$$l = 11 + 16 \cdot 2.$$
$$l = 43.$$

Mean $= \dfrac{11+43}{2} = \dfrac{13+41}{2}$, etc. $= 27.$

19. 10th term $= 32 + 9 \cdot 32 = 320.$

Average velocity $= \dfrac{0+320}{2} = 160.$

20.
$$22 = 1 + 7d.$$
$$d = 3.$$

The distances are 1, 4, 7, 10, 13, 16, 19, 22.

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1. $s = \frac{10}{2}(4 + 9 \cdot 3) = 155.$

2. $s = \frac{18}{2}[20 + 17(-2)] = -126.$

3. $s = \frac{10}{2}(6 + 9 \cdot 1\frac{1}{3}) = 90.$

4. $s = 6[36 + 11 \cdot (-3\frac{1}{2})] = -15.$

5. $s = 50(1 + 100) = 5050.$

6. $s = 50(4 + 99 \cdot 2) = 10{,}100.$

7. $s = 50(2 + 99 \cdot 2) = 10{,}000.$

8. $s = \frac{123}{2}(188 + 432) = 38{,}130.$

9. $s = \dfrac{n}{2}(2 + [n-1]2) = n^2.$

10. $s = \dfrac{n}{2}(4 + [n-1]2) = n^2 + n.$

11. $903 = \dfrac{n}{2}(2 + [n-1]1).$

$1806 = n^2 + n.$

$(n + 43)(n - 42) = 0.$

$n = 42.$

12. $275 = \dfrac{n}{2}(10 + [n-1]4).$

$275 = 2n^2 + 3n.$

$(2n + 25)(n - 11) = 0.$

$n = 11.$

13. $510 = \dfrac{n}{2}(160 + [n-1]2).$

$510 = 79n + n^2.$

$(n - 6)(n + 85) = 0.$

$n = 6.$

14. Let $-7 =$ 1st term.

Then $18 =$ 6th term.

$18 = -7 + 5d.$

$d = 5.$

$-7 = a + 5.$

$a = -12.$

11th term $= -12 + 10 \cdot 5$

$= 38.$

15. $s = \frac{t}{2}\left(\frac{2}{t} + [t-1] \cdot \frac{t-2}{t}\right)$
$= 1 + (t-1) \cdot \frac{t-2}{2}$
$= \frac{t^2 - 3t + 4}{2}.$

16. $l = a + (n-1)d.$
$29 = 2 + (n-1)3.$
$9 = n - 1.$
$n = 10.$
$s = 5(4 + 9 \cdot 3)$
$= 155.$

17. $s = \frac{n}{2}(2a + [n - 1d]).$
$300 = \frac{n}{2}(6 + [n-1]4)$
$= 2n^2 + n.$
$(n-12)(2n+25) = 0.$
$n = 12.$
$l = a + (n-1)d.$
$l = 3 + 11 \cdot 4$
$= 47.$

18. $s = \frac{n}{2}(2a + [n-1]d).$
$0 = \frac{13}{2}(2a + 12 \cdot [-11])$
$= 13(a - 66).$
$a = 66.$
$l = 66 + 12 \cdot (-11)$
$= -66.$

19. $s = \frac{n}{2}(2h + [n-1][k-h]).$

20. $s = \frac{n}{2}(a + l).$
$-33 = \frac{11}{2}(5x + 2 + l).$
$-6 = 5x + 2 + l.$
$l = -5x - 8.$
$l = a + (n-1)d.$
$-5x - 8 = 5x + 2 + 10d.$
$10d = -10x - 10.$
$d = -x - 1.$

21. $s = \frac{n}{2}(2a + [n-1]d).$
$40\sqrt{2} = \frac{n}{2}(-10\sqrt{2} + [n-1]2\sqrt{2}).$
$40\sqrt{2} = \frac{n}{2}(2n\sqrt{2} - 12\sqrt{2}).$
$40\sqrt{2} = n^2\sqrt{2} - 6n\sqrt{2}.$
$n^2 - 6n - 40 = 0.$
$(n-10)(n+4) = 0.$
$n = 10.$
$l = -5\sqrt{2} + 9 \cdot 2\sqrt{2}.$
$l = 13\sqrt{2}.$

22. $s = \frac{12}{2}(1 + 12) = 78.$
$2 \cdot 78 = 156.$

23. 30 miles per hour = 44 feet per second.
$l = a + [n-1]d.$
$0 = 44 + [n-1][-2].$
$46 = 2n.$
$n = 23.$
$s = \frac{23}{2}(44 + 0).$
$s = 506.$

24. $l = a + (n-1)d.$
$l = 4 + 20 \cdot 8.$
$l = 164.$
$s = \frac{21}{2}(4 + 164).$
$s = 21 \cdot 84.$
$s = 1764.$

25. $l = a + (n-1)d.$
$0 = \frac{39}{2} + (n-1)(-\frac{1}{2}).$
$\frac{39}{2} = \frac{1}{2}(n-1).$
$n = 40.$
$s = 20(2 \cdot 39 + 39[-1]) = 780.$
$780 + 20 = 800.$

26. $200 + 10(2 + d) + (2 + 2d)$
$= 17\frac{1}{5}(2 + [2 + d] + [2 + 2d]).$
$222 + 12d = \frac{86}{5}(6 + 3d).$
$1110 + 60d = 516 + 258d.$
$594 = 198d.$
$d = 3.$
The number is 258.

27. $l = 75 \cdot 12 + 10 \cdot 50$
$= 900 + 500 = \$1400$, salary for 11th year.
$$s = \frac{900 + 1400}{2} \cdot 11 = \$12{,}650.$$

28. $s = \frac{n}{2}(a + l) = 6(12 + 1) = 78.$
Interest $= \$.03 \cdot 50 \cdot 78 = \$117.$
$12 \cdot 50 = 600 =$ sum of payments.
$\$600 + \$117 = \$717.$

29. $555 = \frac{n}{2}(2 \cdot 16 + [n - 1]\,32).$
$555 = 16\,n^2.$
$n^2 = 34.6875.$
$n = 5.889$ or 5.9 seconds.
$l = 16 + 4.9 \cdot 32$
$= 172.8.$

30. $256 = \frac{n}{2}(2 \cdot 16 + [n - 1]\,32)$
$= 16\,n^2.$
$n^2 = 16.$
$n = 4$ seconds.
Velocity $= 4 \cdot 32 = 128$ feet per second.

31. Time of rising equals time of falling.
$\therefore\ s = \frac{7}{4}(2 \cdot 16 + [\frac{7}{2} - 1]\,32).$
$s = \frac{7}{4} \cdot 112.$
$s = 196$, height.
Velocity $= \frac{7}{2} \cdot 32 = 112$ feet per second.

32. $s_0 = \frac{10}{2}(2 \cdot 1 + 9 \cdot 1).$
$s_0 = 55.$
$s_1 = \frac{9}{2}(2 \cdot 1 + 8 \cdot 1).$
$s_1 = 45.$
$s_2 = \frac{8}{2}(2 + 7) = 36.$
$s_3 = \frac{7}{2}(2 + 6) = 28.$
$s_4 = \frac{6}{2}(2 + 5) = 21.$
$s_5 = \frac{5}{2}(2 + 4) = 15.$
$s_6 = \frac{4}{2}(2 + 3) = 10.$
$s_7 = \frac{3}{2}(2 + 2) = 6.$
$s_8 = \frac{2}{2}(2 + 1) = 3.$
$s_9 = 1.$
$s\ = 220.$

33. $s = \frac{n}{2}(2\,a + [n - 1]\,d).$
$$\frac{n}{2}(14 + [n - 1]\,\tfrac{1}{2}) = 12\,n.$$
$$14\,n + \frac{n^2}{2} - \frac{n}{2} = 24\,n.$$
$$n^2 - 21\,n = 0.$$
$$n(n - 21) = 0.$$
$$n = 21.$$

34. $\frac{n}{2}[2 \cdot 2 + (n - 1)\,2] = 28\,(n - 5).$
$$\frac{n}{2}(2\,n + 2) = 28\,n - 140.$$
$$n^2 + n = 28\,n - 140.$$
$$n^2 - 27\,n + 140 = 0.$$
$$(n - 7)(n - 20) = 0.$$
$$n = 7 \text{ or } 20.$$
Seven days after A starts, and twenty days after A starts.

Page 415 (First set)

1. Geometrical progression; $r = 3.$
3. Geometrical progression; $r = -\frac{1}{6}.$
5. Geometrical progression; $r = -\frac{1}{\sqrt{2}}.$
6. Geometrical progression; $r = \frac{1}{2}.$
8. Geometrical progression; $r = 3.$
10. Geometrical progression; $r = 5\,a.$
12. Geometrical progression; $r = 2\,y.$
13. Geometrical progression; $r = 4\,x^{-\frac{1}{2}}y^{\frac{1}{2}}.$

14. When $\frac{a}{b} = \frac{b}{c}.$

Page 415 (Second set)

2. $a = 3,\ r = 2,$
$t_{10} = 3 \cdot 2^9 = 1536.$

3. $a = 2,\ r = \frac{3}{2},$
$t_8 = 2 \cdot (\frac{3}{2})^7 = \frac{2187}{64}.$

4. $a = 5,\ r = -2,$
$t_{12} = 5 \cdot (-2)^{11} = -10{,}240.$

5. $a = \$100,\ r = \frac{53}{50} = 1.06,$
$t_6 = \$100 \cdot (1.06)^5 = \$133.82.$

6. $a = 18,\ r = -\frac{1}{3},$
$t_9 = 18 \cdot (-\frac{1}{3})^8 = \frac{2}{729}.$

7. $a = 12\,a,\ r = \frac{3}{4},$
$$t_{10} = 12\,a \cdot \left(\frac{3}{4}\right)^9 = \frac{59049\,a}{65536}.$$

8.
$$a = \frac{-2\,c}{3},$$
$$r = \frac{3}{2\,c},$$
$$t_7 = \frac{-2\,c}{3}\left(\frac{3}{2\,c}\right)^6 = -\frac{243}{32\,c^5}.$$

9.
$$a = 4\sqrt{2},$$
$$r = \frac{1}{\sqrt{2}},$$
$$t_7 = 4\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)^6 = \frac{\sqrt{2}}{2}.$$

10.
$$a = \frac{1}{2\sqrt{2}},$$
$$r = \frac{\sqrt{2}}{3},$$
$$t_6 = \frac{1}{2\sqrt{2}}\left(\frac{\sqrt{2}}{3}\right)^5 = \frac{2}{243}.$$

11.
$$a = \frac{3}{2\sqrt{2}},$$
$$r = \frac{2\sqrt{2}}{3},$$
$$t_8 = \frac{3}{2\sqrt{2}}\left(\frac{2\sqrt{2}}{3}\right)^7 = \frac{512}{729}.$$

12.
$$\frac{ar^{n-1}}{r} = ar^{n-2}.$$
$$\frac{ar^{n-1}}{r^2} = ar^{n-3}.$$
$$\frac{ar^{n-1}}{r^3} = ar^{n-4}.$$
$$(ar^{n-1})\,r = ar^n.$$
$$(ar^{n-1})r^2 = ar^{n+1}.$$

13. $t_1 = h.\ \ t_2 = k.\ \ r = \frac{k}{h}.$
$$t_3 = h\left(\frac{k}{h}\right)^2 = \frac{k^2}{h}.$$
$$t_4 = h\left(\frac{k}{h}\right)^3 = \frac{k^3}{h^2}.$$

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2. $a = 6,\ n = 4,\ t_n = t_4 = 48.$
$t_n = ar^{n-1}.\qquad 48 = 6\,r^3.$
$r^3 = 8.\qquad r = 2.$
The progression is:
6, 12, 24, 48.

3. $a = 6,\ n = 5,\ t_5 = 486.$
$486 = 6\,r^4.$
$r^4 = 81.$
$r = \pm 3.$
The progression is:
$6,\ \pm 18,\ 54,\ \pm 162,\ 486.$

4. $a = 4,\ n = 3,\ t_3 = 9.$
$9 = 4\,r^2.$
$r = \pm \frac{3}{2}.$
The progression is:
$4,\ \pm 6,\ 9.$

5. $a = a^{10},\ n = 3,\ t_3 = a^{20}.$
$a^{20} = a^{10}r^2.$
$r = \pm a^5.$
The progression is:
$a^{10},\ \pm a^{15},\ a^{20}.$

6. $a = -144,\ n = 5,\ t_5 = -9.$

$$-9 = -144\,r^4.$$
$$r^4 = \tfrac{1}{16}.$$
$$r = \pm\tfrac{1}{2}.$$

The progression is:

$-144,\ \pm 72,\ -36,\ \pm 18,\ -9.$

7. $t_5 = 32,\ t_9 = 512.$

Let $32 = t.$

Then $512 = t_5.$

$$512 = 32\,r^4.$$
$$r = 2.$$
$$t_{11} = 2 \cdot 2^{10} = 2048.$$

8. $t_2 = 3\sqrt{2},\ t_5 = \tfrac{3}{16}.$

$$t_5 = t_2r^3.$$
$$\tfrac{3}{16} = 3\sqrt{2}\,r^3.$$
$$r^3 = \frac{1}{16\sqrt{2}} = \frac{1}{2^{\frac{9}{2}}}.$$
$$r = \frac{1}{2^{\frac{3}{2}}} = \frac{1}{2\sqrt{2}}.$$
$$a = \frac{3\sqrt{2}}{\dfrac{1}{2\sqrt{2}}} = 12.$$

9. $a = h,\ n = 3,\ t_3 = k.$

$$k = hr^2.$$
$$r = \pm\sqrt{\frac{k}{h}} = \pm\frac{1}{h}\sqrt{hk}.$$

Geometrical mean is:

$$h\left(\pm\frac{\sqrt{hk}}{h}\right) = \pm\sqrt{hk}.$$

10. $a = h,\ n = 4,\ t_4 = k.$

$$k = hr^3.$$
$$r = \sqrt[3]{\frac{k}{h}} = \frac{1}{h}\sqrt[3]{h^2k}.$$

2d term $= \sqrt[3]{h^2k}.$

3d term $= \dfrac{\sqrt[3]{h^4k^2}}{h} = \sqrt[3]{hk^2}.$

11. $a = a,\ n = 5,\ t_5 = c.$

$$c = ar^4.$$
$$r = \pm\sqrt[4]{\frac{c}{a}}.$$
$$t_2 = \pm a\sqrt[4]{\frac{c}{a}} = \pm\sqrt[4]{a^3c}.$$
$$t_3 = \sqrt[4]{a^2c^2} = \sqrt{ac}.$$
$$t_4 = \pm\sqrt[4]{ac^3}.$$
$$a,\ \pm\sqrt[4]{a^3c},\ \sqrt{ac},\ \pm\sqrt[4]{ac^3},\ c.$$

12.
$$a + ar^2 = 13. \qquad (1)$$
$$ar = 6. \qquad (2)$$

(1) ÷ (2), $\dfrac{1 + r^2}{r} = \dfrac{13}{6}. \qquad (3)$

From (3),

$$6r^2 = 13r + 6 = 0.$$

Whence $r = \tfrac{3}{2}$ or $\tfrac{2}{3}$.

Thus from (2), $a = 4,\ 9.$

1st term $= 4,\ 9.$

2d term $= 6,\ 6.$

3d term $= 9,\ 4.$

13. $a = 4,\ n = 3,\ t_3 = 9.$

(a)
$$9 = 4r^2.$$
$$r = \tfrac{3}{2}.$$
$$t_2 = 6.$$
$$\therefore AD = 6.$$

(b) Let $BD = x,$

and $DC = 26 - x.$

Then $\dfrac{x}{12} = \dfrac{12}{26 - x}. \qquad (1)$

Solving (1), $x = 8$ or $18.$

$$BD = 8,\ 18.$$
$$CD = 18,\ 8.$$

14. $a = 16,\ n = 3,\ t_3 = 9.$

(a)
$$9 = 16r^2.$$
$$r = \tfrac{3}{4}.$$
$$t_2 = 12.$$
$$AB = 12.$$

(b) Let $AC = x,$

and $AD = 24 + x.$

Then $x(24 + x) = (16)^2. \qquad (1)$

Solving (1), $x = 8,\ -32.$

The -32 is rejected.

$$AC = 8.$$
$$AD = 32.$$

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2. $S_n = \dfrac{a - ar^n}{1 - r}$.

$a = 1,\ r = 5,\ n = 7.$

$$S_7 = \frac{1 - 5^7}{1 - 5} = \frac{-78,124}{-4} = 19,531.$$

3. $a = -2,\ r = -2,\ n = 7.$

$$S = \frac{-2 + 2(-2)^7}{1 + 2} = \frac{-258}{3} = -86.$$

4. $a = 50,\ r = \frac{1}{5},\ n = 8.$

$$S = \frac{50 - 50\left(\frac{1}{5}\right)^8}{1 - \frac{1}{5}} = \frac{50 - \frac{2}{15625}}{\frac{4}{5}} = \frac{125}{2} - \frac{1}{6250}$$

$$= \frac{390624}{6250} = 62.49984.$$

5. $a = 180,\ r = -\frac{1}{2},\ n = 9.$

$$S_9 = \frac{180 - 180 \cdot \left(-\frac{1}{2}\right)^9}{1 + \frac{1}{2}} = \frac{180 + \frac{45}{128}}{\frac{3}{2}} = 120\tfrac{15}{64}.$$

6. $a = \frac{2}{5},\ r = \frac{5}{2},\ n = 5.$

$$S = \frac{\frac{2}{5} - \frac{2}{5}\left(\frac{5}{2}\right)^5}{1 - \frac{5}{2}} = \frac{\frac{2}{5} - \frac{625}{16}}{-\frac{3}{2}} = \frac{-\frac{3093}{80}}{-\frac{3}{2}} = \frac{1031}{40} = 25.775.$$

7. $a = c^3,\ r = c^2,\ n = 7.$

$$S = \frac{c^3 - c^3(c^2)^7}{1 - c^2} = \frac{c^3 - c^{17}}{1 - c^2}.$$

8. $a = 3\sqrt{2},\ r = \dfrac{2}{\sqrt{2}},\ n = 5.$

$$S = \frac{3\sqrt{2} - 3\sqrt{2}(\sqrt{2})^5}{1 - \sqrt{2}} = 21\sqrt{2} + 18.$$

9. $a = 81,\ r = -\dfrac{1}{\sqrt{3}},\ n = 6.$

$$S = \frac{81 - 81\left(-\dfrac{1}{\sqrt{3}}\right)^6}{1 + \dfrac{1}{\sqrt{3}}} = \frac{81 - 3}{\dfrac{3 + \sqrt{3}}{3}} = \frac{234}{3 + \sqrt{3}} = 39(3 - \sqrt{3}).$$

10. $a = 3,\ r = 5.$

$$S = \frac{3 - 3(5)^n}{1 - 5} = -\frac{3}{4}(1 - 5^n) = \frac{3}{4}(5^n - 1).$$

11. $a = 2x,\ r = 2x^3,\ n = n - 2.$

$$S = \frac{2x - 2x(2x^3)^{n-2}}{1 - 2x^3} = \frac{2x\left[1 - (2x^3)^{n-2}\right]}{1 - 2x^3}.$$

12. $S = \frac{a - ar^n}{1 - r} = \frac{a - ar^{n-1} \cdot r}{1 - r} = \frac{a - rl}{1 - r}.$

13. $l = \$100\,(1.04)^3 = \$112.486.$
$l = \$100\,(1.02)^6 = 112.616.$

14. $l = ar^{n-1} = 40\,(\frac{2}{5})^5 = \frac{256}{625} = .4096.$
$$S = \frac{32 - 32\,(\frac{2}{5})^5}{1 - \frac{2}{5}} = \frac{\frac{98976}{3125}}{\frac{3}{5}} = \frac{32992}{625} = 52.7872.$$
Total distance $= 40 + 52.7872 = 92.7872$ feet.

15. Amount remaining $= l = c\,(\frac{2}{3})^6 = \frac{64}{729}c$, where c equals the number of gallons at first.

16. $l = 400\left(\frac{9}{10}\right)^{10} = \frac{3486784401}{10^{10}} \cdot 400.$

Then .3486784401 of original contents remain.

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2. $a = 1,\ r = \frac{1}{2}.$
$$S_\infty = \frac{a}{1 - r} = \frac{1}{\frac{1}{2}} = 2.$$

3. $a = 2,\ r = -\frac{1}{2}.$
$$S_\infty = \frac{2}{1 + \frac{1}{2}} = \frac{4}{3}.$$

4. $a = 3,\ r = -\frac{1}{3}.$
$$S_\infty = \frac{3}{1 + \frac{1}{3}} = \frac{9}{4}.$$

5. $a = 2,\ r = \frac{1}{\sqrt{2}}.$
$$S_\infty = \frac{2}{1 - \frac{1}{\sqrt{2}}} = \frac{2\sqrt{2}}{\sqrt{2} - 1}$$
$$= 4 + 2\sqrt{2}.$$

6. $a = 5a,\ r = \frac{1}{4}.$
$$S_\infty = \frac{5a}{1 - \frac{1}{4}} = \frac{20a}{3} = 6\tfrac{2}{3}a.$$

7. $a = 1,\ r = x.$
$$S_\infty = \frac{1}{1 - x}.$$

8. $a = 3,\ r = \frac{1}{\sqrt{3}}.$
$$S_\infty = \frac{3}{1 - \frac{1}{\sqrt{3}}} = \frac{3\sqrt{3}}{\sqrt{3} - 1}$$
$$= \frac{9 + 3\sqrt{3}}{2}.$$

9. $a = 1,\ r = \frac{1}{x}.\quad (x > 1.)$
$$S_\infty = \frac{1}{1 - \frac{1}{x}} = \frac{x}{x - 1}.$$

10. $a = .51,\ r = .01.$
$$S_\infty = \frac{.51}{1 - .01} = \frac{51}{99} = \frac{17}{33}.$$

11. $a = .6,\ r = .1.$
$$S_\infty = \frac{.6}{1 - .1} = \frac{6}{9} = \frac{2}{3}.$$

12. $a = .27,\ r = .01.$
$$S_\infty = \frac{.27}{1 - .01} = \frac{27}{99} = \frac{3}{11}.$$

13. $a = .39,\ r = .01.$
$$S_\infty = \frac{.39}{1 - .01} = \frac{39}{99} = \frac{13}{33}.$$

14. $25.36\dot{3}\dot{6}.$
$$S_\infty = 25 + \frac{.36}{1 - .01} = 25\tfrac{4}{11}.$$

15. $.721\dot{2}\dot{1}$.

$$S_\infty = .7 + \frac{.021}{1 - .01} = \frac{714}{990} = \frac{119}{165}.$$

16. $.3091\dot{0}9\dot{1}$.

$$S_\infty = .3 + \frac{.0091}{1 - .001} = \frac{3088}{9990}$$
$$= \tfrac{1544}{4995}.$$

17. $a = 5 \cdot 80$.

$$S_\infty = \frac{5 \cdot 80}{1 - .99} = \frac{400}{.01} = 40{,}000 \text{ ft.}$$

18. $a = \frac{9}{4}\sqrt{3},\ r = \frac{1}{4}$.

$$S_\infty = \frac{\frac{9}{4}\sqrt{3}}{1 - \frac{1}{4}} = 3\sqrt{3}.$$

19. (*a*) $a = 8,\ r = \dfrac{1}{\sqrt{2}}$.

$$S_\infty = \frac{8}{1 - \dfrac{1}{\sqrt{2}}} = \frac{8\sqrt{2}}{\sqrt{2} - 1}$$
$$= \frac{8\sqrt{2}(\sqrt{2} + 1)}{(\sqrt{2} - 1)(\sqrt{2} + 1)}$$
$$= 16 + 8\sqrt{2}.$$

(*b*) $a = 4,\ r = \frac{1}{2}$.

$$S_\infty = \frac{4}{1 - \frac{1}{2}} = 8.$$

21. $p = \dfrac{Sr(r + 1)^n}{(r + 1)^n - 1}$.

$$p = \frac{\$1000 \times .05(1.05)^3}{(1.05)^3 - 1}$$
$$= \frac{57.88125}{.157625} = \$367.21.$$

22. $p = \dfrac{Sr(1 + r)^n}{(1 + r)^n - 1}$.

$$= \frac{\$5000 \times .06(1.06)^5}{(1.06)^5 - 1}$$
$$= \$1186.98.$$

Page 429

1. $x - y = 1,$
$2y - 2x = -2.$

$$x = \frac{\begin{vmatrix} 1 & -1 \\ -2 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ -2 & 2 \end{vmatrix}} = \frac{2 - 2}{2 - 2} = \frac{0}{0}.$$

The system is indeterminate. The graphs of the two equations are identical. Values for any point on either graph will satisfy both equations.

2. $x - y = 0,$
$y - x = 3.$

$$x = \frac{\begin{vmatrix} 0 & -1 \\ 3 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix}} = \frac{1 + 3}{1 - 1} = \frac{4}{0}.$$

The system is inconsistent. The equations are incompatible and the graphs of the two equations are parallel lines.

3. $x + y + z = 0,$
$x - 2y + 3z = 1,$
$2x - y + 4z = 1.$

$$x = \frac{\begin{vmatrix} 0 & 1 & 1 \\ 1 & -2 & 3 \\ 1 & -1 & 4 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & 3 \\ 2 & -1 & 4 \end{vmatrix}} = \frac{-1 + 3 + 2 - 4}{-8 - 1 + 6 + 4 + 3 - 4} = \frac{0}{0}.$$

The system is indeterminate. An infinite number of sets of values will satisfy the system.

4. $x + y + z = 1,$
$x - y - 2z = 2,$
$0x + 0y + 0z = 0.$

$$x = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & -2 \\ 0 & 0 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & -2 \\ 0 & 0 & 0 \end{vmatrix}} = \frac{0}{0}.$$

The system is indeterminate. An infinite number of sets of values will satisfy the system.

5. The conclusion is that there is an infinite number of values for a system of two linear equations in three variables.

6. $x + y + z = 2,$
$0x + 0y + 0z = 0,$
$0x + 0y + 0z = 0.$

$$x = \frac{\begin{vmatrix} 2 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}} = \frac{0}{0}.$$

The system is indeterminate.

7. Results show that there is an infinite number of sets of roots for one equation in three variables.

8. 0.	10. 0.	12. 1.	14. ∞.	16. 1.	18. 0.	20. 0.
9. 0.	11. 1.	13. 1.	15. 1.	17. ∞.	19. 0.	21. ∞.

22. $\dfrac{1-x}{1-x^2} = \dfrac{1}{1+x} \doteq \dfrac{1}{2}$ as $x \doteq 1.$

23. $\dfrac{x^2-5x+6}{x^2-4} = \dfrac{x-3}{x+2} \doteq -\dfrac{1}{4}$ as $x \doteq 2.$

24. $\dfrac{x-2}{x^3-8} = \dfrac{1}{x^2+2x+4} = \dfrac{1}{12}$ as $x \doteq 2.$

Page 434

1. $\log 2 = .3.$
2. $\log 3 = .47.$
3. $\log 4 = .6.$
4. $\log 5 = .7.$
5. $\log 6.2 = .79.$
6. $\log 7.4 = .86.$
7. $\log 9.6 = .98.$
8. $\log 11 = 1.04.$
9. $\log 15 = 1.17.$
10. $\log .2 = -.7.$
11. $\log .5 = -.3.$
12. $\log .9 = -.04.$
13. $\log 25 = 1.4.$
14. $\log 50 = 1.7.$
15. $\log 64 = 1.81.$
16. $\log 96 = 1.98.$

17. $\log 100 = 2.$

18. $\log 10 = 1.$

19. $\log 1 = 0.$

20. $\log 32 = 1.5.$

21. $\log n = .3,\ n = 2.$

22. $\log n = .4,\ n = 2.5.$

23. $\log n = .6,\ n = 4.$

24. $\log n = .7,\ n = 5.$

25. $\log n = .95,\ n = 9.$

26. $\log n = -.1,\ n = .8.$

27. $\log n = -.4,\ n = .4.$

28. $\log n = -.5,\ n = .3.$

29. $\log n = -.25,\ n = .6.$

30. $\log n = 1.3,\ n = 20.$

31. $\log n = 1.7,\ n = 50.$

32. $\log n = 1.6,\ n = 40.$

33. $\log n = 1.8,\ n = 63.$

34. $\log n = 2,\ n = 100.$

35. $\log n = .84,\ n = 7.$

36. $\log n = 1.2,\ n = 16.$

Page 435 (First set)

The exercises on pages 435–436 are here solved by means of the table on page 448. The pupil should solve them, however, by means of the curve on page 433.

1.
$$\begin{aligned} \log 2 &= .3010 \\ \log 5 &= \underline{.6990} \\ & 1.0000 \\ \log n &= 1,\ n = 10. \end{aligned}$$

2.
$$\begin{aligned} \log 3 &= .4771 \\ & \underline{2} \\ \log n &= .9542 \\ n &= 9. \end{aligned}$$

3.
$$\begin{aligned} \log 4 &= .6021 \\ & \underline{2} \\ \log n &= 1.2042 \\ n &= 16. \end{aligned}$$

4.
$$\begin{aligned} \log 8 &= .9031 \\ & \underline{2} \\ \log n &= 1.8062 \\ n &= 64. \end{aligned}$$

5.
$$\begin{aligned} \log 30 &= 1.4771 \\ \log 2 &= \underline{3010} \\ \log n &= 1.7781 \\ n &= 60. \end{aligned}$$

6.
$$\begin{aligned} \log 25 &= 1.3979 \\ \log 4 &= \underline{.6021} \\ \log n &= 2.0000 \\ n &= 100. \end{aligned}$$

7.
$$\begin{aligned} \log 80 &= 1.9031 \\ \log .5 &= \underline{\bar{1}.6990} \\ \log n &= 1.6021 \\ n &= 40. \end{aligned}$$

8.
$$\begin{aligned} \log 22 &= 1.3424 \\ \log 4.8 &= \underline{.6812} \\ \log n &= 2.0236 \\ n &= 105.6. \end{aligned}$$

9.
$$\begin{aligned} \log 14 &= 1.1461 \\ \log 6 &= \underline{.7782} \\ \log n &= 1.9243 \\ n &= 84. \end{aligned}$$

10.
$$\begin{aligned} \log 80 &= 1.9031 \\ \log .6 &= \underline{\bar{1}.7782} \\ \log n &= 1.6813 \\ n &= 48. \end{aligned}$$

11.
$$\begin{aligned} \log 97 &= 1.9868 \\ \log .7 &= \underline{\bar{1}.8451} \\ \log n &= 1.8319 \\ n &= 67.9. \end{aligned}$$

12.
$$\begin{aligned} \log 7.2 &= .8573 \\ \log 1.5 &= \underline{.1761} \\ \log n &= 1.0334 \\ n &= 10.8. \end{aligned}$$

Page 435 (Second set)

1.
$$\begin{aligned} \log 8 &= .9031 \\ \log 2 &= \underline{.3010} \\ \log n &= .6021 \\ n &= 4. \end{aligned}$$

2.
$$\begin{aligned} \log 6 &= .7782 \\ \log 3 &= \underline{.4771} \\ \log n &= .3011 \\ n &= 2. \end{aligned}$$

3.
$$\begin{aligned} \log 40 &= 1.6021 \\ \log 5 &= \underline{.6990} \\ \log n &= .9031 \\ n &= 8. \end{aligned}$$

4.
$$\begin{aligned} \log 18 &= 1.2553 \\ \log 8 &= \underline{.9031} \\ \log n &= .3522 \\ n &= 2.25. \end{aligned}$$

5.
$$\begin{aligned} \log 16 &= 1.2041 \\ \log 8 &= \underline{.9031} \\ \log n &= .3010 \\ n &= 2. \end{aligned}$$

6.
$$\begin{aligned} \log 48 &= 1.6812 \\ \log 6 &= \underline{.7782} \\ \log n &= .9030 \\ n &= 8. \end{aligned}$$

7.
$$\begin{aligned} \log 22 &= 1.3424 \\ \log 11 &= \underline{1.0414} \\ \log n &= .3010 \\ n &= 2. \end{aligned}$$

8.
$$\begin{aligned} \log 56 &= 1.7482 \\ \log 8 &= \underline{.9031} \\ \log n &= .8451 \\ n &= 7. \end{aligned}$$

9.
$$\begin{aligned} \log 4 &= .6021 \\ \log 6 &= \underline{.7782} \\ \log 24 &= 1.3803 \\ \log 3 &= \underline{.4771} \\ \log n &= .9032 \\ n &= 8. \end{aligned}$$

10.
$$\begin{aligned} \log 80 &= 1.9031 \\ \log 40 &= \underline{1.6021} \\ \log 40 \cdot 80 &= 3.5052 \\ \log 25 &= \underline{1.3979} \\ \log n &= 2.1073 \\ n &= 128. \end{aligned}$$

Page 436 (First set)

1.
$$\begin{aligned} \log 3 &= .4771 \\ & \underline{2} \\ \log n &= .9542 \\ n &= 9. \end{aligned}$$

2.
$$\begin{aligned} \log 5 &= .6990 \\ & \underline{2} \\ \log n &= 1.3980 \\ n &= 25. \end{aligned}$$

3.
$$\begin{aligned} \log 3 &= .4771 \\ & \underline{3} \\ \log n &= 1.4313 \\ n &= 27. \end{aligned}$$

4.
$$\begin{aligned} \log 4 &= .6021 \\ & \underline{3} \\ \log n &= 1.8063 \\ n &= 64. \end{aligned}$$

5.
$$\begin{aligned} \log 5 &= .6990. \\ \log 10 &= 1. \\ \log 25 &= 1.3979. \end{aligned}$$

$$\begin{aligned} 2 \log 5 &= 1.3980 \\ 3 \log 10 &= \underline{3.} \\ & 4.3980 \\ 2 \log 25 &= \underline{2.7958} \\ \log n &= 1.6022 \\ n &= 40. \end{aligned}$$

6. $$\begin{aligned} 2\log 2 &= .3010 \times 2 = .6020 \\ 3\log 3 &= .4771 \times 3 = \underline{1.4313} \\ && 2.0333 \\ \log 6 &= \quad \underline{.7782} \\ \log n &= \quad 1.2551 \\ n &= 18. \end{aligned}$$

7. $$\begin{aligned} 2\log 4 &= 1.2042 \\ 3\log 3 &= \underline{1.4313} \\ \log n &= 2.6355 \\ n &= 432. \end{aligned}$$

Page 436 (Second set)

1. $\dfrac{\log 3}{2} = \dfrac{.4771}{2} = .2385 = \log n.$ Whence $n = 1.732.$

2. $\dfrac{\log 4}{3} = \dfrac{.6021}{3} = .2007 = \log n.$ Whence $n = 1.587.$

3. $\dfrac{\log 81}{4} = \dfrac{1.9085}{4} = .4771 = \log n.$ Whence $n = 3.$

4. $\dfrac{\log 6}{2} = \dfrac{.7782}{2} = .3891 = \log n.$ Whence $n = 2.45.$

5. $\dfrac{\log 7}{3} = \dfrac{.8451}{3} = .2817 = \log n.$ Whence $n = 1.913.$

6. $$\begin{aligned} 2\log 5 &= 1.3980 \\ \frac{\log 3}{2} &= \underline{.2385} \\ \log n &= 1.6365 \\ n &= 43.3. \end{aligned}$$

7. $$\begin{aligned} 3\log 3 &= 1.4313 \\ \frac{\log 2}{2} &= \underline{.1505} \\ && 1.5818 \\ \log .7 &= \underline{\bar{1}.8451} \\ \log n &= 1.7367 \\ n &= 54.53. \end{aligned}$$

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1. $\log 300 = 2.47.$
2. $\log 65 = 1.81.$
3. $\log 4 = .60.$
4. $\log 1 = 0.$
5. $\log .10 = -1.$
6. $\log 1730 = 3.238.$
7. $\log 173 = 2.238.$
8. $\log 1.73 = .238.$
9. $\log .173 = \bar{1}.238.$
10. $\log .0173 = \bar{2}.238.$
11. $3 = 10^{.47}.$
12. $20 = 10^{1.301}.$
13. $4.9 = 10^{.69}.$
14. $490 = 10^{2.69}.$
15. $.0049 = 10^{-3+.69}.$
16. $381 = 10^{2.58}.$

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1. $\log 271 = 2.4330.$
2. $\log 344 = 2.5366.$
3. $\log 982 = 2.9921.$
4. $\log 65 = 1.8129.$
5. $\log 650 = 2.8129.$
6. $\log 27 = 1.4314.$
7. $\log 2.7 = .4314.$
8. $\log 2700 = 3.4314.$
9. $\log 3 = .4771.$
10. $\log 6 = .7782.$

11. $\log 932 = 2.9694.$

12. $\log .932 = \bar{1}.9694.$

13. $\log .643 = \bar{1}.8082.$

14. $\log .0532 = \bar{2}.7259.$

15. $\log .00267 = \bar{3}.4265.$

16. $\log .00579 = \bar{3}.7627.$

17. $\log .0101 = \bar{2}.0043.$

18. $\log 825{,}000 = 5.9165.$

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1.
$$\begin{aligned} \log 4630 &= 3.6656 \\ \log 4620 &= \underline{3.6646} \\ & .0010 \end{aligned}$$
$\therefore \log 4625 = 3.6651.$

2.
$$\begin{aligned} \log 365 &= 2.5623 \\ \log 364 &= \underline{2.5611} \\ & .0012 \end{aligned}$$
$\therefore \log 364.7 = 2.5619.$

3.
$$\begin{aligned} \log 42.8 &= 1.6314 \\ \log 42.7 &= \underline{1.6304} \\ & .0010 \end{aligned}$$
$\therefore \log 42.73 = 1.6307.$

4.
$$\begin{aligned} \log 32.8 &= 1.5159 \\ \log 32.7 &= \underline{1.5145} \\ & .0014 \end{aligned}$$
$\therefore \log 32.75 = 1.5152.$

5.
$$\begin{aligned} \log 547 &= 2.7380 \\ \log 546 &= \underline{2.7372} \\ & .0008 \end{aligned}$$
$\therefore \log 546.8 = 2.7378.$

6.
$$\begin{aligned} \log 72.6 &= 1.8609 \\ \log 72.5 &= \underline{1.8603} \\ & .0006 \end{aligned}$$
$\therefore \log 72.543 = 1.8606.$

7.
$$\begin{aligned} \log 10.2 &= 1.0086 \\ \log 10.1 &= \underline{1.0043} \\ & .0043 \end{aligned}$$
$\therefore \log 10.101 = 1.0047.$

8.
$$\begin{aligned} \log 701 &= 2.8457 \\ \log 700 &= \underline{2.8451} \\ & .0006 \times .35 = .0002+. \end{aligned}$$
$\therefore \log 700.35 = 2.8453.$

9.
$$\begin{aligned} \log 506 &= 2.7042 \\ \log 505 &= \underline{2.7033} \\ & .0009 \end{aligned}$$
$\therefore \log 505.50 = 2.7037.$

10.
$$\begin{aligned} \log 2.01 &= .3032 \\ \log 2.00 &= \underline{.3010} \\ & .0022 \times .75 = .00165. \end{aligned}$$
$\therefore \log 2.0075 = .3026.$

11. $\log .00386 = \bar{3}.5866.$

12.
$$\begin{aligned} \log .000778 &= \bar{4}.8910 \\ \log .000777 &= \underline{\bar{4}.8904} \\ & .0006 \times .7 = .00042. \end{aligned}$$
$\therefore \log .0007777 = \bar{4}.8908.$

13.
$$\begin{aligned} \log 3.15 &= .4983 \\ \log 3.14 &= \underline{.4969} \\ & .0014 \times .16 = .000224. \end{aligned}$$
$\therefore \log 3.1416 = .4971.$

14.
$$\begin{aligned} \log 2.72 &= .4346 \\ \log 2.71 &= \underline{.4330} \\ & .0016 \times .828 = .0013248. \\ \log 2.71828 &= .4343. \end{aligned}$$

15.
$$\begin{aligned} \log .0235 &= \bar{2}.3711 \\ \log .0234 &= \underline{\bar{2}.3692} \\ & .0019 \times .56 = .001065. \end{aligned}$$
$\therefore \log .023456 = \bar{2}.3703.$

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1. antilog 3.9309 = 8530.
2. antilog 1.8162 = 65.5.
3. antilog .6284 = 4.25.
4. antilog $\bar{1}.3541$ = .226.
5. antilog $\bar{2}.5740$ = .0375.
6. antilog 8.5740 – 10 = .0375.
7. antilog 9.7292 – 10 = .536.
8. antilog 4.8136 – 10 = .00000651.
9. antilog .4533 = 2.84.
10. antilog $\bar{4}.6345$ = .000431.
11. antilog 6.9232 = 8,380,000.
12. antilog 8.2148 = 164,000,000.
13. antilog 5.7832 – 6 = .607.
14. antilog $\bar{5}.9996$ = .0000999.

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1. antilog 1.5717 = 37.3.
antilog 1.5729 = 37.4.
∴ antilog 1.5723 = 37.35.

2. antilog 2.3909 = 246.
antilog 2.3927 = 247.
∴ antilog 2.3921 = 246.6.

3. antilog .6684 = 4.66.
antilog .6693 = 4.67.
∴ antilog .6690 = 4.666.

4. antilog 2.5717 = 373.
antilog 2.5729 = 374.
∴ antilog 2.5728 = 373.9.

5. antilog $\bar{1}.2577$ = .181.
antilog $\bar{1}.2601$ = .182.
∴ antilog $\bar{1}.2586$ = .1813.

6. antilog 7.3464 – 10 = .00222.
antilog 7.3483 – 10 = .00223.
∴ antilog 7.3472 – 10 = .002224.

7. antilog 9.8525 – 10 = .712.
antilog 9.8531 – 10 = .713.
∴ antilog 9.8527 – 10 = .7123.

8. antilog 5.9614 – 8 = .00915.
antilog 5.9619 – 8 = .00916.
∴ antilog 5.9616 – 8 = .009154.

9. antilog 9.2648 – 10 = .184.
antilog 9.2672 – 10 = .185.
∴ antilog 9.2654 – 10 = .1842.

10. antilog .7825 = 6.06.
antilog .7832 = 6.07.
∴ antilog .7829 = 6.065.

11. antilog 7.1038 – 10 = .00127.
antilog 7.1072 – 10 = .00128.
∴ antilog 7.1050 – 10 = .001273.

12. antilog 6.2304 – 10 = .000170.
antilog 6.2330 – 10 = .000171.
∴ antilog 6.2308 – 10 = .0001701.

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2.
$$\begin{aligned} \log 37 &= 1.5682 \\ \log 23 &= \underline{1.3617} \\ \log 23 \cdot 37 &= 2.9299 \end{aligned}$$
antilog 2.9299 = 851.

3.
$$\begin{aligned} \log 28 &= 1.4472 \\ \log 8 &= \underline{\ \ .9031} \\ \log 8 \cdot 28 &= 2.3503 \end{aligned}$$
antilog 2.3503 = 224.

4.
$$\begin{aligned}\log 9.8 &= .9912\\ \log 5 &= \underline{.6990}\\ \log 5\cdot 9.8 &= 1.6902\end{aligned}$$
antilog 1.6902 = 49.0.

5.
$$\begin{aligned}\log 42 &= 1.6232\\ \log 2.2 &= \underline{.3424}\\ \log 42\cdot 2.2 &= 1.9656\end{aligned}$$
antilog 1.9656 = 92.38.

6.
$$\begin{aligned}\log 386 &= 2.5866\\ \log 27 &= \underline{1.4314}\\ \log 27\cdot 386 &= 4.0180\end{aligned}$$
antilog 4.0180 = 10,420.

7.
$$\begin{aligned}\log 432 &= 2.6355\\ \log 361 &= \underline{2.5575}\\ \log 432\cdot 361 &= 5.1930\end{aligned}$$
antilog 5.1930 = 155,900.

8.
$$\begin{aligned}\log 589 &= 2.7701\\ \log 734 &= \underline{2.8657}\\ \log 589\cdot 734 &= 5.6358\end{aligned}$$
antilog 5.6358 = 432,300.

9.
$$\begin{aligned}\log 4326 &= 3.6361\\ \log 638 &= \underline{2.8048}\\ \log 638\cdot 4326 &= 6.4409\end{aligned}$$
antilog 6.4409 = 2,760,000.

10.
$$\begin{aligned}\log 2870 &= 3.4579\\ \log 3654 &= \underline{3.5628}\\ \log \text{product} &= 7.0207\end{aligned}$$
antilog 7.0207 = 10,480,000.

11.
$$\begin{aligned}\log 286.7 &= 2.4574\\ \log 2.341 &= \underline{.3694}\\ \log \text{product} &= 2.8268\end{aligned}$$
antilog 2.8268 = 671.1.

12.
$$\begin{aligned}\log 3.412 &= .5331\\ \log 2.596 &= \underline{.4143}\\ \log \text{product} &= .9474\end{aligned}$$
antilog .9474 = 8.860.

14.
$$\begin{aligned}\log 385 &= 2.5855\\ \log .617 &= \underline{\bar{1}.7903}\\ \log \text{product} &= 2.3758\end{aligned}$$
antilog 2.3758 = 237.6.

15.
$$\begin{aligned}\log 541 &= 2.7332\\ \log .073 &= \underline{\bar{2}.8633}\\ \log \text{product} &= 1.5965\end{aligned}$$
antilog 1.5965 = 39.49.

16.
$$\begin{aligned}\log 37.6 &= 1.5752\\ \log .00835 &= \underline{\bar{3}.9217}\\ \log \text{product} &= \bar{1}.4969\end{aligned}$$
antilog $\bar{1}$.4969 = .314.

17.
$$\begin{aligned}\log .0876 &= \bar{2}.9425\\ \log .673 &= \underline{\bar{1}.8280}\\ \log \text{product} &= \bar{2}.7705\end{aligned}$$
antilog $\bar{2}$.7705 = .05895.

18.
$$\begin{aligned}\log .07325 &= \bar{2}.8648\\ \log 6.384 &= \underline{.8051}\\ \log \text{product} &= \bar{1}.6699\end{aligned}$$
antilog $\bar{1}$.6699 = .4676.

19.
$$\begin{aligned}\log .6381 &= \bar{1}.8049\\ \log .01897 &= \underline{\bar{2}.2782}\\ \log \text{product} &= \bar{2}.0831\end{aligned}$$
antilog $\bar{2}$.0831 = .0121.

20.
$$\begin{aligned}\log 675 &= 2.8293\\ \log .0236 &= \underline{\bar{2}.3729}\\ \log \text{product} &= 1.2022\end{aligned}$$
antilog 1.2022 = 15.92.

21.
$$\begin{aligned}\log .437 &= \bar{1}.6405\\ \log .0076 &= \underline{\bar{3}.8808}\\ \log \text{product} &= \bar{3}.5213\end{aligned}$$
antilog $\bar{3}$.5213 = .003321.

23.
$$\begin{aligned}\log 96 &= 1.9823\\ \log 12 &= \underline{1.0792}\\ \log \text{quotient} &= .9031\end{aligned}$$
antilog .9031 = 8.

24.
$$\begin{aligned}\log 888 &= 2.9484\\ \log 37 &= \underline{1.5682}\\ \text{log quotient} &= 1.3802\\ \text{antilog } 1.3802 &= 24.\end{aligned}$$

25.
$$\begin{aligned}\log 976 &= 2.9894\\ \log 321 &= \underline{2.5065}\\ \text{log quotient} &= .4829\\ \text{antilog } .4829 &= 3.04.\end{aligned}$$

26.
$$\begin{aligned}\log 489 &= 2.6893\\ \log 27.1 &= \underline{1.4330}\\ \text{log quotient} &= 1.2563\\ \text{antilog } 1.2563 &= 18.04.\end{aligned}$$

27.
$$\begin{aligned}\log 3460 &= 3.5391\\ \log 4.32 &= \underline{.6355}\\ \text{log quotient} &= 2.9036\\ \text{antilog } 2.9036 &= 801.\end{aligned}$$

28.
$$\begin{aligned}\log 4697 &= 3.6718\\ \log 281 &= \underline{2.4487}\\ \text{log quotient} &= 1.2231\\ \text{antilog } 1.2231 &= 16.71.\end{aligned}$$

29.
$$\begin{aligned}\log 9876 &= 3.9946\\ \log 56.78 &= \underline{1.7542}\\ \text{log quotient} &= 2.2404\\ \text{antilog } 2.2404 &= 173.9.\end{aligned}$$

30.
$$\begin{aligned}\log 6432 &= 3.8083\\ \log 7.81 &= \underline{.8927}\\ \text{log quotient} &= 2.9156\\ \text{antilog } 2.9156 &= 823.4.\end{aligned}$$

32.
$$\begin{aligned}\log 2.35 &= .3711\\ \log .0673 &= \underline{\bar{2}.8280}\\ \text{log quotient} &= 1.5431\\ \text{antilog } 1.5431 &= 34.92.\end{aligned}$$

33.
$$\begin{aligned}\log 4.86 &= .6866\\ \log .721 &= \underline{\bar{1}.8579}\\ \text{log quotient} &= .8287\\ \text{antilog } .8287 &= 6.74.\end{aligned}$$

34.
$$\begin{aligned}\log .0635 &= \bar{2}.8028\\ \log .287 &= \underline{\bar{1}.4579}\\ \text{log quotient} &= \bar{1}.3449\\ \text{antilog } \bar{1}.3449 &= .2212.\end{aligned}$$

35.
$$\begin{aligned}\log .2674 &= \bar{1}.4271\\ \log 3.86 &= \underline{.5866}\\ \text{log quotient} &= \bar{2}.8405\\ \text{antilog } \bar{2}.8405 &= .06926.\end{aligned}$$

36.
$$\begin{aligned}\log 7635 &= 3.8828\\ \log 8692 &= \underline{3.9391}\\ \text{log quotient} &= \bar{1}.9437\\ \text{antilog } \bar{1}.9437 &= .8784.\end{aligned}$$

37.
$$\begin{aligned}\log .07382 &= \bar{2}.8682\\ \log 68.72 &= \underline{1.8371}\\ \text{log quotient} &= \bar{3}.0311\\ \text{antilog } \bar{3}.0311 &= .001074.\end{aligned}$$

38.
$$\begin{aligned}\log 256 &= 2.4082\\ \log 372 &= \underline{2.5705}\\ \text{log product} &= 4.9787\\ \log 128 &= \underline{2.1072}\\ \text{log quotient} &= 2.8715\\ \text{antilog } 2.8715 &= 743.8.\end{aligned}$$

39.
$$\begin{aligned}\log 347 &= 2.5403\\ \log 625 &= \underline{2.7959}\\ \text{log product} &= 5.3362\\ \log 346 &= \underline{2.5391}\\ \text{log quotient} &= 2.7971\\ \text{antilog } 2.7971 &= 626.7.\end{aligned}$$
$$\therefore \frac{347 \cdot (-625)}{346} = -626.7.$$

40.
$$\begin{aligned}\log 463.2 &= 2.6658\\ \log 4.78 &= \underline{.6794}\\ \text{log product} &= 3.3452\\ \log 68.3 &= \underline{1.8344}\\ \text{log quotient} &= 1.5108\\ \text{antilog } 1.5108 &= 32.42.\end{aligned}$$
$$\therefore \frac{463.2 \cdot 4.78}{-68.3} = -32.42.$$

41.

$$\begin{aligned} \log 9.63 &= .9836 \\ \log .0872 &= \bar{2}.9405 \\ \hline \log \text{product} &= \bar{1}.9241 \\ \log .00635 &= \bar{3}.8028 \\ \hline \log \text{quotient} &= 2.1213 \\ \text{antilog } 2.1213 &= 132.2. \end{aligned}$$

42.

$$\begin{aligned} \log .078 &= \bar{2}.8921 \\ \log 4.267 &= .6301 \\ \hline \log \text{quotient} &= \bar{2}.2620 \\ \text{antilog } \bar{2}.2620 &= .01827. \end{aligned}$$

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2.

$$\begin{aligned} \log 6.32 &= .8007 \\ & \quad 4 \\ \hline \log \text{product} &= 3.2028 \\ \text{antilog } 3.2028 &= 1595. \end{aligned}$$

3.

$$\begin{aligned} \log 34.26 &= 1.5348 \\ & \quad 2 \\ \hline \log \text{product} &= 3.0696 \\ \text{antilog } 3.0696 &= 1173. \end{aligned}$$

4.

$$\begin{aligned} \log 6.715 &= .8270 \\ & \quad 3 \\ \hline \log \text{product} &= 2.4810 \\ \text{antilog } 2.4810 &= 302.7. \end{aligned}$$

6.

$$\begin{aligned} \log .352 &= \bar{1}.5465 \\ & \quad 4 \\ \hline \log \text{product} &= \bar{2}.1860 \\ \text{antilog } \bar{2}.1860 &= .01534. \end{aligned}$$

7.

$$\begin{aligned} \log .0672 &= \bar{2}.8274 \\ & \quad 2 \\ \hline \log \text{product} &= \bar{3}.6548 \\ \text{antilog } \bar{3}.6548 &= .004516. \end{aligned}$$

8.

$$\begin{aligned} \log .003567 &= \bar{3}.5522 \\ & \quad 5 \\ \hline \log \text{product} &= \overline{13}.7610 \end{aligned}$$

antilog
$\overline{13}.7610 = .0000000000005767.$

10.

$$\begin{aligned} \log 583 &= 2.7657. \\ \log \sqrt[3]{583} &= .9219. \\ \text{antilog } .9219 &= 8.354. \end{aligned}$$

11.

$$\begin{aligned} \log 1235 &= 3.0916. \\ \log \sqrt[5]{1235} &= .6183. \\ \text{antilog } .6183 &= 4.153. \end{aligned}$$

13.

$$\begin{aligned} \log .0786 &= \bar{2}.8954. \\ \log \sqrt{.0786} &= \bar{1}.4477. \\ \text{antilog } \bar{1}.4477 &= .2803. \end{aligned}$$

14.

$$\begin{aligned} \log .0007324 &= \bar{4}.8647. \\ \log \sqrt[5]{.0007324} &= .3729 - \bar{1}. \\ \text{antilog } \bar{1}.3729 &= .236. \end{aligned}$$

15.

$$\begin{aligned} \log .002679 &= \bar{3}.4279. \\ \log \sqrt[4]{.002679} &= .3569 - 1. \\ \text{antilog } \bar{1}.3569 &= .2274. \end{aligned}$$

16.

$$\begin{aligned} \log 38.4 &= 1.5843 \\ & \quad 2 \\ \hline \log \overline{38.4}^2 &= 3.1686 \\ \log \sqrt[3]{\overline{38.4}^2} &= 1.0562. \\ \text{antilog } 1.0562 &= 11.38. \end{aligned}$$

17.

$$\begin{aligned} \log 4.965 &= .6959 \\ & \quad 3 \\ \hline \log \text{power} &= 2.0877. \\ \log \sqrt{\overline{4.965}^3} &= 1.0439. \\ \text{antilog } 1.0439 &= 11.06. \end{aligned}$$

18.

$$\begin{aligned} \log 6.387 &= .8053. \\ \log \sqrt[3]{(6.387)^5} &= \tfrac{5}{3}(.8053) \\ &= 1.3421. \\ \text{antilog } 1.3421 &= 21.98. \\ \therefore\ (-6.387)^{\frac{5}{3}} &= \sqrt[3]{(-6.387)^5} \\ &= -21.98. \end{aligned}$$

19.
$$\begin{aligned}\log 283 &= 2.4518\\ \log 4.627 &= \underline{\ \ .6653}\\ &\ 3.1171\\ 3\log 8.423 &= \underline{2.7762}\\ \log\text{ quotient} &= \ \ .3409\\ \log\text{ radical} &= .1705.\\ \text{antilog } .1705 &= 1.480.\end{aligned}$$

20.
$$\begin{aligned}2\log 23.56 &= 2.7444\\ \log 7.384 &= \underline{\ \ .8683}\\ &\ 3.6127\\ 3\log 4.623 &= \underline{1.9947}\\ \log\text{ quotient} &= 1.6180\\ \log\text{ radical} &= .8090.\\ \text{antilog } .8090 &= 6.441.\end{aligned}$$

21.
$$\begin{aligned}\log 209 &= 2.3201.\\ \log \sqrt[11]{209} &= 2109.\\ \text{antilog } .2109 &= 1.625.\end{aligned}$$

22.
$$\begin{aligned}\log 163 &= 2.2122.\\ \log \sqrt[4]{163} &= .5430.\\ \text{antilog } .5430 &= 3.4915.\\ 87 - 3.4915 &= 83.5085.\\ \log 83.5085 &= 1.9217.\\ \log\text{ radical} &= .9609.\\ \text{antilog } .9609 &= 9.14.\end{aligned}$$

23.
$$\begin{aligned}\log 127 &= 2.1038\\ \tfrac{1}{3}\log 872 &= \underline{\ \ .9802}\\ &\ 3.0840\\ \log 67 &= \underline{1.8261}\\ \log \tfrac{127}{67}\sqrt[3]{872} &= 1.2579\\ \log \sqrt[5]{\ } &= \ \ .2516\\ \log 25 &= \underline{1.3979}\\ &\ 1.6495\\ \log 361 &= \underline{2.5575}\\ \log\text{ ans.} &= \bar{1}.0920\\ \text{antilog } \bar{1}.0920 &= .1236.\end{aligned}$$

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1. (*a*) Circumference $= 2\pi \cdot 42.$

$$\begin{aligned}\log 2 &= \ \ .3010\\ \log \pi &= \ \ .4971\\ \log 42 &= \underline{1.6232}\\ \log\text{ product} &= 2.4213\\ \text{antilog } 2.4213 &= 263.8.\end{aligned}$$

(*b*) Radius $= \dfrac{6843}{2\cdot\pi}.$

$$\begin{aligned}\log 6843 &= 3.8353\\ \log 2\pi &= \underline{\ \ .7981}\\ \log\text{ quotient} &= 3.0372\\ \text{antilog } 3.0372 &= 1089.\end{aligned}$$

2. (*a*) Area $= \pi \cdot \overline{3.672}^2.$

$$\begin{aligned}\log \pi &= \ \ .4971\\ 2\log 3.672 &= \underline{1.1298}\\ \log\text{ area} &= 1.6269\\ \text{antilog } 1.6269 &= 42.36.\end{aligned}$$

(*b*) $R = \sqrt{\dfrac{\text{area}}{\pi}}.$

$$\begin{aligned}\log 64.37 &= 1.8087\\ \log \pi &= \underline{\ \ .4971}\\ \log\text{ quotient} &= 1.3116\\ \log\text{ radical} &= \ \ .6558.\\ \text{antilog } .6558 &= 4.527.\end{aligned}$$

3. (*a*) Area $= 4\pi \cdot \overline{3958.79}^2.$

$$\begin{aligned}\log 4 &= \ \ .6021\\ \log \pi &= \ \ .4971\\ 2\log\text{ radius} &= \underline{7.1950}\\ \log\text{ area} &= 8.2942\\ \text{antilog } 8.2942 &= 196{,}800{,}000.\end{aligned}$$

(*b*) Equator $= 2\pi R.$

$$\begin{aligned}\log 2 &= \ \ .3010\\ \log \pi &= \ \ .4971\\ \log 3958.79 &= \underline{3.5975}\\ \log\text{ equator} &= 4.3956\\ \text{antilog } 4.3956 &= 24810\text{ miles.}\end{aligned}$$

4. (a) $25 = \dfrac{4\pi R^2}{3}$. $\therefore R = \sqrt[3]{\dfrac{75}{4\pi}}$.

$$\begin{aligned} \log 4 &= .6021 \\ \log \pi &= \underline{.4971} \\ \log 4\pi &= 1.0992 \\ \log 75 &= 1.8750 \\ \log 4\pi &= \underline{1.0992} \\ \log R^3 &= .7758 \\ \log R &= .2586 \\ \text{antilog} &= 1.813 \text{ feet.} \end{aligned}$$

(b) $\dfrac{4\pi \cdot D^3}{3 \cdot 8} = 85$. $D = \sqrt[3]{\dfrac{8 \cdot 85 \cdot 3}{4\pi}}$.

$$\begin{aligned} \log 3 &= .4771 \\ \log 8 &= .9031 \\ \log 85 &= \underline{1.9294} \\ & \quad 3.3096 \\ \log 4\pi &= \underline{1.0992} \\ & \quad 2.2104 \\ \log \sqrt[3]{} &= .7368 \\ \text{antilog } .7368 &= 5.455 \text{ inches.} \end{aligned}$$

5. (a) The log of third side $= \frac{1}{2}\log(377 + 288) + \frac{1}{2}\log(377 - 288)$.

$$\begin{aligned} \frac{\log 665}{2} &= \frac{2.8228}{2} = 1.4114. \\ \frac{\log 89}{2} &= \frac{1.9494}{2} = \underline{.9747} \\ \text{log of third side} &= 2.3861 \\ \text{antilog } 2.3861 &= 243.2. \end{aligned}$$

(b)

$$\begin{aligned} \frac{\log 2025}{2} &= \frac{3.3064}{2} = 1.6532. \\ \frac{\log 961}{2} &= \frac{2.9827}{2} = \underline{1.4913} \\ \log \text{side} &= 3.1445 \\ \text{antilog} &= 1395. \end{aligned}$$

6. (a) Area $= \dfrac{\overline{11.47}^2}{4}\sqrt{3}$.

$$\begin{aligned} 2\log 11.47 &= 2.1190 \\ \tfrac{1}{2}\log 3 &= \underline{.2385} \\ & \quad 2.3575 \\ \log 4 &= .6021 \\ \log 144 &= \underline{2.1584} \\ \log 4 \cdot 144 &= \underline{2.7605} \\ \log \text{area} &= \bar{1}.5970 \\ \text{antilog } \bar{1}.5970 &= .3953. \end{aligned}$$

(b) $60 = \dfrac{s^2}{4}\sqrt{3}$.

$s = \sqrt{\dfrac{4 \cdot 60}{\sqrt{3}}} = \sqrt{80\sqrt{3}}$.

$$\begin{aligned} \log 80 &= 1.9031 \\ \log \sqrt{3} &= \underline{.2385} \\ \log 80\sqrt{3} &= 2.1416 \\ \log s &= 1.0708 \\ \text{antilog } 1.0708 &= 11.75. \end{aligned}$$

7. (a) Area $= \sqrt{23 \cdot 11 \cdot 8 \cdot 4}$.

$$\begin{aligned} \log 23 &= 1.3617 \\ \log 11 &= 1.0414 \\ \log 8 &= .9031 \\ \log 4 &= \underline{.6021} \\ & \quad 3.9083 \\ \log \text{area} &= 1.9542. \end{aligned}$$

antilog $1.9542 = 90.0 =$ area in square inches.

(b) Area $= \sqrt{1203 \cdot 363 \cdot 194 \cdot 646}$.

$$\begin{aligned} \log 1203 &= 3.0802 \\ \log 363 &= 2.5599 \\ \log 194 &= 2.2878 \\ \log 646 &= \underline{2.8102} \\ & \quad 10.7381 \\ \log \text{area} &= 5.3690. \\ \text{antilog } 5.3690 &= 233{,}880. \end{aligned}$$

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2. $x \log 3 = \log 25.$

$$x = \frac{\log 25}{\log 3} = \frac{1.3979}{.4771} = 2.929.$$

3. $x \log 64 = \log 4.$

$$x = \frac{\log 4}{\log 64} = \frac{.6021}{1.8062} = .33+.$$

4. $x \log 16 = \log 1024.$

$$x = \frac{\log 1024}{\log 16} = \frac{3.0103}{1.2041} = 2.50+.$$

5. $x \log 2 = \log 64.$

$$x = \frac{\log 64}{\log 2} = \frac{1.8062}{.3010} = 6.$$

6. $x \log (1.04) = \log 3.$

$$x = \frac{\log 3}{\log (1.04)} = \frac{.4771}{.0170} = 28.0.$$

7. $x \log 2 = \log 64.$

$$x = \frac{\log 64}{\log 2} = \frac{1.8062}{.3010} = 6.$$

8. $2x \log 4 + \log 4 = \log 84.$

$2x \log 4 = \log 84 - \log 4 = \log 21.$

$$x = \frac{\log 21}{2 \log 4} = \frac{1.3222}{1.2041} = 1.09+.$$

9. $\sqrt{81} \cdot 3^{x+1} = (\sqrt[3]{27})^{2x-1}.$

$9 \cdot 3^{x+1} = 3^{2x-1}.$

$3^2 \cdot 3^{x+1} = 3^{2x-1}.$

$\therefore x + 3 = 2x - 1.$

Whence $x = 4.$

10. $x \log 3 + 7 \log 3 = x \log 5.$

$x (\log 5 - \log 3) = 7 \log 3,$

or $$x = \frac{7 \log 3}{\log 5 - \log 3}.$$

$$x = \frac{7 \log 3}{\log \frac{5}{3}} = \frac{3.3397}{.2219} = 15.05.$$

12. Let $x =$ the number.

$(1.05)^x = 2.$

$x \log (1.05) = \log 2.$

$$x = \frac{\log 2}{\log 1.05} = \frac{.3010}{.0212}$$
$= 14.19 +$ years.

13. Let $P =$ amount,

and $x =$ number of years.

$P(1.04)^x = 3P.$

$x \log 1.04 = \log 3.$

$$x = \frac{\log 3}{\log 1.04} = \frac{.4771}{.0170}$$
$= 28.06 +$ years.

14. $265 (1.035)^x = 2 \cdot 265.$

$x \log 1.035 = \log 2.$

$$x = \frac{\log 2}{\log 1.035} = \frac{.3010}{.0149}$$
$= 20.20 +$ years.

15. $4000 \cdot (1.05)^x = 7360.80.$

$(1.05)^x = 1.84020.$

$x \log 1.05 = \log 1.8402.$

$$x = \frac{\log 1.8402}{\log 1.05} = \frac{.2648}{.0212}$$
$= 12.49 +$ years.

16. Let $x =$ amount in dollars.

$24 \cdot (1.04)^{300} = x.$

$300 \log 1.04 + \log 24 = \log x.$

$\log 1.04 = .0170$

$\underline{300}$

5.1000

$\log 24 = \underline{1.3802}$

$\log x = 6.4802$

$x = \$3{,}021{,}000.$

17. Let $x =$ the number of years.

$12 (1.02)^{2x} = 24.$

$2x \log 1.02 = \log 2.$

$$2x = \frac{\log 2}{\log 1.02}$$
$$= \frac{.3010}{.0086} = 35.$$

$x = \frac{35}{2} = 17\frac{1}{2}$ years.

19. (*a*) $x = 5000\,(1.08)^4$
$= 5000\,(1.359)$
$= \$6795$, annually.

$$\begin{aligned} \log 1.08 &= .0334 \\ &\underline{\quad\ \ 4} \\ &\ \ .1336 \end{aligned}$$

antilog $.1336 = 1.359$.

(*b*) $x = 5000\,(1.04)^8$
$= 5000\,(1.367)$
$= \$6835$, semiannually.

$$\begin{aligned} \log 1.04 &= .0170 \\ &\underline{\quad\ \ 8} \\ &\ \ .1360 \end{aligned}$$

antilog $.1360 = 1.367$.

(*c*) $x = 5000\,(1.02)^{16}$
$= 5000\,(1.373)$
$= \$6865$, quarterly.

$\log 1.02 = .0086$ (1)

$\quad 16$ (2)

(1) × (2), $.1376$

antilog $.1376 = 1.373$.

20. From Exercise 20, page 174, $p = \dfrac{Sr\,(1+r)^n}{(1+r)^n - 1}$. (1)

Substituting, $\$50 = \dfrac{\$6000\,(.005)\,(1.005)^n}{(1.005)^n - 1}$. (2)

Solving (2), $n = \dfrac{\log 2.5}{\log 1.005} = \dfrac{.3979}{.00215} = 185 + .$

21. The amount of the first four payments would be:

$$\$50\,[(1.06)^{15} \times 1.02 + (1.06)^{15} \times 1.015 + (1.06)^{15} \times 1.01 + (1.06)^{15} \times 1.005]$$
$$= \$50\,(1.06)^{15} \times 4.05 = \$485.30.$$

Similarly, the next twelve payments would amount to:

$$\$50\,[(1.06)^{14} \times 1.06 + (1.06)^{14} \times 1.055 + \cdots + (1.06)^{14} \times 1.01 + (1.06)^{14} \times 1.005]$$
$$= \$50 \times 1.06^{14} \times 12.39.$$

The amount of each succeeding group of twelve payments would be a similar expression. Therefore the last 180 payments would amount to:

$$\$50 \times 12.39\,[(1.06)^{14} + (1.06)^{13} + \cdots + (1.06)^2 + 1.06]$$
$$= \$50 \times 12.39 \times \frac{1.06 - (1.06)^{15}}{1 - 1.06} = \$13{,}790.$$

The last payment is \$50. Then the whole amount is approximately $\$485.30 + \$13{,}790 + \$50 = \$14{,}325.30$.

22. From \$6000 the final return is $\$14{,}325.30 - \$6000$, or \$8325.30. One year's income will be $\frac{12}{185}$ of \$8325.30, or \$540.01. Dividing 540.01 by 6000 gives 9.00%.

23.
(*a*)
$$\log 2^9 = 9(.3010) = 2.7090$$
$$\log 3^8 = 8(.4771) = 3.8168$$
$$\log 5^7 = 7(.6990) = \underline{4.8930}$$
$$11.4188. \text{ Twelve digits.}$$

(*b*) $\log 3^{52} = 52(.4771) = 24.8092$. Twenty-five digits.

(*c*) $\log 2^{340} = 340(.3010) = 102.3400$. One hundred and three digits.

24. No. Most logarithms are incommensurable numbers and therefore neither odd nor even. Hence the signs of the antilogarithms to a negative base could not be determined. Without certainty on this point correct computation is impossible.

25. $\log_3 9 = x.$
$$3^x = 9 = 3^2.$$
$$\therefore x = 2.$$

26. $2^x = 8 = 2^3.$
$$x = 3.$$

27. $8^x = 2.$
$$2^{3x} = 2.$$
$$x = \tfrac{1}{3}.$$

28. $9^x = 27.$
$$3^{2x} = 3^3.$$
$$x = \tfrac{3}{2}.$$

29. $27^x = 9.$
$$3^{3x} = 3^2.$$
$$x = \tfrac{2}{3}.$$

30. Let $\log_4 8 = x$, $\log_8 4 = y.$
Then $4^x = 8$, and $8^y = 4.$
Whence $x = \frac{3}{2}$, and $y = \frac{2}{3}.$
Therefore $x + y = \frac{3}{2} + \frac{2}{3} = \frac{13}{6}.$

31. Let $\log_{27} 81 = x$, and $\log_{81} 27 = y.$
Then $27^x = 81$, and $81^y = 27.$
Whence $x = \frac{4}{3}$, and $y = \frac{3}{4}.$
Therefore $x - y = \frac{4}{3} - \frac{3}{4} = \frac{7}{12}.$

32. Let $\log_{25} 125 = x$, $\log_5 25 = y$, and $\log_{125} 5 = z.$
Then $25^x = 125$, $5^y = 25$, and $125^z = 5.$
Whence $x = \frac{3}{2}$, $y = 2$, and $z = \frac{1}{3}.$
$$\therefore x + y - z = \tfrac{3}{2} + 2 - \tfrac{1}{3} = \tfrac{19}{6}.$$

33. Let $\log_3 (\frac{1}{3}) = x$, $\log_9 (\frac{1}{27}) = y$, and $\log_{27} 9 = z.$
Then $3^x = \frac{1}{3}$, $9^y = \frac{1}{27}$, and $27^z = 9.$
Whence $x = -1$, $y = -\frac{3}{2}$, and $z = \frac{2}{3}.$
$$\therefore x - y + z = -1 + \tfrac{3}{2} + \tfrac{2}{3} = \tfrac{7}{6}.$$

34. $\operatorname{Log} \frac{5}{8} + \log \frac{24}{25} = \log (\frac{5}{8} \cdot \frac{24}{25}) = \log \frac{3}{5}.$

35. $\operatorname{Log} \frac{7}{32} - \log \frac{35}{64} = \log (\frac{7}{32} \div \frac{35}{64}) = \log \frac{2}{5}.$

36. $\operatorname{Log} \frac{25}{4} + \log \frac{36}{50} - \log \frac{3}{4} = \log (\frac{25}{4} \cdot \frac{36}{50} \div \frac{3}{4}) = \log 6.$

37. $2 \log 3 + 3 \log 2 = \log (3^2 \cdot 2^3) = \log 72.$

38. $3 \log 4 + 4 \log 3 - 2 \log 6 = \log \dfrac{4^3 \cdot 3^4}{6^2} = \log 144.$

39.

$$a^x = c^{x-1}.$$
$$x \log a = (x-1)\log c.$$
$$x \log a = x \log c - \log c.$$
$$x(\log a - \log c) = -\log c.$$
$$x = \frac{-\log c}{\log a - \log c} = \frac{\log c}{\log c - \log a} = \frac{\log c}{\log \frac{c}{a}}.$$

40.

$$a^{x-1} = c^{x-2}.$$
$$(x-1)\log a = (x-2)\log c.$$
$$x \log a - \log a = x \log c - 2\log c.$$
$$x(\log a - \log c) = \log a - 2\log c.$$
$$x = \frac{\log a - 2\log c}{\log a - \log c} = \frac{\log \frac{a}{c^2}}{\log \frac{a}{c}}.$$

41.

$$a^{x-1} \cdot b^x = c^{2x}.$$
$$x \log a - \log a + x \log b = 2x \log c.$$
$$x(\log a + \log b - 2\log c) = \log a.$$
$$x = \frac{\log a}{\log a + \log b - 2\log c} = \frac{\log a}{\log \frac{ab}{c^2}}.$$

42.

$$3^x \cdot 2^{\frac{1}{x}} = 6.$$
$$x \log 3 + \frac{1}{x}\log 2 = \log 6.$$
$$x^2 \log 3 - x \log 6 + \log 2 = 0.$$
$$x = \frac{\log 6 \pm \sqrt{(\log 6)^2 - 4\log 2 \log 3}}{2\log 3}$$
$$= \frac{\log 2 + \log 3 \pm \sqrt{(\log 2 + \log 3)^2 - 4\log 2\log 3}}{2\log 3}$$
$$= \frac{\log 2 + \log 3 \pm (\log 2 - \log 3)}{2\log 3}$$
$$= \frac{2\log 2}{2\log 3} = \frac{\log 2}{\log 3}, \text{ and } = \frac{2\log 3}{2\log 3} = 1.$$

43.

$$e^x = e^{-x}.$$
$$x \log e = -x \log e.$$
$$x = -x.$$
$$2x = 0.$$
$$x = 0.$$

44.

$$2^{2x} \cdot 3^{\frac{2}{x}} = 36.$$
$$2x \log 2 + \frac{2}{x}\log 3 = \log 6^2 = 2\log 6.$$
$$x^2 \log 2 - x \log 6 + \log 3 = 0.$$
$$x^2 \log 2 - x \log 2 - x \log 3 + \log 3 = 0.$$
$$(x-1)(x \log 2 - \log 3) = 0.$$
$$\therefore\ x = 1, \frac{\log 3}{\log 2}.$$

45.

$$a^{x+1} = b^{2x} \div c^{x-1}.$$
$$(x+1)\log a = 2x\log b - (x-1)\log c.$$
$$x\log a + \log a = 2x\log b - x\log c + \log c.$$
$$x(\log a + \log c - 2\log b) = \log c - \log a.$$
$$x = \frac{\log c - \log a}{\log a - 2\log b + \log c} = \frac{\log \dfrac{c}{a}}{\log ac - \log b^2}$$

46.

$$a^{4x} + 8a^{2x} = 6a^{3x}.$$
$$a^{4x} + 8a^{2x} - 6a^{3x} = 0.$$
$$a^{2x}(a^{2x} - 6a^x + 8) = 0.$$
$$\therefore\ a^{2x} = 0. \quad \text{Whence } x \doteq -\infty.$$
$$a^{2x} - 6a^x + 8 = 0.$$
$$(a^x - 4)(a^x - 2) = 0.$$
$$\therefore\ a^x - 4 = 0. \quad \text{Whence } x = \frac{\log 4}{\log a}.$$
$$a^x - 2 = 0. \quad \text{Whence } x = \frac{\log 2}{\log a}.$$

47.

$$a^{5x} + a^{4x} = 6a^{4x} - 6a^{3x}.$$
$$a^{5x} + a^{4x} - 6a^{4x} + 6a^{3x} = 0.$$
$$a^{3x}(a^{2x} - 5a^x + 6) = 0.$$
$$\therefore\ a^{3x} = 0. \quad \text{Whence } x \doteq -\infty.$$

And
$$a^{2x} - 5a^x + 6 = 0.$$
$$(a^x - 3)(a^x - 2) = 0.$$
$$a^x - 3 = 0. \quad \text{Whence } x = \frac{\log 3}{\log a}.$$
$$a^x - 2 = 0. \quad \text{Whence } x = \frac{\log 2}{\log a}.$$

48.

$$2^x = 3^y, \qquad (1)$$
$$3^{x-1} = 4^y. \qquad (2)$$

From (1), $x\log 2 = y\log 3.$ (3)

From (2), $x\log 3 - \log 3 = 2y\log 2.$ (4)

$$x\log 2 - y\log 3 = 0. \qquad (5)$$
$$x\log 3 - 2y\log 2 = \log 3. \qquad (6)$$

(5) · log 3, $x\log 2\log 3 - y(\log 3)^2 = 0.$ (7)

(6) · log 2, $x\log 2\log 3 - 2y(\log 2)^2 = \log 2\log 3.$ (8)

(7) – (8), $y[2(\log 2)^2 - (\log 3)^2] = \log 2\log 3.$ (9)

$$y = \frac{\log 2\log 3}{2(\log 2)^2 - (\log 3)^2}. \qquad (10)$$

From (10) and (3),
$$x = \frac{y\log 3}{\log 2}$$
$$= \frac{(\log 3)^2}{[2(\log 2)^2 - (\log 3)^2]}.$$

49.

$$2x - y = 5, \qquad (1)$$
$$3^x \cdot 9^{3y} = 9^{11}. \qquad (2)$$
$$x = \frac{y+5}{2}. \qquad (3)$$
$$3^{\frac{y+5}{2}} \cdot 9^{3y} = 9^{11}. \qquad (4)$$
$$3^{\frac{y+5}{2}} \cdot 3^{6y} = 3^{22}. \qquad (5)$$
$$3^{\frac{13y+5}{2}} = 3^{22}. \qquad (6)$$
$$\therefore 13y + 5 = 44. \qquad (7)$$
$$y = 3. \qquad (8)$$

From (3) and (8), $x = 4$.

50.

$$3x + y = 9,$$
$$2^x \cdot 8^{2y} = 4^{10}.$$
$$y = 9 - 3x.$$
$$2^x \cdot 8^{18-6x} = 4^{10}.$$
$$2^x \cdot 2^{54-18x} = 2^{20}.$$
$$2^{54-17x} = 2^{20}.$$
$$\therefore 54 - 17x = 20.$$

Whence $x = 2$.

Then $y = 3$.

51.

$$8^x \cdot 5^y = 50, \qquad (1)$$
$$2^{6x} \cdot 3^{2y} = 328. \qquad (2)$$
$$x \log 8 + y \log 5 = \log 50. \qquad (3)$$
$$6x \log 2 + 2y \log 3 = \log 328.$$

Now $6x \log 2 = 2x \log 2^3 = 2x \log 8. \qquad (4)$

(3) · 2, $2x \log 8 + 2y \log 5 = 2 \log 50. \qquad (5)$

From (4) and (3), $2x \log 8 + 2y \log 3 = \log 328. \qquad (6)$

(5) – (6), $2y(\log 5 - \log 3) = 2 \log 50 - \log 328. \qquad (7)$

$$y = \frac{2 \log 50 - \log 328}{2(\log 5 - \log 3)} = \frac{\log \frac{625}{82}}{\log \frac{25}{9}}. \qquad (8)$$

From (3), $$x = \frac{\log 50}{\log 8} - \frac{y \log 5}{\log 8}. \qquad (9)$$

From (9) and (8), $$x = \frac{\log 50}{\log 8} - \frac{\log 5 \cdot \log \frac{625}{82}}{\log 8 \cdot \log \frac{25}{9}}.$$

52.

$$3^x - 6^y = 0, \qquad (1)$$
$$3^{x+1} - 6^x = 0. \qquad (2)$$

From (2), $(x+1)\log 3 = x \log 6. \qquad (3)$

$$x(\log 3 - \log 6) = -\log 3. \qquad (4)$$
$$x = \frac{\log 3}{\log 6 - \log 3} = \frac{\log 3}{\log 2}. \qquad (5)$$

(1) · 3, $3^{x+1} - 3 \cdot 6^y = 0 \qquad (6)$

$$3^{x+1} - 6^x = 0 \qquad (7)$$
$$6^x - 3 \cdot 6^y = 0 \qquad (8)$$
$$6^x = 3 \cdot 6^y. \qquad (9)$$

From (9), $$x = \frac{\log 3 + y \log 6}{\log 6}. \qquad (10)$$

From (5) and (10), $$\frac{\log 3}{\log 2} = \frac{\log 3 + y \log 6}{\log 6}.$$

$$\frac{\log 3}{\log 2} = \frac{\log 3}{\log 6} + y.$$

$$y = \frac{\log 3 \log 6 - \log 3 \log 2}{\log 2 \log 6}$$

$$= \frac{\log 3 (\log 2 + \log 3) - \log 3 \log 2}{\log 2 \log 6}$$

$$= \frac{2 \log 3}{\log 6 \cdot \log 2}.$$

53. $$x^y = y^x, \quad (1)$$

$$y = x^2. \quad (2)$$

Substituting from (2) in (1), $(x)^{x^2} = (x^2)^x.$

$$x^2 \log x = 2x \log x.$$

$$\log x (x^2 - 2x) = 0.$$

$x^2 - 2x = 0.$ Whence $x = 0$ and 2.

$\log x = 0.$ Whence $x = 1.$

If $x = 0$, from (2), $y = 0.$

If $x = 1$, from (2), $y = 1.$

If $x = 2$, from (2), $y = 4.$

These all check except $x = 0, y = 0.$

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1. $42 : 28 = \frac{42}{28} = \frac{3}{2}.$
2. $24a^3 : 56a^2 = \dfrac{24a^3}{56a^2} = \dfrac{3a}{7}.$
3. $(x^2 - y^2) : (x - y) = \dfrac{x^2 - y^2}{x - y} = x + y.$
4. $(x^3 + 8y^3) : (x + 2y) = \dfrac{x^3 + 8y^3}{x + 2y} = x^2 - 2xy + 4y^2.$
5. $\left(1 - \frac{4}{a^2}\right) : \left(1 - \frac{2}{a}\right) = \dfrac{1 - \dfrac{4}{a^2}}{1 - \dfrac{2}{a}} = 1 + \dfrac{2}{a} = \dfrac{a+2}{a}.$
6. $\left(a - \frac{16}{a}\right) : \left(\frac{24}{a^4} + \frac{10}{a^3} + \frac{1}{a^2}\right) = \dfrac{\dfrac{a^2 - 16}{a}}{\dfrac{24 + 10a + a^2}{a^4}} = \dfrac{a^4 - 4a^3}{a + 6}.$
7. (*a*) 4 weeks : 12 hours = 672 hours : 12 hours = $\frac{672}{12} = 56.$

(*b*) 48000 inches : 2 miles = 4000 feet : 10560 feet = $\frac{4000}{10560} = \frac{25}{66}.$

8. 1 mile : 1 kilometer $= 63{,}360$ inches : $39{,}370$ inches $= \frac{63360}{39370} = 1.6+$.

9. Let $4x$, $6x$, and $2x$ be the numbers.

Then $4x + 6x + 2x = 150.$

Whence $x = 12\frac{1}{2},$

$4x = 50,$

$6x = 75,$

and $2x = 25.$

10. $$\frac{5+3a}{5+4a} = \frac{(5+3a)(5+5a)}{(5+4a)(5+5a)} = \frac{15a^2+40a+25}{20a^2+45a+25}.$$

$$\frac{5+4a}{5+5a} = \frac{(5+4a)(5+4a)}{(5+5a)(5+4a)} = \frac{16a^2+40a+25}{20a^2+45a+25}.$$

$\therefore \dfrac{5+3a}{5+4a} < \dfrac{5+4a}{5+5a}$. Similarly for ($b$).

11. See page 180.

12. See page 180.

13. $a : b = c : d.$

$$\frac{a}{b} = \frac{c}{d}.$$

$$\therefore \frac{a^n}{b^n} = \frac{c^n}{d^n}.$$

14. $a : b = c : d.$

$$\frac{a}{b} = \frac{c}{d}.$$

$$\therefore \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \frac{\sqrt[n]{c}}{\sqrt[n]{d}}.$$

15. See page 181.

16. See page 182.

17. See page 182.

18. See page 182.

19. See page 182.

20. See page 183.

21. $1.44 : b = b : .0256.$

$b = \pm\sqrt{.0256 \times 1.44}$

$= \pm .192.$

22. $15 : 125 = 125 : a.$

$15a = 125^2.$

$a = 1041\frac{2}{3}.$

23. $16\frac{1}{4} : 8\frac{1}{3} = 62\frac{1}{2} : a.$

$16\frac{1}{4}a = 8\frac{1}{3} \cdot 62\frac{1}{2}.$

$a = 32.05.$

24. $5 : 15 = 8 : 24.$

By addition,

$20 : 5 = 32 : 8,$

or $20 : 15 = 32 : 24.$

By subtraction,

$10 : 5 = 16 : 8,$

or $10 : 15 = 16 : 24.$

By addition and subtraction,

$20 . 10 = 32 : 16,$

or $10 : 20 = 16 : 32.$

By alternation,

$5 : 8 = 15 : 24.$

By inversion,

$15 : 5 = 24 : 8.$

25. $8 : 12 = (3 - x) : 7.$

$12(3 - x) = 56.$

$x = -\frac{5}{3}.$

26. $4 : x = x : 169.$

$x = \pm\sqrt{169.4} = \pm 26.$

27. $3 : 5 = \dfrac{1}{x} : 2.$

$\dfrac{5}{x} = 6.$

$x = \frac{5}{6}.$

28. $8 : x = 12 : (10 - x).$

$80 - 8x = 12x.$

$x = 4.$

29. $3 : 5 = (x - 3) : (2x + 18).$

$6x + 54 = 5x - 15.$

$x = -69.$

30.
$$20 : x = x : (10 - x).$$
$$x^2 = 200 - 20x.$$
$$x^2 + 20x - 200 = 0.$$
$$x = 7.32 \text{ and } -27.32.$$

31.
$$S_1 = 4\pi R_1^2. \quad S_2 = 4\pi R_2^2.$$
$$\frac{S_1}{S_2} = \frac{4\pi R_1^2}{4\pi R_2^2} = \frac{R_1^2}{R_2^2}.$$
$$\frac{S_1}{S_2} = \frac{4\pi\left(\frac{D_1}{2}\right)^2}{4\pi\left(\frac{D_2}{2}\right)^2} = \frac{D_1^2}{D_2^2}.$$
$$\frac{S_1}{S_2} = \frac{R_1^2}{R_2^2} = \frac{D_1^2}{D_2^2}.$$

32.
$$V_1 = \frac{4\pi R_1^3}{3}. \quad V_2 = \frac{4\pi R_2^3}{3}.$$
$$\frac{V_1}{V_2} = \frac{\frac{4\pi R_1^3}{3}}{\frac{4\pi R_2^3}{3}} = \frac{R_1^3}{R_2^3}.$$
$$\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi\left(\frac{D_1}{2}\right)^3}{\frac{4}{3}\pi\left(\frac{D_2}{2}\right)^3} = \frac{D_1^3}{D_2^3}.$$
$$\frac{V_1}{V_2} = \frac{R_1^3}{R_2^3} = \frac{D_1^3}{D_2^3}.$$

33. Let S_1 = surface of the earth,

and S_2 = surface of the moon.
$$\frac{S_1}{S^2} = \frac{D_1^2}{D_2^2} = \frac{(7920)^2}{(2160)^2} = \left(\frac{11}{3}\right)^2 = \frac{121}{9}.$$

34. Let V_1 = the volume of the earth,

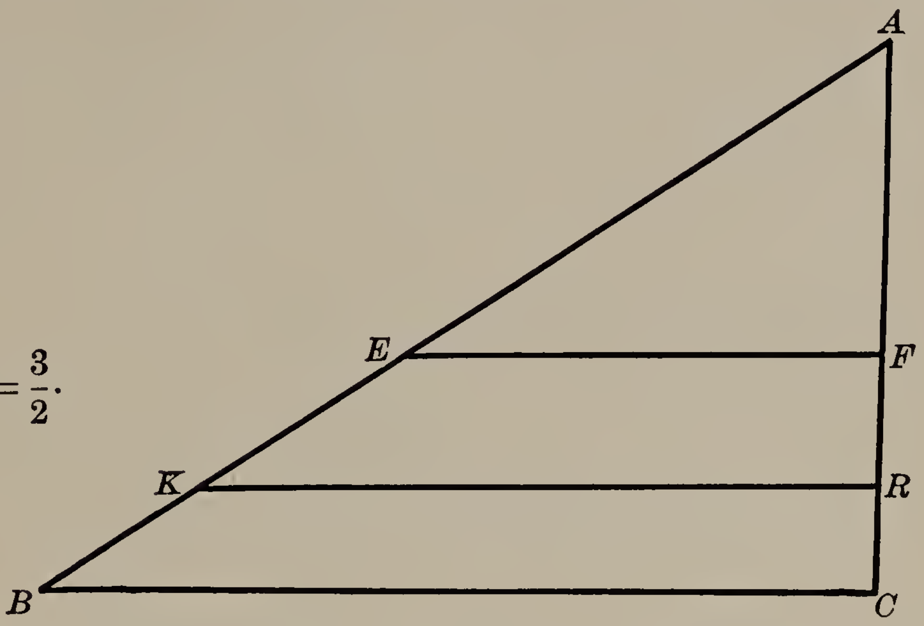
and V_2 = the volume of the moon.
$$\frac{V_1}{V_2} = \frac{P_1^3}{P_1^3} = \frac{(7920)^3}{(2160)^3} = \left(\frac{11}{3}\right)^3 = \frac{1331}{27}.$$

35.
$$\frac{\text{area } ABC}{\text{area } AKR} = \frac{\overline{AB}^2}{\overline{AK}^2}.$$
$$\frac{100 \text{ square inches}}{25 \text{ square inches}} = \frac{100}{\overline{AK}^2}.$$
$$AK = 5.$$

36.
$$\frac{\overline{AB}^2}{\overline{AK}^2} = \frac{2\,AKR}{AKR} = \frac{2}{1}.$$
$$\frac{144}{\overline{AK}^2} = \frac{2}{1}.$$
$$AK = 6\sqrt{2}.$$

37.
$$KRCB = 8\,AKR.$$
$$\therefore ABC = 8\,AKR + AKR = 9\,AKR.$$
$$\frac{\overline{AC}^2}{\overline{AR}^2} = \frac{9\,AKR}{AKR}.$$
$$\frac{1600}{\overline{AR}^2} = \frac{9}{1}.$$
$$AR = 13\tfrac{1}{3}.$$

38. Let EF and KR be lines ∥ to BC, cutting AB and AC at points E and K, F and R, respectively, and dividing ABC into three parts of equal area.



$$\frac{\overline{AB}^2}{\overline{AE}^2}=\frac{3\,AEF}{AEF}.$$

$$\frac{1024}{\overline{AE}^2}=\frac{3}{1}.$$

$$AE = 18.474.$$

$$\frac{\overline{AB}^2}{\overline{AK}^2}=\frac{3\,AEF}{2\,AEF}=\frac{3}{2}.$$

$$AK = 26.127.$$

$$EK = 7.65.$$

$$KB = 5.87.$$

39. $$\frac{D\text{-}ABC}{D\text{-}KRL}=\frac{\overline{DH}^3}{\overline{DS}^3}.$$

$$\frac{32}{4}=\frac{8}{1}=\frac{(18)^3}{\overline{DS}^3}.$$

$$\frac{2}{1}=\frac{18}{DS}.$$

$$DS = 9.$$

40. $$\frac{2\,(D-KRL)}{D-KRL}=\frac{\overline{DH}^3}{\overline{DS}^3}.$$

$$\frac{2}{1}=\frac{12^3}{\overline{DS}^3}.$$

$$\overline{DS}^3=\frac{1728}{2}.$$

$$DS = 6\sqrt[3]{4}$$
$$= 9.52.$$

41. Volume of small pyramid $=\frac{8}{27}$ of the volume of large pyramid.

$$\frac{D\text{-}ABC}{\frac{8}{27}(D\text{-}ABC)}=\frac{36^3}{\overline{DS}^3}.$$

$$\frac{3}{2}=\frac{36}{DS}.$$

$$DS = 24.$$

42. Let x be the point where DH cuts the first plane below the vertex.

$$\frac{\overline{Dx}^3}{100^3}=\frac{1}{3}.$$

$$\frac{Dx}{100}=\frac{1}{\sqrt[3]{3}}.$$

$$Dx=\frac{100}{\sqrt[3]{3}}=69.33.$$

Let y be the point where DH cuts the second plane below the vertex.

$$\frac{\overline{Dy}^3}{100^3}=\frac{2}{3}.$$

$$Dy = 100\sqrt[3]{\tfrac{2}{3}} = 87.33.$$
$$xy = 18.00.$$
$$yH = 100 - Dy$$
$$= 100 - \tfrac{100}{3}\sqrt[3]{18}$$
$$= 12.66.$$

Ans. 69.33, 18.00, 12.66.

43. If $a : b = c : d$,

$$\frac{a}{3b}=\frac{c}{3d}.$$

Then $$\frac{a+3b}{a-3b}=\frac{c+3d}{c-3d}.$$

44. If
$$\frac{a}{b}=\frac{c}{d},$$
then
$$\frac{a^2}{b^2}=\frac{c^2}{d^2}, \text{ and } \frac{a^2}{2b^2}=\frac{c^2}{2d^2}.$$
Therefore
$$\frac{a^2+2b^2}{a^2}=\frac{c^2+2d^2}{c^2}.$$

45. If
$$\frac{a}{b}=\frac{c}{d},$$
by addition and subtraction,
$$\frac{a+b}{a-b}=\frac{c+d}{c-d}.$$
Squaring,
$$\frac{a^2+2ab+b^2}{a^2-2ab+b^2}=\frac{c^2+2cd+d^2}{c^2-2cd+d^2}.$$
By addition and subtraction,
$$\frac{2a^2+2b^2}{4ab}=\frac{2c^2+2d^2}{4cd},$$
or
$$\frac{a^2+b^2}{2ab}=\frac{c^2+d^2}{2cd}.$$

46. If
$$\frac{a}{b}=\frac{c}{d},$$
$$\frac{a^3}{b^3}=\frac{c^3}{d^3},$$
and
$$\frac{5a^3}{b^3}=\frac{5c^3}{d^3},$$
then
$$\frac{5a^3-b^3}{b^3}=\frac{5c^3-d^3}{d^3}.$$

47. If $\frac{a}{b}=\frac{c}{d}$, by Exercise 45,
$$\frac{a^2+b^2}{ab}=\frac{c^2+d^2}{cd}.$$
By subtraction,
$$\frac{a^2-ab+b^2}{ab}=\frac{c^2-cd+d^2}{cd}.$$
$$\therefore \frac{a^3+b^3}{ab(a+b)}=\frac{c^3+d^3}{cd(c+d)},$$
or
$$\frac{a^3+b^3}{3a^2b+3ab^2}=\frac{c^3+d^3}{3c^2d+3cd^2}.$$

48. From Exercise 47,
$$\frac{a^3+b^3}{3a^2b+3ab^2}=\frac{c^3+d^3}{3c^2d+3cd^2},$$
or
$$\frac{a^3+b^3}{3ab(a+b)}=\frac{c^3+d^3}{3cd(c+d)}.$$
Then
$$\frac{a^2-ab+b^2}{3ab}=\frac{c^2-cd+d^2}{3cd}.$$

40. If $$\frac{h}{H}=\frac{a}{A},$$

then $$\frac{h}{a}=\frac{H}{A}.$$

Squaring, $$\frac{h^2}{a^2}=\frac{H^2}{A^2}.$$

By subtraction, $$\frac{h^2-a^2}{a^2}=\frac{H^2-A^2}{A^2}.$$

But $$h^2-a^2=b^2,$$
and $$H^2-A^2=B^2.$$
$$\therefore\ \frac{b^2}{a^2}=\frac{B^2}{A^2}.$$

Whence $$\frac{a}{A}=\frac{B}{b}.$$

Then $$\frac{a}{A}=\frac{b}{B}=\frac{c}{C},$$ and the triangles are similar.

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2. $$\frac{6}{15}=\frac{10}{y}.$$
$$y=\frac{150}{6}=25.$$

3. $$\frac{h}{m}=\frac{k}{y}.$$
$$y=\frac{mk}{h}.$$

5. $$4:x=\tfrac{1}{100}:\tfrac{1}{10}.$$
$$x=40.$$

6. $$h:y=\frac{1}{k}:\frac{1}{m}.$$
$$y=\frac{hk}{m}.$$

8. $$\frac{3}{x}=\frac{4\cdot 5}{20\cdot 2}.$$
$$x=6.$$

9. $$10:x=\tfrac{4}{9}:\tfrac{2}{6}.$$
$$x=7\tfrac{1}{2}.$$

10. $$\frac{64}{d}=\frac{4}{16}.$$
$$d=256.$$

11. $$80:P=\tfrac{400}{15}:\tfrac{450}{45}.$$
$$P=30.$$

12. Let x equal the weight of the object.

(*a*) $$\frac{100}{x}=\frac{4000}{3000}.$$
$$x=75.$$

(*b*) $$\frac{100}{x}=\frac{4000}{2000}.$$
$$x=50.$$

(*c*) $$\frac{x}{100}=\frac{0}{4000}.$$
$$x=0.$$

13. $$4\tfrac{2}{3}:5=\frac{1}{360}:\frac{1}{r}.$$
$$r=336.$$

14. $$\frac{1}{d}=\frac{4\frac{2}{5}}{2}.$$
$$d=\frac{2}{4.4}=\tfrac{5}{11}\text{ miles}$$
$$=2400\text{ feet.}$$

15. $\frac{24}{20} = \frac{36}{30}$.

$v : v_1 = \frac{1}{P} : \frac{1}{P_1}$ (inverse).

$\frac{24}{v} = \frac{48}{30}$.

$v = 15$ cubic inches.

16. $\frac{16}{d} = \frac{1}{36}$.

$d = 576$ feet.

17. $\frac{x}{1} = \frac{36}{16}$.

$x = 2\frac{1}{4}$.

The page is $2\frac{1}{4}$ times as bright 4 feet from the lamp as it is 6 feet away.

18. $\frac{1}{4} = \frac{d^2}{81}$.

$d^2 = \frac{81}{4}$.

$d = \frac{9}{2} = 4.5$ feet away.

$\frac{1}{2} = \frac{d^2}{81}$.

$d = \frac{9}{2}\sqrt{2}$

$= 6.36 +$ feet away.

19. (*a*)

$100 : x = \frac{1}{(4000)^2} : \frac{1}{(5000)^2}$.

$\therefore x = 64$ pounds.

(*b*) $100 : x = \frac{1}{(4000)^2} : \frac{1}{(6000)^2}$.

$\therefore x = 44\frac{4}{9}$ pounds.

$100 : x = \frac{1}{(4000)^2} : \frac{1}{(8000)^2}$.

$\therefore x = 25$ pounds.

20. $\frac{154}{594} = \frac{49}{r^2}$.

$r^2 = \frac{594 \cdot 49}{154} = 189$.

$r = 13.74$ inches.

21. $\frac{6}{x} = \frac{2^3}{6^3}$.

$x = 162$ pounds.

22. $\frac{1}{t} = \frac{\sqrt{100}}{\sqrt{64}}$.

$t = .8$ second.

23. $\frac{1}{2} = \frac{\sqrt{100}}{\sqrt{l}}$.

$l = 400$ centimeters.

$\frac{1}{5} = \frac{\sqrt{100}}{\sqrt{l}}$.

$l = 2500$ centimeters.

24. $P_1 : P_2 = A_1V_1^2 : A_2V_2^2$.

$.9 : 18 = 1 \cdot (15)^2 : 9 \cdot V_2^2$.

$V_2^2 = \frac{18 \cdot (15)^2}{.9 \cdot 9} = 500$.

$V_2 = \sqrt{500} = 22.36$ miles per hour.

25. $\frac{62.5}{P} = \frac{1 \cdot 1}{96 \cdot 6}$.

(*a*) $P = 36{,}000$ pounds.

(*b*) The pressure at the top of the tank on one end is zero and the pressure at the bottom on one end is equal to the pressure of a column of water 6 feet high. The average pressure per square foot on sides would be equal to the weight of a column of water 3 feet high having a cross section of 1 square foot.

$\therefore P$ on end $= 8 \times 6 \times (3 \cdot 62.5) = 9000$ pounds.

P on side $= 12 \times 6 \times (3 \cdot 62.5) = 13{,}500$ pounds.

26. $1320 : c = \frac{1}{2} : \frac{20}{2\frac{1}{2}}$.

$\therefore c = \$21,120.$

28.

$$w \propto r^3.$$
$$w = kr^3.$$
$$6 = k(2)^3.$$
$$k = \tfrac{3}{4}.$$

29. In Problem 22,

$$t = k\sqrt{l}.$$
$$1 = k\sqrt{100}.$$
$$k = \tfrac{1}{10}.$$

30.

$$1 = k \cdot 4\tfrac{2}{5}.$$
$$k = .227.$$

k is the distance in miles which sound travels in 1 second.

31. $24 \cdot 30 = k.$

$$k = 720.$$

32.

$$a \propto b \cdot h.$$
$$a = k \cdot bh = \frac{bh}{2}.$$
$$\therefore k = \tfrac{1}{2}.$$

33.

$$a \propto d^2.$$
$$a = kd^2.$$
$$a = \frac{\pi d^2}{4}.$$
$$\therefore k = .7854.$$

34.

$$v \propto d^3.$$
$$v = kd^3.$$
$$v = \frac{\pi d^3}{6}.$$
$$\therefore k = .5236.$$

35. (Direct.) At any time the heights of two objects are directly proportional to the lengths of the shadows they cast on a horizontal plane.

(Inverse.) The resistance of a wire carrying a current of electricity varies inversely as the square of its diameter.

(Joint.) The amount of light or heat received from the sun by a planet varies inversely as the square of the planet's distance from the sun, and directly as the square of the planet's diameter.

36. If

$$x^2 + y^2 \propto x^2 - y^2, \quad (1)$$
$$x^2 + y^2 = K(x^2 - y^2). \quad (2)$$

Then

$$\frac{x^2 + y^2}{x^2 - y^2} = \frac{K}{1}. \quad (3)$$

By addition and subtraction (3) becomes

$$\frac{x^2}{y^2} = \frac{K+1}{K-1}. \quad (4)$$
$$\frac{x}{y} = \frac{\sqrt{K+1}}{\sqrt{K-1}}. \quad (5)$$

By addition and subtraction (5) becomes

$$\frac{x+y}{x-y} = \frac{\sqrt{K+1} + \sqrt{K-1}}{\sqrt{K+1} - \sqrt{K-1}}. \quad (6)$$

But the right member of (6) is a constant. Call it c.

Then

$$\frac{x+y}{x-y} = c, \text{ or } x + y \propto x - y.$$

37. If

$$x^3 + y^3 \propto x^3 - y^3, \quad (1)$$
$$x^3 + y^3 = K(x^3 - y^3). \quad (2)$$
$$\frac{x^3 + y^3}{x^3 - y^3} = \frac{K}{1}. \quad (3)$$

By addition and subtraction (3) becomes

$$\frac{x^3}{y^3}=\frac{K+1}{K-1}. \tag{4}$$

$$\frac{x}{y}=\frac{\sqrt[3]{K+1}}{\sqrt[3]{K-1}}. \tag{5}$$

By addition and subtraction

$$\frac{x+y}{x-y}=\frac{\sqrt[3]{K+1}+\sqrt[3]{K-1}}{\sqrt[3]{K+1}-\sqrt[3]{K-1}}. \tag{6}$$

But the right member of (6) is a constant.

$$\therefore\ x+y \propto x-y.$$

38. From the solution of Exercise 37 it follows that

$$x^3+y^3=K(x^3-y^3), \tag{1}$$

and

$$x+y=m(x-y). \tag{2}$$

(1) ÷ (2),

$$x^2-xy+y^2=\frac{K}{m}(x^2+xy+y^2). \tag{3}$$

But $\frac{K}{m}$ is a constant.

Therefore $x^2-xy+y^2 \propto x^2+xy+y^2.$

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1. $3\sqrt{-1}+4\sqrt{-1}-2\sqrt{-1}=5\sqrt{-1}.$

2. $\sqrt{-4}+\sqrt{-9}=2\sqrt{-1}+3\sqrt{-1}=5\sqrt{-1}.$

3. $\sqrt{-25}-\sqrt{-16}=5\sqrt{-1}-4\sqrt{-1}=\sqrt{-1}.$

4. $5\sqrt{-1}+\sqrt{-9}=5\sqrt{-1}+3\sqrt{-1}=8\sqrt{-1}.$

5. $\sqrt{-4}+\sqrt{-16}=2\sqrt{-1}+4\sqrt{-1}=6\sqrt{-1}.$

6. $5\sqrt{-36x^2}-2\sqrt{-49x^2}=30x\sqrt{-1}-14x\sqrt{-1}=16x\sqrt{-1}.$

7. $\sqrt{-18}+\sqrt{-8}=3\sqrt{-2}+2\sqrt{-2}=5\sqrt{-2}.$

8. $(-12)^{\frac{1}{2}}+(-27)^{\frac{1}{2}}=2\sqrt{-3}+3\sqrt{-3}=5\sqrt{-3}.$

9. $3+2\sqrt{-1}+5-6\sqrt{-1}=8-4\sqrt{-1}.$

10. $5\sqrt{-x^2}-7a-3\sqrt{-x^2}=5x\sqrt{-1}-7a-3x\sqrt{-1}=2x\sqrt{-1}-7a.$

11. $6-2\sqrt{-64x^2}-3\sqrt{-25x^2}+8=14-16x\sqrt{-1}-15x\sqrt{-1}$
$=14-31x\sqrt{-1}.$

12. $5\sqrt{-3}+3\sqrt{-2}-\sqrt{-27}+2\sqrt{-8}$
$=5\sqrt{-3}+3\sqrt{-2}-3\sqrt{-3}+4\sqrt{-2}$
$=2\sqrt{-3}+7\sqrt{-2}.$

13. $6\sqrt{-4a^4}-7a^2\sqrt{-9}+3\sqrt{-6}-5\sqrt{-24}$
$=12a^2\sqrt{-1}-21a^2\sqrt{-1}+3\sqrt{-6}-10\sqrt{-6}$
$=-9a^2\sqrt{-1}-7\sqrt{-6}.$

14. $(12-6\sqrt{-9})-(15+2\sqrt{-36})=12-18\sqrt{-1}-15-12\sqrt{-1}$
$=-3-30\sqrt{-1}.$

15. $3a-2x-(2a\sqrt{-a^2}-5ra^2\sqrt{-1})$
$=3a-2x-2a^2\sqrt{-1}+5ra^2\sqrt{-1}$
$=3a-2x-a^2(2-5r)\sqrt{-1}.$

16. $(x-iy)-(n-iv)=x-iy-n+iv=x-n-(y-v)i.$

17. $\sqrt{-10}=\sqrt{10}\cdot\sqrt{-1}.$

18. $\sqrt{-6}=\sqrt{6}\cdot\sqrt{-1}.$

19. $2\sqrt{-3}=2\sqrt{3}\cdot\sqrt{-1}.$

20. $\sqrt{-a}=\sqrt{a}\cdot\sqrt{-1}.$

21. $a\sqrt{-b}=a\sqrt{b}\cdot\sqrt{-1}.$

22. $\sqrt{-a-b}=\sqrt{-1(a+b)}$
$=\sqrt{a+b}\cdot\sqrt{-1}.$

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1. $(\sqrt{-1})^5=(\sqrt{-1})^2\cdot(\sqrt{-1})^2\cdot\sqrt{-1}=\sqrt{-1}.$
2. $(\sqrt{-1})^6=(\sqrt{-1})^2\cdot(\sqrt{-1})^2\cdot(\sqrt{-1})^2=-1.$
3. $(\sqrt{-1})^7=-1\sqrt{-1}.$
4. $(\sqrt{-1})^8=1.$
5. $2\sqrt{-1}\cdot3\sqrt{-1}=-6.$
6. $\sqrt{-9}\cdot\sqrt{-16}=3\sqrt{-1}\cdot4\sqrt{-1}=-12.$
7. $\sqrt{-5}(-\sqrt{-6})=\sqrt{5}\cdot\sqrt{-1}(-\sqrt{6}\cdot\sqrt{-1})=-1(-\sqrt{30})=\sqrt{30}.$
8. $\sqrt{-25}\cdot\sqrt{3}=5\sqrt{-1}\cdot\sqrt{3}=5\sqrt{-3}.$

9. $2\sqrt{-3}\cdot 3\sqrt{-2}=2\sqrt{3}\sqrt{-1}\cdot 3\sqrt{2}\sqrt{-1}=-6\sqrt{6}.$

10. $\sqrt{-m}\cdot\sqrt{-n}=\sqrt{m}\sqrt{-1}\cdot\sqrt{n}\sqrt{-1}=-\sqrt{mn}.$

11. $4\sqrt{-5}(-3\sqrt{-6})=4\sqrt{5}\sqrt{-1}(-3\sqrt{6}\sqrt{-1})=12\sqrt{30}.$

12. $\sqrt{a+b}\cdot\sqrt{-a-b}=\sqrt{a+b}\cdot\sqrt{a+b}\sqrt{-1}=(a+b)\sqrt{-1}.$

13. $(2+\sqrt{-1})(2-\sqrt{-1})=4-(-1)=5.$

14. $(3+\sqrt{-2})(3-\sqrt{-2})=(3+\sqrt{2}\sqrt{-1})(3-\sqrt{2}\sqrt{-1})=11.$

15. $(4-2\sqrt{3}\,i)(4+2\sqrt{3}\,i)=16-4\cdot 3(-1)=28.$

16. $(3+\sqrt{-1})(6-\sqrt{-2})=(3+\sqrt{-1})(6-\sqrt{2}\sqrt{-1})$
$=18-3\sqrt{2}\sqrt{-1}+6\sqrt{-1}-\sqrt{2}(-1)$
$=18-3\sqrt{-2}+6\sqrt{-1}+\sqrt{2}.$

17. $(4-2i)(3-2\sqrt{3}\,i)=12-6i-8\sqrt{3}\,i+4\sqrt{3}(-1)$
$=12-6\sqrt{-1}-8\sqrt{-3}-4\sqrt{3}.$

18. $(a+ib)(c+id)=ac+iad+ibc+bd(-1)$
$=ac+ad\sqrt{-1}+bc\sqrt{-1}-bd.$

19. $(a+ib)(a+ib)=a^2+2\,abi+i^2b^2=a^2+2\,ab\sqrt{-1}-b^2.$

20. $(a+bi)(a-bi)=a^2-b^2i^2=a^2+b^2.$

21. $(-\frac{1}{2}+\frac{1}{2}\sqrt{-3})^2=(-\frac{1}{2}+\frac{1}{2}i\sqrt{3})^2=\frac{1}{4}-2\cdot\frac{1}{4}i\sqrt{3}+\frac{1}{4}(-1)3$
$=\frac{1}{4}-\frac{1}{2}\sqrt{-3}-\frac{3}{4}=-\frac{1}{2}-\frac{1}{2}\sqrt{-3}.$

22. $(-\frac{1}{2}-\frac{1}{2}\sqrt{-3})^2=(-\frac{1}{2}-\frac{1}{2}i\sqrt{3})^2=\frac{1}{4}+2\cdot\frac{1}{4}i\sqrt{3}+\frac{1}{4}(-1)3$
$=\frac{1}{4}+\frac{1}{2}\sqrt{-3}-\frac{3}{4}=-\frac{1}{2}+\frac{1}{2}\sqrt{-3}.$

23. $(x-iy)^2-(x+iy)^2=(x^2-2\,ixy+i^2y^2)-(x^2+2\,ixy+i^2y^2)$
$=-4\,ixy=-4\,xy\sqrt{-1}.$

24. $(-\frac{1}{2}+\frac{1}{2}\sqrt{-3})^3-(-\frac{1}{2}-\frac{1}{2}\sqrt{-3})^3=(-\frac{1}{2}+\frac{1}{2}i\sqrt{3})^3-(-\frac{1}{2}-\frac{1}{2}i\sqrt{3})^3$
$=1-1=0.$

25. $(a+i\sqrt{1-x^2})(a-i\sqrt{1-x^2})=a^2-(-1)(1-x^2)=a^2+1-x^2.$

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1. $\dfrac{\sqrt{-8}}{\sqrt{-2}}=\dfrac{\sqrt{8}\,i^2\sqrt{2}}{-2}=\dfrac{\sqrt{16}(-1)}{-2}=2.$

2. $\dfrac{\sqrt{-6}}{\sqrt{-3}}=\dfrac{\sqrt{-6}\sqrt{-3}}{-3}=\dfrac{-\sqrt{18}}{-3}=\sqrt{2}.$

3. $\dfrac{2\sqrt{-3}}{3\sqrt{-1}}=\dfrac{2\sqrt{3}\sqrt{-1}\sqrt{-1}}{3(-1)}=\dfrac{-2\sqrt{3}}{-3}=\dfrac{2\sqrt{3}}{3}.$

4. $\dfrac{\sqrt[4]{-1}}{\sqrt[4]{-4}}=\dfrac{\sqrt[4]{-1}}{\sqrt[4]{4}\sqrt[4]{-1}}=\dfrac{1}{\sqrt[4]{4}}=\dfrac{1}{\sqrt{2}}=\dfrac{\sqrt{2}}{2}.$

5. $\dfrac{\sqrt{6}}{\sqrt{-2}}=\dfrac{\sqrt{6}\sqrt{2}\sqrt{-1}}{-2}=\dfrac{\sqrt{-12}}{-2}=-\sqrt{-3}.$

6. $\dfrac{(-25)^{\frac{1}{4}}}{(-81)^{\frac{1}{4}}}=\dfrac{(-1)^{\frac{1}{4}}(25)^{\frac{1}{4}}}{(-1)^{\frac{1}{4}}(81)^{\frac{1}{4}}}=\dfrac{\sqrt{5}}{3}.$

7. $\dfrac{\sqrt{ax}}{\sqrt{-x}}=\dfrac{\sqrt{ax}\sqrt{-x}}{-x}=\dfrac{x\sqrt{-a}}{-x}=-\sqrt{-a}.$

8. $\dfrac{\sqrt{-a}}{\sqrt{-b}}=\dfrac{i\sqrt{a}}{i\sqrt{b}}=\dfrac{\sqrt{a}}{\sqrt{b}}=\dfrac{\sqrt{ab}}{b}.$

9. $\dfrac{(-5\,ax)^{\frac{1}{2}}}{(-2\,x)^{\frac{1}{2}}}=\dfrac{(-5\,ax)^{\frac{1}{2}}(-2\,x)^{\frac{1}{2}}}{-2\,x}=\dfrac{-x(10\,a)^{\frac{1}{2}}}{-2\,x}=\dfrac{\sqrt{10\,a}}{2}.$

10. $\dfrac{(-a^6)^{\frac{1}{4}}-(-a^2)^{\frac{1}{4}}}{(-a)^{\frac{1}{4}}}=\dfrac{(-1)^{\frac{1}{4}}a^{\frac{6}{4}}-(-1)^{\frac{1}{4}}a^{\frac{2}{4}}}{(-1)^{\frac{1}{4}}a^{\frac{1}{4}}}=\dfrac{a^{\frac{6}{4}}-a^{\frac{2}{4}}}{a^{\frac{1}{4}}}=a^{\frac{5}{4}}-a^{\frac{1}{4}}.$

11. $\dfrac{2}{1-\sqrt{-1}}=\dfrac{2(1+\sqrt{-1})}{1-(-1)}=\dfrac{2+2\sqrt{-1}}{2}=1+\sqrt{-1}.$

12. $\dfrac{3}{2-\sqrt{-2}}=\dfrac{3(2+\sqrt{-2})}{4-(-2)}=\dfrac{6+3\sqrt{-2}}{6}=1+\dfrac{\sqrt{-2}}{2}.$

13. $\dfrac{2\sqrt{-1}}{\sqrt{-1}+3}=\dfrac{2\sqrt{-1}(\sqrt{-1}-3)}{-1-9}=\dfrac{-2-6\sqrt{-1}}{-10}=\dfrac{1+3\sqrt{-1}}{5}.$

14. $\dfrac{3\sqrt{-2}}{2\sqrt{-3}+2}=\dfrac{3\sqrt{-2}(2\sqrt{-3}-2)}{(2\sqrt{-3}+2)(2\sqrt{-3}-2)}=\dfrac{-6\sqrt{6}-6\sqrt{-2}}{4(-3)-4}$

$=\dfrac{3\sqrt{6}+3\sqrt{-2}}{8}.$

15. $\dfrac{-1+\sqrt{-3}}{-1-\sqrt{-3}}=\dfrac{(1-\sqrt{-3})(1-\sqrt{-3})}{(1+\sqrt{-3})(1-\sqrt{-3})}=\dfrac{1-2\sqrt{-3}+(-3)}{1-(-3)}$

$=\dfrac{-1-\sqrt{-3}}{2}.$

16. $\dfrac{1+i}{1-i}=\dfrac{(1+i)^2}{1+1}=\dfrac{1+2i+(-1)}{2}=i=\sqrt{-1}.$

17. $\dfrac{a}{a+bi}=\dfrac{a(a-bi)}{a^2-b^2(-1)}=\dfrac{a^2-abi}{a^2+b^2}.$

18. $\dfrac{a+ib}{c+id}=\dfrac{(a+ib)(c-id)}{c^2-d^2(-1)}=\dfrac{ac-iad+ibc+bd}{c^2+d^2}.$

19. $$\frac{(2+3i)}{(2i-1)(5i-3)}=\frac{2+3i}{-7-11i}=\frac{(2+3i)(-7+11i)}{(-7-11i)(-7+11i)}$$
$$=\frac{-47+i}{(-7)^2-(11i)^2}=\frac{-47+i}{170}.$$

20. $(i\sqrt{3}-1)^3=8$. Yes.

21. $(1-\sqrt{-3})^3=-8$. Yes.

22. Yes.

23. Yes.

24. The sum is 4 and the product 13.

25. Let $a+b\sqrt{-1}$ and $a-b\sqrt{-1}$ be the conjugate complex numbers.
$a+b\sqrt{-1}+a-b\sqrt{-1}=2a$, a real number.
$(a+b\sqrt{-1})(a-b\sqrt{-1})=[a^2-(-1)b^2]=a^2+b^2$, a real number.

26. $$\frac{a+b\sqrt{-1}}{a-b\sqrt{-1}}=\frac{(a+b\sqrt{-1})(a+b\sqrt{-1})}{(a-b\sqrt{-1})(a+b\sqrt{-1})}=\frac{a^2+2ab\sqrt{-1}-b^2}{a^2+b^2},$$
a complex number.

27. The error consists in asserting that equation (1) is an identity when it is really an equation of condition true for any real value of x which is less than y. This can be seen by putting 5 for x and 2 for y, and then putting 2 for x and 5 for y.

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1. $x^2+4x+12=0$.
$$x=\frac{-4\pm\sqrt{-32}}{2}=-2\pm2\sqrt{-2}.$$

2. $x^2-6x+36=0$.
$$x=\frac{6\pm\sqrt{-108}}{2}=3\pm3\sqrt{-3}.$$

3. $x^2+5x+7=0$.
$$x=\frac{-5\pm\sqrt{25-28}}{2}=\frac{-5\pm\sqrt{-3}}{2}.$$

4. $x^2-3x+10=0$.
$$x=\frac{3\pm\sqrt{9-40}}{2}=\frac{3\pm\sqrt{-31}}{2}.$$

5. $2x^2+6x+5=0$.
$$x=\frac{-6\pm\sqrt{36-40}}{4}=\frac{-6\pm2\sqrt{-1}}{4}=\frac{-3\pm\sqrt{-1}}{2}.$$

6. $3x^2-7x+6=0$.
$$x=\frac{7\pm\sqrt{49-72}}{6}=\frac{7\pm\sqrt{-23}}{6}.$$

8. $x^3 = 8.$

$$x^3 - 8 = 0.$$
$$(x-2)(x^2+2x+4) = 0.$$
$$\therefore\ x - 2 = 0.$$
$$x = 2.$$
$$\therefore\ x^2 + 2x + 4 = 0.$$
$$x = -1 \pm \sqrt{-3}.$$

9. $x^3 = 27.$

$$x^3 - 27 = 0.$$
$$(x-3)(x^2+3x+9) = 0.$$
$$\therefore\ x - 3 = 0.$$
$$x = 3.$$
$$\therefore\ x^2 + 3x + 9 = 0.$$
$$x = \frac{-3 \pm 3\sqrt{-3}}{2}.$$

10. $x^3 = -8.$

$$x^3 + 8 = 0.$$
$$(x+2)(x^2 - 2x + 4) = 0.$$
$$\therefore\ x + 2 = 0.$$
$$x = -2.$$
$$\therefore\ x^2 - 2x + 4 = 0.$$
$$x = 1 \pm \sqrt{-3}.$$

11. $x^4 = 1.$

$$x^4 - 1 = 0.$$
$$(x-1)(x+1)(x^2+1) = 0.$$
$$\therefore\ x + 1 = 0.$$
$$x = -1.$$
$$\therefore\ x^2 + 1 = 0.$$
$$x = \pm\sqrt{-1}.$$
$$\therefore\ x - 1 = 0.$$
$$x = 1.$$

12. $x^4 = 16.$

$$x^4 - 16 = 0.$$
$$(x-2)(x+2)(x^2+4) = 0.$$
$$\therefore\ x - 2 = 0.$$
$$x = 2.$$
$$\therefore\ x + 2 = 0.$$
$$x = -2.$$
$$\therefore\ x^2 + 4 = 0.$$
$$x = \pm 2\sqrt{-1}.$$

13. $x^6 = 1.$

$$x^6 - 1 = 0.$$
$$(x^3 - 1)(x^3 + 1) = 0.$$
$$(x-1)(x^2+x+1)(x+1)(x^2-x+1) = 0.$$
$$\therefore\ x = 1, \text{ and } x = -1.$$
$$\therefore\ x^2 + x + 1 = 0.$$
$$x = \frac{-1 \pm \sqrt{-3}}{2}.$$
$$\therefore\ x^2 - x + 1 = 0.$$
$$x = \frac{1 \pm \sqrt{-3}}{2}.$$

14. $x^6 = 64.$

$$(x^3 - 8)(x^3 + 8) = 0.$$
$$(x-2)(x^2+2x+4)(x+2)(x^2-2x+4) = 0.$$
$$\therefore\ x - 2 = 0, \text{ and } x + 2 = 0.$$
$$x = 2, \text{ and } x = -2.$$
$$\therefore\ x^2 + 2x + 4 = 0.$$
$$x = -1 \pm \sqrt{-3}.$$
$$\therefore\ x^2 - 2x + 4 = 0.$$
$$x = 1 \pm \sqrt{-3}.$$

15. Two; three; four; six.

16. Any real number has n nth roots.

17. $8x^3 - 27 = 0.$

$$(2x-3)(4x^2+6x+9)=0.$$
$$\therefore\ 2x-3=0.$$
$$x=\tfrac{3}{2}.$$
$$\therefore\ 4x^2+6x+9=0.$$
$$x=\frac{-3\pm3\sqrt{-3}}{4}.$$

18. $125x^3+64=0.$

$$(5x+4)(25x^2-20x+16)=0.$$
$$\therefore\ 5x+4=0.$$
$$x=-\tfrac{4}{5}.$$
$$\therefore\ 25x^2-20x^2+16=0.$$
$$x=\frac{2\pm2\sqrt{-3}}{5}.$$

19. $(x^2+5)(x^2-7)+27=0.$

$$\therefore\ x^4-2x^2-35+27=0.$$
$$(x^2+2)(x^2-4)=0.$$
$$(x^2+2)(x+2)(x-2)=0.$$
$$x=\pm\sqrt{-2},\ -2,\ 2.$$

20. $x^3-x^2+2x-2=0.$

$$(x^2+2)(x-1)=0.$$
$$x=1,\ \pm\sqrt{-2}.$$

21. $x^6+7x^3-8=0.$

$$(x^3-1)(x^3+8)=0.$$
$$(x-1)(x^2+x+1)(x+2)(x^2-2x+4)=0.$$
$$x=1,\ \frac{-1\pm\sqrt{-3}}{2},\ -2,\ 1\pm\sqrt{-3}.$$

22. $4x^4+20x^2+21=0.$

$$(2x^2+7)(2x^2+3)=0.$$
$$2x^2+7=0.$$
$$x=\pm\frac{\sqrt{-14}}{2}.$$
$$2x^2+3=0.$$
$$x=\pm\frac{\sqrt{-6}}{2}.$$

23. $64x^4-12x^2-27=0.$

$$x^2=\frac{12\pm\sqrt{144+6912}}{128}=\frac{12\pm\sqrt{7056}}{128}=\frac{12\pm84}{128}.$$
$$x^2=\tfrac{3}{4},\ -\tfrac{9}{16}.$$
$$x=\pm\tfrac{1}{2}\sqrt{3},\ \pm\tfrac{3}{4}\sqrt{-1}.$$

24. $9x^4 + 18x^2 + 8 = 0.$

$$(3x^2 + 4)(3x^2 + 2) = 0.$$
$$3x^2 + 4 = 0.$$
$$3x^2 + 2 = 0.$$
$$x = \frac{\pm 2\sqrt{-3}}{3}, \ \frac{\pm\sqrt{-6}}{3}.$$

25. $50x^4 + 135x^2 + 36 = 0.$

$$(5x^2 + 12)(10x^2 + 3) = 0.$$
$$x = \frac{\pm 2\sqrt{-15}}{5}, \ \frac{\pm\sqrt{-30}}{10}.$$

26. $(x^2 + 9)(x^2 + 2x + 8) = 0.$

$$x^2 + 9 = 0.$$
$$x^2 + 2x + 8 = 0.$$
$$x = \pm 3\sqrt{-1}, \ -1 \pm \sqrt{-7}.$$

27. $(x^2 + x)^2 + 13(x^2 + x) + 36 = 0.$

$$(x^2 + x)^2 + 9(x^2 + x) + 4(x^2 + x) + 36 = 0.$$
$$(x^2 + x)(x^2 + x + 9) + 4(x^2 + x + 9) = 0.$$
$$(x^2 + x + 4)(x^2 + x + 9) = 0.$$
$$x = \frac{-1 \pm \sqrt{-15}}{2}, \ \frac{-1 \pm \sqrt{-35}}{2}.$$

28. $(x^2 + 5x)^2 + 17(x^2 + 5x) + 66 = 0.$

$$(x^2 + 5x + 6)(x^2 + 5x + 11) = 0.$$
$$x = \frac{-5 \pm 1}{2} = -2, \ -3, \text{ and } \frac{-5 \pm \sqrt{-19}}{2}.$$

29.

$$x + y = 4, \qquad (1)$$
$$x^2 - 3xy - y^2 = -39. \qquad (2)$$

From (1) and (2),

$$16 - 8y + y^2 - 3y(4 - y) - y^2 + 39 = 0. \qquad (3)$$
$$3y^2 - 20y + 55 = 0. \qquad (4)$$
$$y = \frac{10 \pm \sqrt{-65}}{3}. \qquad (5)$$

From (1) and (6), $x = 4 - \dfrac{10 \pm \sqrt{-65}}{3} = \dfrac{2 \mp \sqrt{-65}}{3}.$

30.

$$z^2 + x^2 = 130. \qquad (1)$$
$$z + x + 2\sqrt{z + x} = 2. \qquad (2)$$
$$z + x + 2\sqrt{z + x} + 1 = 3. \qquad (3)$$
$$\sqrt{z + x} + 1 = \pm\sqrt{3}. \qquad (4)$$
$$\sqrt{z + x} = -1 \pm \sqrt{3}. \qquad (5)$$
$$z + x = 4 \pm 2\sqrt{3}. \quad 4 + 2\sqrt{3} \text{ is impossible.} \qquad (6)$$

Squaring in (6),

$$z^2 + 2\,zx + x^2 = 28 - 16\sqrt{3}. \tag{7}$$

(7) – (1), $$2\,zx = -102 - 16\sqrt{3}. \tag{8}$$

(1) – (8), $$z^2 - 2\,zx + x^2 = 232 + 16\sqrt{3}. \tag{9}$$

$$z - x = \pm\sqrt{232 + 16\sqrt{3}} = \pm 2\sqrt{58 + 4\sqrt{3}}. \tag{10}$$

$$z + x = 4 - 2\sqrt{3}. \tag{11}$$

From (10) and (11), $$z = 2 - \sqrt{3} \pm \sqrt{58 + 4\sqrt{3}}. \tag{12}$$

From (10) and (11), $$x = 2 - \sqrt{3} \mp \sqrt{58 - 4\sqrt{3}}. \tag{13}$$

31. See page 381. **32.** See pages 397–398.

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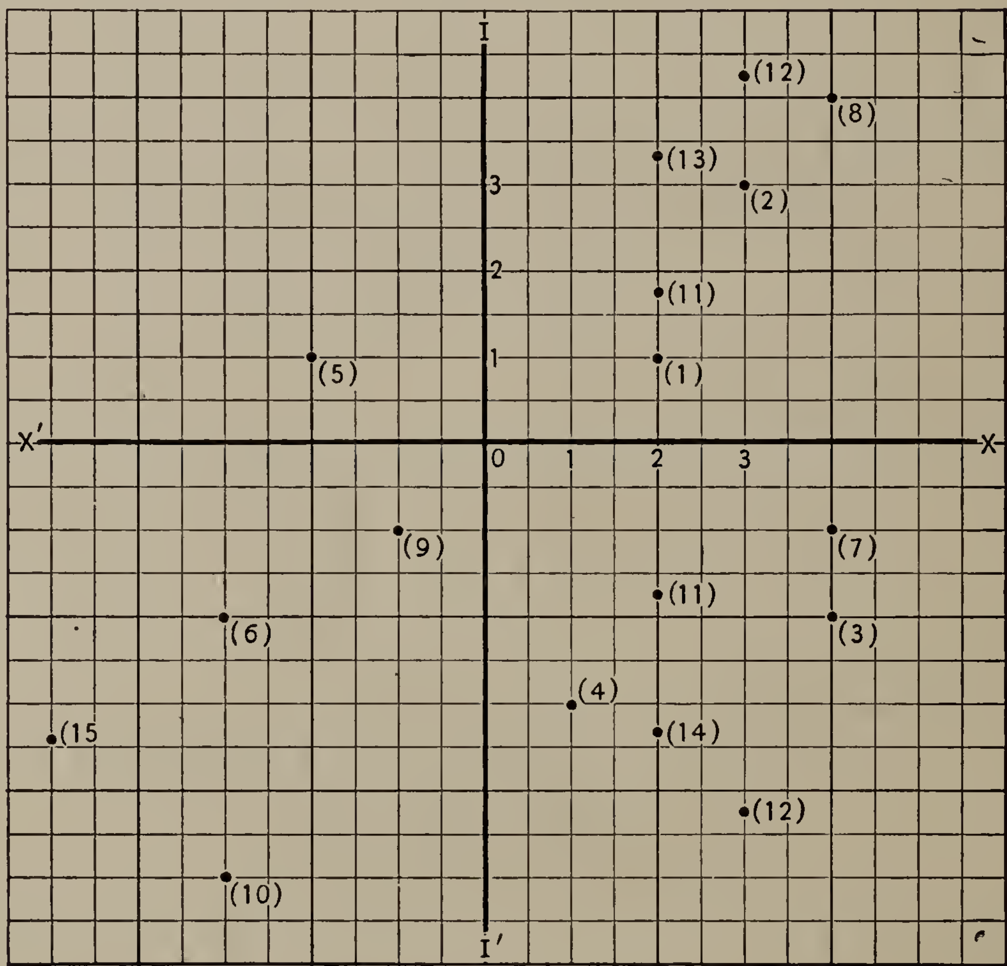
For Exercises 1–15, see the graph below :



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2. $x^2 - 5x + 6 = 0.$

$b^2 - 4\,ac = (-5)^2 - 4 \cdot 1 \cdot 6 = 25 - 24 = 1.$

The roots are real, unequal, and rational.

3. $5x^2 - 11x + 2 = 0.$

$b^2 - 4\,ac = (-11)^2 - 4 \cdot 5 \cdot 2 = 121 - 40 = 81.$

The roots are real, unequal, and rational.

4. $4x^2 - 20x + 25 = 0.$
$b^2 - 4ac = (-20)^2 - 4 \cdot 4 \cdot 25 = 400 - 400.$
There is only one root; it is real and rational.

5. $5x^2 - 3x - 3 = 0.$
$b^2 - 4ac = (-3)^2 - 4 \cdot 5 \cdot (-3) = 9 + 60 = 69.$
The roots are real, unequal, and irrational.

6. $7x^2 - 2x + 10 = 0.$
$b^2 - 4ac = (2)^2 - 4 \cdot 7 \cdot 10 = 4 - 280 = -276.$
The roots are imaginary.

7. $x^2 - 6x + 6 = 0.$
$b^2 - 4ac = (-6)^2 - 4 \cdot 1 \cdot 6 = 36 - 24 = 12.$
The roots are real, unequal, and irrational.

8. $4x^2 = 9 - 9x$, or $4x^2 + 9x - 9 = 0.$
$b^2 - 4ac = 9^2 - 4 \cdot 4 \cdot (-9) = 81 + 144 = 225 = (15)^2.$
The roots are real, unequal, and rational.

9. $5x = x^2 + 5$, or $-x^2 + 5x - 5 = 0.$
$b^2 - 4ac = (5)^2 - 4 \cdot (-1) \cdot (-5) = 25 - 20 = 5.$
The roots are real, unequal, and irrational.

10. $x^2 - 5x + 7 = 0.$
$b^2 - 4ac = (-5)^2 - 4 \cdot 1 \cdot 7 = 25 - 28 = -3.$
The roots are imaginary.

11. $12x^2 - 7x + 6 = 0.$
$b^2 - 4ac = (-7)^2 - 4 \cdot 12 \cdot 6 = 49 - 288 = -239.$
The roots are imaginary.

12. $x(x - 5) = x - 16$, or $x^2 - 6x + 16 = 0.$
$b^2 - 4ac = (-6)^2 - 4 \cdot 1 \cdot 16 = 36 - 64 = -28.$
The roots are imaginary.

13. $\frac{3}{x^2} + \frac{11}{x} = 20$, or $20x^2 - 11x - 3 = 0.$
$b^2 - 4ac = (-11)^2 - 4 \cdot 20 \cdot (-3) = 121 + 240 = 361 = (19)^2.$
The roots are real, unequal, and rational.

14. $5x - \frac{11}{7} - \frac{6}{7x} = 0$, or $35x^2 - 11x - 6 = 0.$
$b^2 - 4ac = (-11)^2 - 4 \cdot 35 \cdot (-6) = 121 + 840 = 961 = (31)^2.$
The roots are real, unequal, and rational.

16.
$$x^2 - Kx + 36 = 0.$$
$$a = 1,\ b = -K,\ c = 36.$$
$$b^2 - 4ac = K^2 - 144.$$
In order that there be but one root, $b^2 - 4ac$ must equal 0.
$$\therefore\ K^2 - 144 = 0.$$
$$K = \pm 12.$$

17. $x^2 - 3Kx + 81 = 0.$

$$a = 1,\ b = -3K,\ c = 81.$$
$$b^2 - 4ac = 9K^2 - 324.$$
$$9K^2 = 324.$$
$$K = \pm 6.$$

18. $2x^2 + 4Kx + 98 = 0.$

$$a = 2,\ b = 4K,\ c = 98.$$
$$b^2 - 4ac = 16K^2 - 784.$$
$$16K^2 = 784.$$
$$K = \pm 7.$$

19. $x^2 - 10x + K = 0.$

$$a = 1,\ b = -10,\ c = K.$$
$$b^2 - 4ac = 100 - 4K.$$
$$K = 25.$$

20. $2x^2 + 8x + K = 0.$

$$a = 2,\ b = 8,\ c = K.$$
$$b^2 - 4ac = 64 - 8K.$$
$$K = 8.$$

21. $9x^2 + 30x + K + 9 = 0.$

$$a = 9,\ b = 30,\ c = (K + 9).$$
$$b^2 - 4ac = 900 - (36K + 324)$$
$$= 576 - 36K.$$
$$K = 16.$$

22. $4Kx^2 - 60x + 25 = 0.$

$$a = 4K,\ b = -60,\ c = 25.$$
$$b^2 - 4ac = 3600 - 400K.$$
$$K = 9.$$

23. $9K^2x^2 - 84x + 49 = 0.$

$$a = 9K^2,\ b = -84,\ c = 49.$$
$$b^2 - 4ac = 7056 - 1764K^2.$$
$$7056 - 1764K^2 = 0.$$
$$K = \pm 2.$$

24. $49x^2 - (K + 3)x + 4 = 0.$

$$a = 49,\ b = -(K + 3),\ c = 4.$$
$$b^2 - 4ac = K^2 + 6K + 9 - 784.$$
$$K^2 + 6K - 775 = 0.$$
$$K = 25 \text{ or } -31.$$

25. $(K^2 + 5)x^2 - 30x + 25 = 0.$

$$a = K^2 + 5,\ b = -30,\ c = 25.$$
$$b^2 - 4ac = 900 - (100K^2 + 500).$$
$$400 - 100K^2 = 0.$$
$$K = \pm 2.$$

26. $(K^2 + 17)x^2 + (5K - 4)x + 4 = 0.$

$$a = K^2 + 17,\ b = 5K - 4,\ c = 4.$$
$$b^2 - 4ac = (25K^2 - 40K + 16) - (16K^2 + 272).$$
$$9K^2 - 40K - 256 = 0.$$
$$K = 8, \text{ or } -\tfrac{32}{9}.$$

27. $k^2x^2 + 6hx + 9 = 0.$

$$a = k^2,\ b = 6h,\ c = 9.$$
$$b^2 - 4ac = 36h^2 - 36k^2.$$
$$36h^2 - 36k^2 = 0.$$
$$h^2 = k^2.$$
$$h = \pm k.$$

28. $kx^2 - 2hx + 16 = 0.$

$$a = k,\ b = -2h,\ c = 16.$$
$$b^2 - 4ac = 4h^2 - 64k.$$
$$4h^2 = 64k.$$
$$h^2 = 16k.$$
$$h = \pm 4\sqrt{k}.$$

29. $x^2 + 4kx + 4h = 0.$

$$a = 1,\ b = 4k,\ c = h.$$
$$b^2 - 4ac = 16k^2 - 16h.$$
$$16k^2 - 16h = 0.$$
$$k = \pm\sqrt{h}.$$

30. $kx^2 - 2hx + 6 = 0.$

$$a = k,\ b = -2h,\ c = 6.$$
$$b^2 - 4ac = 4h^2 - 24k.$$
$$4h^2 - 24k = 0.$$
$$h = \pm\sqrt{6k}.$$

31.
$$y^2 = ax. \tag{1}$$
$$y = x + 1. \tag{2}$$
Squaring (2),
$$y^2 = x^2 + 2x + 1. \tag{3}$$
(3) – (1),
$$x^2 + 2x - ax + 1 = 0.$$
$$a = 1,\ b = (2 - a),\ c = 1.$$
$$b^2 - 4ac = (4 - 4a + a^2) - 4.$$
$$a^2 - 4a = 0.$$
$$a = 0,\ 4.$$

32.
$$y^2 = 2x. \tag{1}$$
$$y = x + a. \tag{2}$$
Squaring (2),
$$y^2 = x^2 + 2ax + a^2.$$
$$x^2 + 2ax - 2x + a^2 = 0.$$
$$a = 1,\ b = 2a - 2,\ c = a^2.$$
$$b^2 - 4ac = 4a^2 - 8a + 4 - 4a^2.$$
$$4 - 8a = 0.$$
$$a = \tfrac{1}{2}.$$

33.
$$x^2 + y^2 = a^2. \tag{1}$$
$$y = x + 1. \tag{2}$$
Squaring (2),
$$y^2 = x^2 + 2x + 1. \tag{3}$$
(3) – (1),
$$2x^2 + 2x - a^2 + 1 = 0.$$
$$a = 2,\ b = 2,\ c = 1 - a^2.$$
$$b^2 - 4ac = 4 - 4\cdot 2\cdot(1 - a^2) = 8a^2 - 4.$$
$$8a^2 - 4 = 0.$$
$$a = \pm\frac{\sqrt{2}}{2}.$$

34.
$$x^2 + y^2 = 2x. \tag{1}$$
$$y = x + a. \tag{2}$$
Squaring (2),
$$y^2 = x^2 + 2ax + a^2. \tag{3}$$
(3) – (1),
$$2x^2 + (2a - 2)x + a^2 = 0.$$
$$a = 2,\ b = 2a - 2,\ c = a^2.$$
$$b^2 - 4ac = 4a^2 - 8a + 4 - 8a^2.$$
$$a^2 + 2a - 1 = 0.$$
$$a = -1 \pm \sqrt{2}.$$

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2. 2, 7.

$$r_1 + r_2 \text{ with the sign changed is } -9.$$
$$r_1 \times r_2 = 2 \cdot 7 = 14.$$
$$\therefore\ x^2 - 9x + 14 = 0.$$

3. -3, 10.

$$-(r_1 + r_2) = -(-3 + 10) = -7.$$
$$r_1 \times r_2 = -30.$$
$$\therefore\ x^2 - 7x - 30 = 0.$$

4. -4, -5.

$$-(r_1 + r_2) = -(-4 - 5) = 9.$$
$$r_1 \times r_2 = -4 \cdot (-5) = 20.$$
$$\therefore\ x^2 + 9x + 20 = 0.$$

5. -12, -1.

$$-(r_1 + r_2) = -(-12 - 1) = 13.$$
$$r_1 \times r_2 = -12 \cdot (-1) = 12.$$
$$\therefore\ x^2 + 13x + 12 = 0.$$

6. $\frac{2}{3}$, 5.

$$-(r_1 + r_2) = -(\tfrac{2}{3} + 5) = -5\tfrac{2}{3}, \text{ or } -\tfrac{17}{3}.$$
$$r_1 \times r_2 = \tfrac{10}{3}.$$
$$\therefore\ 3x^2 - 17x + 10 = 0.$$

7. 10, $-\frac{1}{5}$.

$$-(r_1 + r_2) = -(10 - \tfrac{1}{5}) = -9\tfrac{4}{5}, \text{ or } -\tfrac{49}{5}.$$
$$r_1 \times r_2 = -\tfrac{10}{5}.$$
$$\therefore\ 5x^2 - 49x - 10 = 0.$$

8. $2 + \sqrt{3}$, $2 - \sqrt{3}$.

$$-(r_1 + r_2) = -(2 + \sqrt{3} + 2 - \sqrt{3}) = -4.$$
$$r_1 \times r_2 = (2 + \sqrt{3}) \cdot (2 - \sqrt{3}) = 4 - 3 = 1.$$
$$\therefore\ x^2 - 4x + 1 = 0.$$

9. $-3 \pm \sqrt{5}$.

$$-(r_1 + r_2) = -(-3 + \sqrt{5} - 3 - \sqrt{5}) = 6.$$
$$r_1 \times r_2 = (-3 + \sqrt{5})(-3 - \sqrt{5}) = 4.$$
$$\therefore\ x^2 + 6x + 4 = 0.$$

10. $\frac{4}{3} \pm \sqrt{7}$.

$$-(r_1 + r_2) = -(\tfrac{4}{3} + \sqrt{7} + \tfrac{4}{3} - \sqrt{7}) = -\tfrac{8}{3}.$$
$$r_1 \times r_2 = (\tfrac{4}{3} + \sqrt{7})(\tfrac{4}{3} - \sqrt{7}) = -\tfrac{47}{9}.$$
$$\therefore\ 9x^2 - 24x - 47 = 0.$$

11. $\frac{3}{2} \pm \frac{1}{2}\sqrt{6}$.

$$-(r_1 + r_2) = -(\tfrac{3}{2} + \tfrac{1}{2}\sqrt{6} + \tfrac{3}{2} - \tfrac{1}{2}\sqrt{6}) = -3.$$
$$r_1 \times r_2 = (\tfrac{3}{2} + \tfrac{1}{2}\sqrt{6})(\tfrac{3}{2} - \tfrac{1}{2}\sqrt{6}) = \tfrac{3}{4}.$$
$$\therefore\ 4x^2 - 12x + 3 = 0.$$

12. $\dfrac{-6 \pm 2\sqrt{3}}{5}$.

$$-(r_1 + r_2) = -\left(-\frac{6}{5} + \frac{2\sqrt{3}}{5} - \frac{6}{5} - \frac{2\sqrt{3}}{5}\right) = \frac{12}{5}.$$
$$r_1 \times r_2 = \left(-\frac{6}{5} + \frac{2\sqrt{3}}{5}\right)\left(-\frac{6}{5} - \frac{2\sqrt{3}}{5}\right) = \frac{24}{25}.$$
$$\therefore\ 25x^2 + 60x + 24 = 0.$$

13. $\sqrt{5},\ -3\sqrt{5}$.

$$-(r_1 + r_2) = -(\sqrt{5} - 3\sqrt{5}) = 2\sqrt{5}.$$
$$r_1 \times r_2 = \sqrt{5} \cdot (-3\sqrt{5}) = -15.$$
$$\therefore\ x^2 + 2\sqrt{5}x - 15 = 0.$$

14. $3 - \sqrt{2},\ 2 + \sqrt{2}$.

$$-(r_1 + r_2) = -(3 - \sqrt{2} + 2 + \sqrt{2}) = -5.$$
$$r_1 \times r_2 = 4 + \sqrt{2}.$$
$$\therefore\ x^2 - 5x + (4 + \sqrt{2}) = 0.$$

15. $a,\ c$.

$$-(r_1 + r_2) = -(a + c).$$
$$r_1 \times r_2 = ac.$$
$$\therefore\ x^2 - ax - cx + ac = 0.$$

16. $a,\ \dfrac{1}{a}$.

$$-(r_1 + r_2) = -\left(a + \frac{1}{a}\right) = -\frac{a^2 + 1}{a}.$$
$$r_1 \times r_2 = a \cdot \frac{1}{a} = 1.$$
$$\therefore\ ax^2 - a^2x - x + a = 0.$$

17. $3a,\ \dfrac{2a}{3}$.

$$-(r_1 + r_2) = -\left(3a + \frac{2a}{3}\right) = -\frac{11a}{3}.$$
$$r_1 \times r_2 = 3a \cdot \frac{2a}{3} = 2a^2.$$
$$\therefore\ 3x^2 - 11ax + 6a^2 = 0.$$

18. $a + 1,\ \dfrac{1}{a - 1}$.

$$-(r_1 + r_2) = -\left(a + 1 + \frac{1}{a - 1}\right) = -\frac{a^2}{a - 1}.$$
$$r_1 \times r_2 = \frac{a + 1}{a - 1}.$$
$$\therefore\ ax^2 - x^2 - a^2x + a + 1 = 0.$$

19. $x^2 - 12x - 13 = 0$.

$$x = \frac{12 \pm \sqrt{144 + 52}}{2} = \frac{12 \pm 14}{2} = 13 \text{ or } -1.$$
$$-(r_1 + r_2) = -(13 - 1) = -12.$$
$$r_1 \times r_2 = 13 \cdot (-1) = -13.$$

20. $x^2 - 10x + 16 = 0.$

$$x = \frac{10 \pm \sqrt{100 - 64}}{2} = \frac{10 \pm 6}{2} = 8 \text{ or } 2.$$

$$-(r_1 + r_2) = -(8 + 2) = -10.$$

$$r_1 \times r_2 = 16.$$

21. $x^2 + 3x + 3 = 0.$

$$x = \frac{-3 \pm \sqrt{9 - 12}}{2} = \frac{-3 \pm \sqrt{-3}}{2}.$$

$$-(r_1 + r_2) = -\left(\frac{-3 + \sqrt{-3} - 3 - \sqrt{-3}}{2}\right) = 3.$$

$$r_1 \times r_2 = \left(\frac{-3 + \sqrt{-3}}{2}\right)\left(\frac{-3 - \sqrt{-3}}{2}\right) = 3.$$

22. $x^2 - 5x + 20 = 0.$

$$x = \frac{5 \pm \sqrt{25 - 80}}{2} = \frac{5 \pm \sqrt{-55}}{2}.$$

$$-(r_1 + r_2) = -\left(\frac{5 + \sqrt{-55} + 5 - \sqrt{-55}}{2}\right) = -5.$$

$$r_1 \times r_2 = \frac{(5 + \sqrt{-55})}{2} \cdot \frac{(5 - \sqrt{-55})}{2} = 20.$$

23. $x^2 + 2x + 2 = 0.$

$$x = \frac{-2 \pm \sqrt{-4}}{2} = -1 \pm \sqrt{-1}.$$

$$-(r_1 + r_2) = -(-1 + \sqrt{-1} - 1 - \sqrt{-1}) = 2.$$

$$r_1 \times r_2 = (-1 + \sqrt{-1})(-1 - \sqrt{-1}) = 2.$$

25. $x^2 + 7x - 18 = 0.\quad r_1 = -9.$

$$-(r_1 + r_2) = -(-9 + r_2) = 7.$$

$$r_2 = 2.$$

26. $x^2 + 2x - c = 0.\quad r_1 = 3.$

$$-(r_1 + r_2) = -(3 + r_2) = 2.$$

$$r_2 = -5.$$

$$r_1 \times r_2 = 2 \cdot (-5) = -10 = c.$$

27. $x^2 - x - c = 0.\quad r_1 = 10.$

$$-(r_1 + r_2) = -(10 + r_2) = -1.$$

$$r_2 = -9.$$

$$r_1 \times r_2 = -90 = c.$$

28. $x^2 + 8x - c = 0.\quad r_1 = -2.$

$$-(r_1 + r_2) = -(-2 + r_2) = 8.$$

$$r_2 = -6.$$

$$r_1 \times r_2 = (-2) \cdot (-6) = 12 = c.$$

29. $x^2 - cx - 70 = 0.\quad r_1 = 10.$

$$r_1 \times r_2 = 10\,r_2 = -70.$$
$$r_2 = -7.$$
$$c = -(r_1 + r_2) = -3.$$

30. $x^2 + 2\,bx + 25 = 0.\quad r_1 = -5.$

$$r_1 \times r_2 = -5\,r_2 = 25.$$
$$r_2 = -5.$$
$$2\,b = -(-5 - 5) = 10.$$
$$b = 5.$$

31. $x^2 - 3\,ax - 52 = 0.\quad r_1 = 4.$

$$r_1 \times r_2 = 4\,r_2 = -52.$$
$$r_2 = -13.$$
$$-3\,a = -(r_1 + r_2) = -(4 - 13) = 9.$$
$$a = -3.$$

32. $2\,x^2 - 11\,x + c = 0.\quad r_1 = 5.$

$$-(r_1 + r_2) = -\tfrac{11}{2}.$$
$$5 + r_2 = \tfrac{11}{2}.$$
$$r_2 = \tfrac{1}{2}.$$
$$r_1 \times r_2 = c.$$
$$c = 5.$$

33. $ax^2 - 20\,x + 12 = 0.\quad r_1 = \tfrac{2}{3}.$

$$-(r_1 + r_2) = -\frac{20}{a}.$$
$$\frac{2}{3} + r_2 = \frac{20}{a}.$$
$$r_2 = \frac{60 - 2\,a}{3\,a}.$$
$$r_1 r_2 = \frac{12}{a}.$$
$$\frac{60 - 2\,a}{3\,a} \cdot \frac{2}{3} = \frac{12}{a}.$$
$$a = 3.$$

34. $ax^2 - 6\,x - 21.\quad r_1 = -3.$

$$-(r_1 + r_2) = -\frac{6}{a}.$$
$$-3 + r_2 = \frac{6}{a}.$$
$$r_2 = 3 + \frac{6}{a}.$$
$$r_1 r_2 = -\frac{21}{a}.$$
$$\left(3 + \frac{6}{a}\right)(-3) = -\frac{21}{a}.$$
$$a = \tfrac{1}{3}.$$

35. $x^2 - 8\,x + c = 0.\quad r_1 = 3\,r_2.$

$$-(3\,r_2 + r_2) = -8.$$
$$r_2 = 2.$$
$$c = r_1 \times r_2 = 2 \cdot 6 = 12.$$

36. $x^2 + 7\,x + c = 0.\quad r_1 + 1 = r_2.$

$$-(r_1 + r_1 + 1) = 7.$$
$$r_1 = -4.$$
$$r_2 = -3.$$
$$c = r_1 \times r_2 = 12.$$

37. $x^2 + 11\,x + b = 0.\quad r_1 - r_2 = 9.$

$$-(r_1 + r_1 - 9) = 11.$$
$$r_1 = -1.$$
$$r_2 = -10.$$
$$b = (r_1 \times r_2) = 10.$$

38. $x^2 - 5\,x - c = 0.\quad r_1 - r_2 = 7.$

$$-(r_1 + r_1 - 7) = -5.$$
$$r_1 = 6.$$
$$r_2 = -1.$$
$$c = r_1 \times r_2 = -6.$$

39. $x^2 - 5\,x - a = 0.\quad r_1 - r_2 = -13.$

$$-(r_1 + r_1 + 13) = -5.$$
$$r_1 = -4.$$
$$r_2 = 9.$$
$$a = r_1 \times r_2 = -36.$$

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1. 3, 7.

$$(x-3)=0,\ (x-7)=0.$$
$$\therefore\ (x-3)(x-7)=0.$$
$$x^2-10x+21=0.$$

2. 4, -5, 6.

$$(x-4)=0,\ (x+5)=0,\ (x-6)=0.$$
$$\therefore\ (x-4)(x+5)(x-6)=0.$$
$$x^3-5x^2-26x+120=0.$$

3. $1+\sqrt{3}$, $1-\sqrt{3}$.

$$[x-(1+\sqrt{3})]=0,\ [x-(1-\sqrt{3})]=0.$$
$$\therefore\ [x-(1+\sqrt{3})][x-(1-\sqrt{3})]=0.$$
$$x^2-2x-2=0.$$

4. $2\pm\sqrt{5}$.

$$[x-(2+\sqrt{5})]=0,\ [x-(2-\sqrt{5})]=0.$$
$$\therefore\ [x-(2+\sqrt{5})][x-(2-\sqrt{5})]=0.$$
$$x^2-4x-1=0.$$

5. $\dfrac{3\pm\sqrt{7}}{2}$.

$$\left(x-\frac{3+\sqrt{7}}{2}\right)=0,\left(x-\frac{3-\sqrt{7}}{2}\right)=0.$$
$$\therefore\ \left(x-\frac{3+\sqrt{7}}{2}\right)\left(x-\frac{3-\sqrt{7}}{2}\right)=0.$$
$$2x^2-6x+1=0.$$

6. 3, -3, 8.

$$(x-3)=0,\ (x+3)=0,\ (x-8)=0.$$
$$\therefore\ (x-3)(x+3)(x-8)=0, \text{ or } x^3-8x^2-9x+72=0.$$

7. 1, $\frac{3}{2}$, -2.

$$(x-1)=0,\ (x-\tfrac{3}{2})=0,\ (x+2)=0.$$
$$\therefore\ (x-1)(x-\tfrac{3}{2})(x+2)=0, \text{ or } 2x^3-x^2-7x+6=0.$$

8. $1\pm\sqrt{3}$, 3.

$$x-(1+\sqrt{3})=0,\ x-(1-\sqrt{3})=0,\ x-3=0.$$
$$\therefore\ (x-1-\sqrt{3})(x-1+\sqrt{3})(x-3)=0.$$
$$x^3-5x^2+4x+6=0.$$

9. $a+b$, $a-b$.

$$x-(a+b)=0,\ x-(a-b)=0.$$
$$\therefore\ (x-a-b)(x-a+b)=0.$$
$$x^2-2ax+a^2-b^2=0.$$

10. $\frac{1}{a}$, $5a$.

$$x - \frac{1}{a} = 0, x - 5a = 0.$$

$$\therefore \left(x - \frac{1}{a}\right)(x - 5a) = 0.$$

$$ax^2 - x - 5a^2x + 5 = 0.$$

11. $3c \pm \sqrt{2a}$.

$$x - (3c + \sqrt{2a}) = 0, x - (3c - \sqrt{2a}) = 0.$$

$$\therefore (x - 3c - \sqrt{2a})(x - 3c + \sqrt{2a}) = 0.$$

$$\therefore x^2 - 6cx + 9c^2 - 2a = 0.$$

12. $\frac{4a \pm \sqrt{3c}}{2}$.

$$x - \frac{4a + \sqrt{3c}}{2} = 0, x - \frac{4a - \sqrt{3c}}{2} = 0.$$

$$\therefore \left(x - \frac{4a + \sqrt{3c}}{2}\right)\left(x - \frac{4a - \sqrt{3c}}{2}\right) = 0.$$

$$4x^2 - 16ax + 16a^2 - 3c = 0.$$

13. r_1, r_2, r_3.

$$x - r_1 = 0, x - r_2 = 0, x - r_3 = 0.$$

$$\therefore (x - r_1)(x - r_2)(x - r_3) = 0.$$

$$x^3 - (r_1 + r_2 + r_3)x^2 + (r_1r_3 + r_2r_3 + r_1r_2)x - r_1r_2r_3 = 0.$$

14. $3, 2 \pm \sqrt{a}$.

$$(x - 3) = 0, x - (2 + \sqrt{a}) = 0, x - (2 - \sqrt{a}) = 0.$$

$$\therefore (x - 3)(x - 2 - \sqrt{a})(x - 2 + \sqrt{a}) = 0.$$

$$x^3 - 7x^2 - 8x - ax - 12 + 3a = 0.$$

15. $-5, -7, 6, 8$.

$$x + 5 = 0, x + 7 = 0, x - 6 = 0, x - 8 = 0.$$

$$\therefore (x + 5)(x + 7)(x - 6)(x - 8) = 0.$$

$$x^4 - 2x^3 - 85x^2 + 86x + 1680 = 0.$$

16. $2 \pm \sqrt{3}, 3 \pm \sqrt{2}$.

$$x - (2 + \sqrt{3}) = 0, x - (2 - \sqrt{3}) = 0, x - (3 + \sqrt{2}) = 0, x - (3 - \sqrt{2}) = 0.$$

$$\therefore (x - 2 - \sqrt{3})(x - 2 + \sqrt{3})(x - 3 - \sqrt{2})(x - 3 + \sqrt{2}) = 0.$$

$$x^4 - 10x^3 + 32x^2 - 34x + 7 = 0.$$

17. $1, -2, a \pm \sqrt{a}$.

$$(x - 1) = 0, (x + 2) = 0, x - (a + \sqrt{a}) = 0, x - (a - \sqrt{a}) = 0.$$

$$\therefore (x - 1)(x + 2)(x - a - \sqrt{a})(x - a + \sqrt{a}) = 0.$$

$$x^4 - 2ax^3 + x^3 + a^2x^2 - 3ax^2 - 2x^2 + a^2x + 3ax - 2a^2 + 2a = 0.$$

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1. $x^2 - 3x - 40.$

$$a = 1, \quad b = -3, \quad c = -40.$$
$$b^2 - 4ac = 9 + 160 = 169.$$

The equation has rational factors.

2. $2x^2 + 5x - 7.$

$$a = 2, \quad b = 5, \quad c = -7.$$
$$b^2 - 4ac = 25 + 56 = 81.$$

The equation has rational factors.

3. $7x^2 - 9x + 18.$

$$a = 7, \quad b = -9, \quad c = 18.$$
$$b^2 - 4ac = 81 - 504 = -423.$$

The equation has imaginary factors.

4. $24x^2 - x - 10.$

$$a = 24, \quad b = -1, \quad c = -10.$$
$$b^2 - 4ac = 1 + 960 = 961 = (31)^2.$$

The equation has rational factors.

5. $72x^2 - 17x + 1.$

$$a = 72, \quad b = -17, \quad c = 1.$$
$$b^2 - 4ac = 289 - 288 = 1.$$

The equation has rational factors.

6. $5x^2 + 3x - 20.$

$$a = 5, \quad b = 3, \quad c = -20.$$
$$b^2 - 4ac = 9 + 400 = 409.$$

The equation has irrational factors.

7. $3x^2 - 9x + 28.$

$$a = 3, \quad b = -9, \quad c = 28.$$
$$b^2 - 4ac = 81 - 336 = -255.$$

The equation has imaginary factors.

8. $33h^2 - 233h - 6.$

$$a = 33, \quad b = -233, \quad c = -6.$$
$$b^2 - 4ac = 54{,}289 + 792 = 55{,}081.$$

The equation has irrational factors.

9. $x^2 - 2ax + (a^2 - b^2).$

$$a = 1, \quad b = -2a, \quad c = a^2 - b^2.$$
$$\text{The discriminant} = 4a^2 - 4a^2 + 4b^2 = 4b^2.$$

The equation has rational factors.

10. $abx^2 - (b^2 + a^2)x + ab.$

$$a = ab, \quad b = b^2 + a^2, \quad c = ab.$$
$$\text{The discriminant} = b^4 + 2a^2b^2 + b^4 - 4a^2b^2$$
$$= b^4 - 2a^2b^2 + a^4.$$

The equation has rational factors.

12. $x^2 - 7x - 30.$

Let $x^2 - 7x - 30 = 0.$

Solving, $$x = \frac{7 \pm \sqrt{49 + 120}}{2} = 10 \text{ or } -3.$$

Then $r_1 = 10$, and $r_2 = -3.$

$$\therefore\ x^2 - 7x - 30 = (x - 10)(x + 3).$$

13. $x^2 - 4x - 1.$

Let $x^2 - 4x - 1 = 0.$

$$x = \frac{4 \pm \sqrt{16 + 4}}{2} = 2 \pm \sqrt{5}.$$

Then $r_1 = 2 + \sqrt{5}$, and $r_2 = 2 - \sqrt{5}.$

$$\therefore\ x^2 - 4x - 1 = \left(x - 2 - \sqrt{5}\right)\left(x - 2 + \sqrt{5}\right).$$

14. $x^2 + 2x + 2.$

Let $x^2 + 2x + 2 = 0.$

$$x = \frac{-2 \pm \sqrt{4 - 8}}{2} = -1 \pm \sqrt{-1}.$$

Then $r_1 = -1 + \sqrt{-1}$, and $r_2 = -1 - \sqrt{-1}.$

$$\therefore\ x^2 + 2x + 2 = \left(x + 1 - \sqrt{-1}\right)\left(x + 1 + \sqrt{-1}\right).$$

15. $x^2 + 4x - 9.$

Let $x^2 + 4x - 9 = 0.$

$$x = \frac{-4 \pm \sqrt{16 + 36}}{2} = -2 \pm \sqrt{13}.$$

Then $r_1 = -2 + \sqrt{13}$, and $r_2 = -2 - \sqrt{13}.$

$$\therefore\ x^2 + 4x - 9 = \left(x + 2 - \sqrt{13}\right)\left(x + 2 + \sqrt{13}\right).$$

16. $4x^2 - 12x - 9.$

Let $4x^2 - 12x - 9 = 0.$

$$x = \frac{12 \pm \sqrt{144 + 144}}{8} = \frac{3 \pm 3\sqrt{2}}{2}.$$

Then $r_1 = \dfrac{3 + 3\sqrt{2}}{2}$, and $r_2 = \dfrac{3 - 3\sqrt{2}}{2}.$

$$\therefore\ 4x^2 - 12x - 9 = \left(2x - 3 - 3\sqrt{2}\right)\left(2x - 3 + 3\sqrt{2}\right).$$

17. $25x^2 + 20x + 4.$

Let $25x^2 + 20x + 4 = 0.$

$$x = \frac{-20 \pm \sqrt{400 - 400}}{50} = -\frac{2}{5}.$$

Then $r_1 = -\frac{2}{5}$, and $r_2 = -\frac{2}{5}.$

$$\therefore\ 25x^2 + 20x + 4 = (5x + 2)(5x + 2).$$

18. $6x^2 + 14x - 40.$

Let $6x^2 + 14x - 40 = 0.$

$$x = \frac{-14 \pm \sqrt{196 + 960}}{12} = \frac{-14 \pm 34}{12}$$

$$= \tfrac{5}{3} \text{ or } -4.$$

Then $r_1 = \frac{5}{3}$, and $r_2 = -4.$

$$\therefore 6x^2 + 14x - 40 = 2(3x - 5)(x + 4).$$

19. $10 - 9x - 9x^2.$

Let $10 - 9x - 9x^2 = 0.$

$$x = \frac{9 \pm \sqrt{81 + 360}}{-18} = -\tfrac{5}{3} \text{ or } \tfrac{2}{3}.$$

Then $r_1 = -\frac{5}{3}$, and $r_2 = \frac{2}{3}.$

$$\therefore 10 - 9x - 9x^2 = (-3x - 5)(3x - 2).$$

20. $10x^2 + 12 - 26x.$

Let $10x^2 + 12 - 26x = 0.$

$$x = \frac{26 \pm \sqrt{676 - 480}}{20} = 2 \text{ or } \tfrac{3}{5}.$$

Then $r_1 = 2$, and $r_2 = \frac{3}{5}.$

$$\therefore 10x^2 + 12 - 26x = 2(x - 2)(5x - 3).$$

21. $x^2 + 7x + 8.$

Let $x^2 + 7x + 8 = 0.$

$$x = \frac{-7 \pm \sqrt{49 - 32}}{2} = \frac{-7 \pm \sqrt{17}}{2}.$$

Then $r_1 = \dfrac{-7 + \sqrt{17}}{2}$, and $r_2 = \dfrac{-7 - \sqrt{17}}{2}.$

$$\therefore x^2 + 7x + 8 = \left(\frac{2x + 7 - \sqrt{17}}{2}\right)\left(\frac{2x + 7 + \sqrt{17}}{2}\right).$$

22. $x^2 + x + 1.$

Let $x^2 + x + 1 = 0.$

$$x = \frac{-1 \pm \sqrt{1 - 4}}{2} = \frac{-1 \pm \sqrt{-3}}{2}.$$

Then $r_1 = \dfrac{-1 + \sqrt{-3}}{2}$, and $r_2 = \dfrac{-1 - \sqrt{-3}}{2}.$

$$\therefore x^2 + x + 1 = \left(\frac{2x + 1 - \sqrt{-3}}{2}\right)\left(\frac{2x + 1 + \sqrt{-3}}{2}\right).$$

23. $x^2 + 1$.

Let $$x^2 + 1 = 0.$$
$$x = \pm\sqrt{-1}.$$
Then $$r_1 = \sqrt{-1}, \text{ and } r_2 = -\sqrt{-1}.$$
$$\therefore x^2 + 1 = (x - \sqrt{-1})(x + \sqrt{-1}).$$

24. $x^2 + 9$.

Let $$x^2 + 9 = 0.$$
$$x = \pm 3\sqrt{-1}.$$
Then $$r_1 = 3\sqrt{-1}, \text{ and } r_2 = -3\sqrt{-1}.$$
$$\therefore x^2 + 9 = (x - 3\sqrt{-1})(x + 3\sqrt{-1}).$$

25. $x^2 - 2ax + a^2 - b$.

Let $$x^2 - 2ax + a^2 - b = 0.$$
$$x = \frac{2a \pm \sqrt{4a^2 - 4a^2 + 4b}}{2} = a \pm \sqrt{b}.$$
Then $$r_1 = a + \sqrt{b}, \text{ and } r_2 = a - \sqrt{a}.$$
$$\therefore x^2 - 2ax + a^2 - b = (x - a - \sqrt{b})(x - a + \sqrt{b}).$$

26. $x^2 + 6ax + 9a^2 - 4b$.

Let $$x^2 + 6ax + 9a^2 - 4b = 0.$$
$$x = \frac{-6a \pm \sqrt{36a^2 - 36a^2 + 16b}}{2} = -3a \pm 2\sqrt{b}.$$
Then $$r_1 = -3a + 2\sqrt{b}, \text{ and } r_2 = -3a - 2\sqrt{b}.$$
$$\therefore x^2 + 6ax + 9a^2 - 4b = (x + 3a - 2\sqrt{b})(x + 3a + 2\sqrt{b}).$$

27. $4x^2 + 4ax + a^2 - 4c$.

Let $$4x^2 + 4ax + a^2 - 4c = 0.$$
$$x = \frac{-4a \pm \sqrt{16a^2 - 16a^2 + 64c}}{8} = \frac{-a \pm 2\sqrt{c}}{2}.$$
Then $$r_1 = \frac{-a + 2\sqrt{c}}{2}, \text{ and } r_2 = \frac{-a - 2\sqrt{c}}{2}.$$
$$\therefore 4x^2 + 4ax + a^2 - 4c = (2x + a - 2\sqrt{c})(2x + a + 2\sqrt{c}).$$

28. $x^2 - 4ax + 4a^2 + c$.

Let $$x^2 - 4ax + 4a^2 + c = 0.$$
$$x = \frac{4a \pm \sqrt{16a^2 - 16a^2 - 4c}}{2} = 2a \pm \sqrt{-c}.$$
Then $$r_1 = 2a + \sqrt{-c}, \text{ and } r_2 = 2a - \sqrt{-c}.$$
$$\therefore x^2 - 4ax + 4a^2 + c = (x - 2a - \sqrt{-c})(x - 2a + \sqrt{-c}).$$

29. $ax^2 + bx + c.$

Let $$ax^2 + bx + c = 0.$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Then $$r_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \text{ and } r_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

$$\therefore ax^2 + bx + c = a\left[x - \left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right)\right]\left[x - \left(\frac{-b - \sqrt{b^2 - 4ac}}{2a}\right)\right]$$

$$= a\left(\frac{2ax + b - \sqrt{b^2 - 4ac}}{2a}\right)\left(\frac{2ax + b + \sqrt{b^2 - 4ac}}{2a}\right).$$

31. $3x^2 - 6xy + 14x - 4y + 8.$

Let $$3x^2 - 6xy + 14x - 4y + 8 = 0.$$

Then $$3x^2 - (6y - 14)x - (4y - 8) = 0.$$

$$x = \frac{(6y - 14) \pm \sqrt{(6y - 14)^2 + 4 \cdot 3 \cdot (4y - 8)}}{6}$$

$$= \frac{3y - 7 \pm \sqrt{9y^2 - 30y + 25}}{3}.$$

Then $$r_1 = \frac{3y - 7 + 3y - 5}{3} = 2y - 4.$$

$$r_2 = \frac{3y - 7 - 3y + 5}{3} = -\tfrac{2}{3}.$$

$$\therefore 3x^2 - 6xy + 14x - 4y + 8 = 3(x - 2y + 4)(x + \tfrac{2}{3})$$
$$= (x - 2y + 4)(3x + 2).$$

32. $x^2 - xy - 2y^2 + 3x - 6y.$

Let $$x^2 - xy - 2y^2 + 3x - 6y = 0.$$

Then $$x^2 - (y - 3)x - 2y^2 - 6y = 0.$$

$$x = \frac{(y - 3) \pm \sqrt{(y - 3)^2 + 8y^2 + 24y}}{2} = 2y \text{ or } (-3 - y).$$

Then $$r_1 = 2y, \text{ and } r_2 = (-3 - y).$$

$$\therefore x^2 - xy - 2y^2 + 3x - 6y = (x - 2y)(x + 3 + y).$$

33. $x^2 - 4xy - y + 3y^2 - 2 - x.$

Let $$x^2 - (4y + 1)x + (3y^2 - y - 2) = 0.$$

$$x = \frac{4y + 1 \pm \sqrt{(4y + 1)^2 - 4(3y^2 - y - 2)}}{2}.$$

Then $$r_1 = \frac{4y + 1 + \sqrt{4y^2 + 12y + 9}}{2} = \frac{4y + 1 + 2y + 3}{2} = 3y + 2,$$

and $$r_2 = \frac{4y + 1 - \sqrt{4y^2 + 12y + 9}}{2} = \frac{4y + 1 - 2y - 3}{2} = y - 1.$$

$$\therefore x^2 - 4xy - y + 3y^2 - 2 - x = (x - y + 1)(x - 3y - 2).$$

34. $x^2 - 2y^2 - xy + 2x + 5y - 3.$

Let $$x^2 - (y-2)x - (2y^2 - 5y + 3) = 0.$$

$$x = \frac{y - 2 \pm \sqrt{(y-2)^2 + 4(2y^2 - 5y + 3)}}{2} = 2y - 3 \text{ or } -y + 1.$$

Then $$r_1 = (2y - 3), \text{ and } r_2 = (-y + 1).$$

$$\therefore x^2 - 2y^2 - xy + 2x + 5y - 3 = (x - 2y + 3)(x + y - 1).$$

35. $6x^2 + xy - 12y^2 + x + 10y - 2.$

Let $$6x^2 + (y+1)x - (12y^2 - 10y + 2) = 0.$$

$$x = \frac{-(y+1) \pm \sqrt{(y+1)^2 + 24(12y^2 - 10y + 2)}}{12}$$

$$= \frac{-(y+1) \pm \sqrt{289y^2 - 238y + 49}}{12}.$$

Then $$r_1 = \frac{-y - 1 + 17y - 7}{12} = \frac{4y - 2}{3},$$

and $$r_2 = \frac{-y - 1 - 17y + 7}{12} = \frac{-3y + 1}{2}.$$

$$\therefore 6x^2 + xy - 12y^2 + x + 10y - 2 = 6\left(x - \frac{4y-2}{3}\right)\left(x - \frac{-3y+1}{2}\right)$$

$$= (3x - 4y + 2)(2x + 3y - 1).$$

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1. $$(a+b)^6 = a^6 + 6a^5b + \frac{30}{\lfloor 2}a^4b^2 + \frac{120}{\lfloor 3}a^3b^3 + \frac{360}{\lfloor 4}a^2b^4 + \frac{720}{\lfloor 5}ab^5 + \frac{720}{\lfloor 6}b^6$$

$$= a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6.$$

2. $$(a-1)^6 = a^6 - 6a^5 \cdot 1 + \frac{30}{\lfloor 2}a^4 \cdot 1^2 - \frac{120}{\lfloor 3}a^3 \cdot 1^3 + \frac{360}{\lfloor 4}a^2 \cdot 1^4 - \frac{720}{\lfloor 5}a \cdot 1^5 + \frac{720}{\lfloor 6} \cdot 1^6$$

$$= a^6 - 6a^5 + 15a^4 - 20a^3 + 15a^2 - 6a + 1.$$

3. $$(a+1)^7 = a^7 + 7a^6 \cdot 1 + \frac{42}{\lfloor 2}a^5 \cdot 1^2 + \frac{210}{\lfloor 3}a^4 \cdot 1^3 + \frac{840}{\lfloor 4}a^3 \cdot 1^4 + \frac{2520}{\lfloor 5}a^2 \cdot 1^5 + \frac{5040}{\lfloor 6}a \cdot 1^6 + \frac{5040}{\lfloor 7} \cdot 1^7$$

$$= a^7 + 7a^6 + 21a^5 + 35a^4 + 35a^3 + 21a^2 + 7a + 1$$

4. $(a+2)^6 = a^6 + 6a^5 \cdot 2 + \frac{30}{\underline{|2}} a^4 \cdot 2^2 + \frac{120}{\underline{|3}} a^3 \cdot 2^3 + \frac{360}{\underline{|4}} a^2 \cdot 2^4$
$+ \frac{720}{\underline{|5}} a \cdot 2^5 + \frac{720}{\underline{|6}} \cdot 2^6$
$= a^6 + 12a^5 + 60a^4 + 160a^3 + 240a^2 + 192a + 64.$

5. $(a+3)^7 = a^7 + 7a^6 \cdot 3 + \frac{42}{\underline{|2}} a^5 \cdot 3^2 + \frac{210}{\underline{|3}} a^4 \cdot 3^3 + \frac{840}{\underline{|4}} a^3 \cdot 3^4$
$+ \frac{2520}{\underline{|5}} a^2 \cdot 3^5 + \frac{5040}{\underline{|6}} a \cdot 3^6 + \frac{5040}{\underline{|7}} \cdot 3^7$
$= a^7 + 21a^6 + 189a^5 + 945a^4 + 2835a^3 + 5103a^2$
$+ 5103a + 2187.$

6. $(2-a)^6 = 2^6 - 6 \cdot 2^5 a + \frac{30}{\underline{|2}} \cdot 2^4 a^2 - \frac{120}{\underline{|3}} \cdot 2^3 a^3$
$+ \frac{360}{\underline{|4}} \cdot 2^2 a^4 - \frac{720}{\underline{|5}} \cdot 2a^5 + \frac{720}{\underline{|6}} a^6$
$= 64 - 192a + 240a^2 - 160a^3 + 60a^4 - 12a^5 + a^6.$

7. $(a+b)^{20} = a^{20} + 20a^{19}b + \frac{380}{\underline{|2}} a^{18}b^2 + \frac{6840}{\underline{|3}} a^{17}b^3 + \cdots$
$= a^{20} + 20a^{19}b + 190a^{18}b^2 + 1140a^{17}b^3 + \cdots.$

8. $(a+b)^{30} = a^{30} + 30a^{29}b + \frac{870}{\underline{|2}} a^{28}b^2 + \frac{24360}{\underline{|3}} a^{27}b^3 + \cdots$
$= a^{30} + 30a^{29}b + 435a^{28}b^2 + 4060a^{27}b^3 + \cdots.$

9. $(a+1)^{40} = a^{40} + 40a^{39} \cdot 1 + \frac{1560}{\underline{|2}} a^{38} \cdot 1^2 + \frac{59280}{\underline{|3}} a^{37} \cdot 1^3 + \cdots$
$= a^{40} + 40a^{39} + 780a^{38} + 9880a^{37} + \cdots.$

10. $(a-2)^{20} = a^{20} - 20a^{19} \cdot 2 + \frac{380}{\underline{|2}} a^{18} \cdot 2^2 - \frac{6840}{\underline{|3}} a^{17} \cdot 2^3 + \cdots$
$= a^{20} - 40a^{19} + 760a^{18} - 9120a^{17} + \cdots.$

11. $(a^2+2b)^5 = (a^2)^5 + 5(a^2)^4(2b) + \frac{20}{\underline{|2}}(a^2)^3(2b)^2 + \frac{60}{\underline{|3}}(a^2)^2(2b)^3$
$+ \frac{120}{\underline{|4}}(a^2)(2b)^4 + \frac{120}{\underline{|5}}(2b)^5$
$= a^{10} + 10a^8b + 40a^6b^2 + 80a^4b^3 + 80a^2b^4 + 32b^5.$

12. $$(a^2-2)^6 = (a^2)^6 - 6(a^2)^5\cdot 2 + \frac{30}{\underline{|2}}(a^2)^4\cdot 2^2 - \frac{120}{\underline{|3}}(a^2)^3\cdot 2^3$$
$$+\frac{360}{\underline{|4}}(a^2)^2\cdot 2^4 - \frac{720}{\underline{|5}}(a^2)\cdot 2^5 + \frac{720}{\underline{|6}}\cdot 2^6$$
$$= a^{12} - 12a^{10} + 60a^8 - 160a^6 + 240a^4 - 192a^2 + 64.$$

13. $$(a^2+2b)^7 = (a^2)^7 + 7(a^2)^6(2b) + \frac{42}{\underline{|2}}(a^2)^5(2b)^2 + \frac{210}{\underline{|3}}(a^2)^4(2b)^3$$
$$+\frac{840}{\underline{|4}}(a^2)^3(2b)^4 + \frac{2520}{\underline{|5}}(a^2)^2(2b)^5$$
$$+\frac{5040}{\underline{|6}}(a^2)(2b)^6 + \frac{5040}{\underline{|7}}(2b)^7$$
$$= a^{14} + 14a^{12}b + 84a^{10}b^2 + 280a^8b^3 + 560a^6b^4$$
$$+ 672a^4b^5 + 448a^2b^6 + 128b^7.$$

14. $$\left(a^2+\frac{1}{b}\right)^5 = (a^2)^5 + 5(a^2)^4\left(\frac{1}{b}\right)^1 + \frac{20}{\underline{|2}}(a^2)^3\left(\frac{1}{b}\right)^2 + \frac{60}{\underline{|3}}(a^2)^2\left(\frac{1}{b}\right)^3$$
$$+\frac{120}{\underline{|4}}(a^2)^1\left(\frac{1}{b}\right)^4 + \frac{120}{\underline{|5}}\left(\frac{1}{b}\right)^5$$
$$= a^{10} + \frac{5a^8}{b} + \frac{10a^6}{b^2} + \frac{10a^4}{b^3} + \frac{5a^2}{b^4} + \frac{1}{b^5}.$$

15. $$\left(a^2-\frac{1}{a^3}\right)^6 = (a^2)^6 - 6(a^2)^5\left(\frac{1}{a^3}\right)^1 + \frac{30}{\underline{|2}}(a^2)^4\left(\frac{1}{a^3}\right)^2 - \frac{120}{\underline{|3}}(a^2)^3\left(\frac{1}{a^3}\right)^3$$
$$+\frac{360}{\underline{|4}}(a^2)^2\left(\frac{1}{a^3}\right)^4 - \frac{720}{\underline{|5}}(a^2)^1\left(\frac{1}{a^3}\right)^5 + \frac{720}{\underline{|6}}\left(\frac{1}{a^3}\right)^6$$
$$= a^{12} - 6a^7 + 15a^2 - \frac{20}{a^3} + \frac{15}{a^8} - \frac{6}{a^{13}} + \frac{1}{a^{18}}.$$

16. $$(a^2+2b)^{20} = (a^2)^{20} + 20(a^2)^{19}(2b)^1 + \frac{380}{\underline{|2}}(a^2)^{18}(2b)^2$$
$$+\frac{6840}{\underline{|3}}(a^2)^{17}(2b)^3 + \cdots$$
$$= a^{40} + 40a^{38}b + 760a^{36}b^2 + 9120a^{34}b^3 + \cdots.$$

17. $$\left(a^2-\frac{2}{a}\right)^{30} = (a^2)^{30} - 30(a^2)^{29}\left(\frac{2}{a}\right)^1$$
$$+\frac{870}{\underline{|2}}(a^2)^{28}\left(\frac{2}{a}\right)^2 - \frac{24360}{\underline{|3}}(a^2)^{27}\left(\frac{2}{a}\right)^3 + \cdots$$
$$= a^{60} - 60a^{57} + 1740a^{54} - 32{,}480a^{51} + \cdots.$$

18. $$\left(\frac{a}{b}+\frac{3\,b}{a}\right)^{20}=\left(\frac{a}{b}\right)^{20}+20\left(\frac{a}{b}\right)^{19}\left(\frac{3\,b}{a}\right)^{1}+\frac{380}{\underline{|2}}\left(\frac{a}{b}\right)^{18}\left(\frac{3\,b}{a}\right)^{2}$$
$$+\frac{6840}{\underline{|3}}\left(\frac{a}{b}\right)^{17}\left(\frac{3\,b}{a}\right)^{3}+\cdots$$
$$=\frac{a^{20}}{b^{20}}+60\,\frac{a^{18}}{b^{18}}+1710\,\frac{a^{16}}{b^{16}}+30{,}780\,\frac{a^{14}}{b^{14}}+\cdots.$$

19. $$\left(\frac{2\,x}{y^3}-\frac{y^4}{6\,x^5}\right)^{7}=\left(\frac{2\,x}{y^3}\right)^{7}-7\left(\frac{2\,x}{y^3}\right)^{6}\left(\frac{y^4}{6\,x^5}\right)^{1}$$
$$+\frac{42}{\underline{|2}}\left(\frac{2\,x}{y^3}\right)^{5}\left(\frac{y^4}{6\,x^5}\right)^{2}-\frac{210}{\underline{|3}}\left(\frac{2\,x}{y^3}\right)^{4}\left(\frac{y^4}{6\,x^5}\right)^{3}+\cdots$$
$$=128\,\frac{x^7}{y^{21}}-\frac{224\,x}{3\,y^{14}}+\frac{56}{3\,x^5y^7}-\frac{70}{27\,x^{11}}+\cdots.$$

20. $$(a^2-3\,b^2)^{10}=(a^2)^{10}-10\,(a^2)^9\,(3\,b^2)^1$$
$$+\frac{90}{\underline{|2}}(a^2)^8\,(3\,b^2)^2-\frac{720}{\underline{|3}}(a^2)^7\,(3\,b^2)^3+\cdots$$
$$=a^{20}-30\,a^{18}b^2+405\,a^{16}b^4-3240\,a^{14}b^6+\cdots.$$

21. $$\left(\frac{3\,x^5}{y^3}-\frac{2\,y^{15}}{9\,x^{12}}\right)^{6}=\left(\frac{3\,x^5}{y^3}\right)^{6}-6\left(\frac{3\,x^5}{y^3}\right)^{5}\left(\frac{2\,y^{15}}{9\,x^{12}}\right)^{1}$$
$$+\frac{30}{\underline{|2}}\left(\frac{3\,x^5}{y^3}\right)^{4}\left(\frac{2\,y^{15}}{9\,x^{12}}\right)^{2}-\frac{120}{\underline{|3}}\left(\frac{3\,x^5}{y^3}\right)^{3}\left(\frac{2\,y^{15}}{9\,x^{12}}\right)^{3}\cdots$$
$$=\frac{729\,x^{30}}{y^{18}}-324\,x^{13}+\frac{60\,y^{18}}{x^4}-\frac{160\,y^{36}}{27\,x^{21}}+\cdots.$$

22. $$\left(\frac{a^2}{b^3}-\frac{2\,b^2}{a^4}\right)^{12}=\left(\frac{a^2}{b^3}\right)^{12}-12\left(\frac{a^2}{b^3}\right)^{11}\left(\frac{2\,b^2}{a^4}\right)^{1}$$
$$+\frac{132}{\underline{|2}}\left(\frac{a^2}{b^3}\right)^{10}\left(\frac{2\,b^2}{a^4}\right)^{2}-\frac{1320}{\underline{|3}}\left(\frac{a^2}{b^3}\right)^{9}\left(\frac{2\,b^2}{a^4}\right)^{3}+\cdots$$
$$=\frac{a^{24}}{b^{36}}-\frac{12\,a^{22}\cdot 2\,b^2}{b^{33}a^4}+\frac{132\,a^{20}\cdot 4\,b^4}{2\,b^{30}a^8}-\frac{1320\,a^{18}\cdot 8\,b^6}{6\,b^{27}a^{12}}+\cdots$$
$$=\frac{a^{24}}{b^{36}}-24\,\frac{a^{18}}{b^{31}}+264\,\frac{a^{12}}{b^{26}}-1760\,\frac{a^6}{b^{21}}+\cdots.$$

23. $$\left(\frac{\sqrt{x}}{y}+\frac{\sqrt{y}}{x}\right)^{12}=\left(\frac{\sqrt{x}}{y}\right)^{12}+12\left(\frac{\sqrt{x}}{y}\right)^{11}\left(\frac{\sqrt{y}}{x}\right)^{1}+\frac{132}{\underline{|2}}\left(\frac{\sqrt{x}}{y}\right)^{10}\left(\frac{\sqrt{y}}{x}\right)^{2}$$
$$+\frac{1320}{\underline{|3}}\left(\frac{\sqrt{x}}{y}\right)^{9}\left(\frac{\sqrt{y}}{x}\right)^{3}+\cdots$$
$$=\frac{x^6}{y^{12}}+12\,\frac{x^{\frac{9}{2}}}{y^{\frac{21}{2}}}+66\,\frac{x^3}{y^9}+220\,\frac{x^{\frac{3}{2}}}{y^{\frac{15}{2}}}+\cdots.$$

24. $$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{\underline{|2}}a^{n-2}b^2$$
$$+\frac{n(n-1)(n-2)}{\underline{|3}}a^{n-3}b^3$$
$$+\frac{n(n-1)(n-2)(n-3)}{\underline{|4}}a^{n-4}b^4$$
$$+\frac{n(n-1)(n-2)(n-3)(n-4)}{\underline{|5}}a^{n-5}b^5+\cdots.$$

$$(a+b)^3 = a^3 + 3a^{3-1}b + \frac{3(3-1)}{\underline{|2}}a^{3-2}b^2$$
$$+\frac{3(3-1)(3-2)}{\underline{|3}}a^{3-3}b^3 + 0 = a^3+3a^2b+3ab^2+b^3.$$

For $n = 1, 2,$ or 4, $(a+b)^n = a+b,\ a^2+2ab+b^2,$
and $a^4+4a^3b+6a^2b^2+4ab^3+b^4$, respectively.

The number of terms in each expansion is $n+1$.

The value of each coefficient after the $(n+1)$st is zero.

The expansion does not extend to more than six terms when $n = 5$, because $(5-5)$ or 0 appears as a factor in each coefficient after the sixth.

25. $$\left(1+\frac{1}{n}\right)^n = 1^n + n\cdot 1^{n-1}\left(\frac{1}{n}\right)^1 + \frac{n(n-1)}{\underline{|2}}\cdot 1^{n-2}\left(\frac{1}{n}\right)^2$$
$$+\frac{n(n-1)(n-2)}{\underline{|3}}\cdot 1^{n-3}\left(\frac{1}{n}\right)^3+\cdots$$
$$=1+1+\frac{n(n-1)}{n^2\underline{|2}}+\frac{n(n-1)(n-2)}{n^3\underline{|3}}+\cdots.$$

26. $$(1+.1)^{10} = 1^{10} + 10\cdot 1^9\cdot .1 + \frac{90}{\underline{|2}}\cdot 1^8\cdot .1^2 + \frac{720}{\underline{|3}}\cdot 1^7\cdot .1^3$$
$$+\frac{5040}{\underline{|4}}\cdot 1^6\cdot .1^4+\cdots$$
$$=1+1+.450+.120+.021+\cdots = 2.59+.$$

27. $$(1.-.02)^{11} = 1^{11} - 11\cdot 1^{10}\cdot .02 + \frac{110}{\underline{|2}}\cdot 1^9(.02)^2 - \frac{990}{\underline{|3}}\cdot 1^8\cdot(.02)^3+\cdots$$
$$=1-.220+.022-.00132+\cdots = .80+.$$

28. $$(3-.1)^8 = 3^8 - 8\cdot 3^7\cdot .1 + \frac{56}{\underline{|2}}3^6\cdot .1^2 - \frac{336}{\underline{|3}}3^5\cdot .1^3$$
$$+\frac{1680}{\underline{|4}}3^4\cdot .1^4 - \frac{6720}{\underline{|5}}3^3\cdot .1^5+\cdots$$
$$=6561-1749.6+204.12-13.608+.567-.01512+\cdots$$
$$=5002.46+.$$

29. $(1+.06)^6 = 1^6 + 6\cdot 1^5\cdot .06 + \dfrac{30}{\underline{|2}}1^4(.06)^2 + \cdots$
$= 1 + .360 + .054 + \cdots = 1.41+.$

30. $\underline{|6} = 6\cdot 5\cdot 4\cdot 3\cdot 2 = 720.$

31. $\underline{|2}\cdot\underline{|4} = 2\cdot 4\cdot 3\cdot 2 = 48.$

32. $\underline{|6} \div \underline{|3} = \dfrac{6\cdot 5\cdot 4\cdot 3\cdot 2}{3\cdot 2} = 120.$

33. $\underline{|4} - \underline{|3}\cdot\underline{|2}\cdot\underline{|2} = 24 - 24 = 0.$

34. $2 + \dfrac{1}{\underline{|2}} + \dfrac{1}{\underline{|3}} + \dfrac{1}{\underline{|4}} + \dfrac{1}{\underline{|5}} = 2 + \dfrac{1}{2} + \dfrac{1}{6} + \dfrac{1}{24} + \dfrac{1}{120} = \dfrac{326}{120} = 2\tfrac{43}{60} = 2.71+.$

35. $\dfrac{15\cdot 14\cdot 13\cdot 12\cdot 11\cdot 10\cdot 9}{\underline{|7}} = \dfrac{15\cdot 14\cdot 13\cdot 12\cdot 11\cdot 10\cdot 9}{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2} = 6435.$

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1. $(26)^{\frac{1}{2}} = (25+1)^{\frac{1}{2}} = 25^{\frac{1}{2}} + \tfrac{1}{2}\cdot 25^{-\frac{1}{2}}\cdot 1 - \tfrac{1}{8}\cdot 25^{-\frac{3}{2}}\cdot 1^2 + \cdots$
$= 5 + \tfrac{1}{10} - \tfrac{1}{1000} + \cdots = 5.09+.$

2. $(38)^{\frac{1}{2}} = (36+2)^{\frac{1}{2}} = 36^{\frac{1}{2}} + \tfrac{1}{2}\cdot 36^{-\frac{1}{2}}\cdot 2 - \cdots$
$= 6 + \tfrac{1}{6} - \cdots = 6.16+.$

3. $(79)^{\frac{1}{2}} = (81-2)^{\frac{1}{2}} = 81^{\frac{1}{2}} - \tfrac{1}{2}\cdot 81^{-\frac{1}{2}}\cdot 2 + \cdots$
$= 9 - \tfrac{1}{9} + \cdots$
$= 8.88+.$

4. $(120)^{\frac{1}{2}} = (121-1)^{\frac{1}{2}} = 121^{\frac{1}{2}} - \tfrac{1}{2}\cdot 121^{-\frac{1}{2}}\cdot 1 + \cdots$
$= 11 - \tfrac{1}{22} + \cdots$
$= 10.95+.$

5. $(28)^{\frac{1}{3}} = (27+1)^{\frac{1}{3}} = 27^{\frac{1}{3}} + \tfrac{1}{3}\cdot 27^{-\frac{2}{3}}\cdot 1 - \cdots$
$= 3 + \tfrac{1}{27} - \cdots$
$= 3.03+.$

6. $(66)^{\frac{1}{3}} = (64+2)^{\frac{1}{3}} = 64^{\frac{1}{3}} + \tfrac{1}{3}\cdot 64^{-\frac{2}{3}}\cdot 2 - \cdots$
$= 4 + \tfrac{1}{24} - \cdots$
$= 4.04+.$

7. $(25)^{\frac{1}{3}} = (27-2)^{\frac{1}{3}} = 27^{\frac{1}{3}} - \tfrac{1}{3}\cdot 27^{-\frac{2}{3}}\cdot 2 + \cdots$
$= 3 - \tfrac{2}{27} + \cdots$
$= 2.92+.$

8. $(720)^{\frac{1}{3}} = (729-9)^{\frac{1}{3}} = 729^{\frac{1}{3}} - \tfrac{1}{3}(729)^{-\frac{2}{3}}\cdot 9 +$
$= 9 - .037 +$
$= 8.96+.$

9. $(1+x)^{\frac{1}{2}} = 1^{\frac{1}{2}} + \tfrac{1}{2}\cdot 1^{-\frac{1}{2}}\cdot x - \tfrac{1}{8}\cdot 1^{-\frac{3}{2}}\cdot x^2 + \tfrac{1}{16}\cdot 1^{-\frac{5}{2}}\cdot x^3 - \cdots$
$= 1 + \dfrac{x}{2} - \dfrac{x^2}{8} + \dfrac{x^3}{16} - \cdots.$

10. $(2+x)^{\frac{1}{2}} = 2^{\frac{1}{2}} + \frac{1}{2}\cdot 2^{-\frac{1}{2}}\cdot x - \frac{1}{8}\cdot 2^{-\frac{3}{2}}\cdot x^2 + \frac{1}{16}\cdot 2^{-\frac{5}{2}}\cdot x^3 - \cdots$
$= \sqrt{2} + \frac{x}{2\sqrt{2}} - \frac{x^2}{16\sqrt{2}} + \frac{x^3}{64\sqrt{2}} - \cdots.$

11. $(3-x)^{\frac{1}{2}} = 3^{\frac{1}{2}} - \frac{1}{2}\cdot 3^{-\frac{1}{2}}\cdot x + \frac{1}{8}\cdot 3^{-\frac{3}{2}}\cdot x^2 + \frac{1}{16}\cdot 3^{-\frac{5}{2}}\cdot x^3 - \cdots$
$= \sqrt{3} - \frac{x}{2\sqrt{3}} + \frac{x^2}{24\sqrt{3}} + \frac{x^3}{144\sqrt{3}} - \cdots.$

12. $(1+x)^{\frac{1}{3}} = 1^{\frac{1}{3}} + \frac{1}{3}\cdot 1^{-\frac{2}{3}}\cdot x - \frac{1}{9}\cdot 1^{-\frac{5}{3}}\cdot x^2 + \frac{5}{81}\cdot 1^{-\frac{8}{3}}\cdot x^3 - \cdots$
$= 1 + \frac{x}{3} - \frac{x^2}{9} + \frac{5x^3}{81} - \cdots.$

13. $(2+x)^{\frac{1}{3}} = 2^{\frac{1}{3}} + \frac{1}{3}\cdot 2^{-\frac{2}{3}}\cdot x - \frac{1}{9}\cdot 2^{-\frac{5}{3}}\cdot x^2 + \frac{5}{81}\cdot 2^{-\frac{8}{3}}\cdot x^3 - \cdots$
$= \sqrt[3]{2} + \frac{x}{3\sqrt[3]{4}} - \frac{x^2}{18\sqrt[3]{4}} + \frac{5x^3}{324\sqrt[3]{4}} - \cdots.$

14. $(3-x)^{\frac{1}{3}} = 3^{\frac{1}{3}} - \frac{1}{3}\cdot 3^{-\frac{2}{3}}\cdot x + \frac{1}{9}\cdot 3^{-\frac{5}{3}}\cdot x^2 - \frac{5}{81}\cdot 3^{-\frac{8}{3}}\cdot x^3 + \cdots$
$= \sqrt[3]{3} - \frac{x}{3\sqrt[3]{9}} + \frac{x^2}{27\sqrt[3]{9}} - \frac{5x^3}{729\sqrt[3]{9}} + \cdots.$

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2. 6th term of $(a+b)^9 = \frac{9\cdot 8\cdot 7\cdot 6\cdot 5}{5\cdot 4\cdot 3\cdot 2}a^4b^5 = 126\,a^4b^5.$

3. 4th term of $(a+b)^{20} = \frac{20\cdot 19\cdot 18}{3\cdot 2}a^{17}b^3 = 1140\,a^{17}b^3.$

4. 7th term of $(a-b)^{10} = \frac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5}{6\cdot 5\cdot 4\cdot 3\cdot 2}a^4b^6 = 210\,a^4b^6.$

5. 8th term of $(a-b)^{15} = -\frac{15\cdot 14\cdot 13\cdot 12\cdot 11\cdot 10\cdot 9}{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2}a^8b^7 = -6435\,a^8b^7.$

6. 4th term of $\left(a+\frac{1}{a}\right)^{30} = \frac{30\cdot 29\cdot 28}{3\cdot 2}a^{27}\left(\frac{1}{a}\right)^3 = 4060\,a^{24}.$

7. 5th term of $(a^2-b)^{20} = \frac{20\cdot 19\cdot 18\cdot 17}{4\cdot 3\cdot 2}(a^2)^{16}b^4 = 4845\,a^{32}b^4.$

8. 6th term of $\left(\frac{a}{b}-\frac{b^2}{a}\right)^{18} = -\frac{18\cdot 17\cdot 16\cdot 15\cdot 14}{5\cdot 4\cdot 3\cdot 2}\left(\frac{a}{b}\right)^{13}\left(\frac{b^2}{a}\right)^5 = -8658\frac{a^8}{b^3}.$

9. 7th term of $\left(\frac{a^2}{b}-\frac{2b^2}{a}\right)^{14} = \frac{14\cdot 13\cdot 12\cdot 11\cdot 10\cdot 9}{6\cdot 5\cdot 4\cdot 3\cdot 2}\left(\frac{a^2}{b}\right)^8\left(\frac{2b^2}{a}\right)^6$
$= 192{,}192\,a^{10}b^4.$

10. 9th term of $(x^2-x)^{16} = \frac{16\cdot 15\cdot 14\cdot 13\cdot 12\cdot 11\cdot 10\cdot 9}{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2}(x^2)^8x^8$
$= 12{,}870\,x^{24}.$

11. 5th term of $\left(\sqrt{x}-\sqrt{\frac{y}{x}}\right)^{15}=\frac{15\cdot 14\cdot 13\cdot 12}{4\cdot 3\cdot 2}(\sqrt{x})^{11}\left(\sqrt{\frac{y}{x}}\right)^{4}$
$=1365\,x^{\frac{7}{2}}y^2.$

12. If $x^{r-1}=x^5$, $r=6$. The coefficient of $x^5=\frac{10\cdot 9\cdot 8\cdot 7\cdot 6}{5\cdot 4\cdot 3\cdot 2}=252.$

13. If $(x^2)^{r-1}=x^8$, $r=5$. The coefficient of $x^8=\frac{16\cdot 15\cdot 14\cdot 13}{4\cdot 3\cdot 2}=1820.$

14. If $(x^3)^{15-r+1}=x^{15}$, $r=11$. The coefficient of x^{15}

$$=\frac{15\cdot 14\cdot 13\cdot 12\cdot 11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6}{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2}=3003.$$

15. If $(x^2)^{14-r+1}(x^{-1})^{r-1}=x^{10}$, $r=7$. The coefficient of x^{10}

$$=\frac{14\cdot 13\cdot 12\cdot 11\cdot 10\cdot 9}{6\cdot 5\cdot 4\cdot 3\cdot 2}=3003.$$

16. $(3+1)^{-1}=3^{-1}-1\cdot 3^{-2}\cdot 1+\frac{2}{\underline{|2}}\cdot 3^{-3}\cdot 1^2-\frac{6}{\underline{|3}}\cdot 3^{-4}\cdot 1^3+\cdots$
$=\frac{1}{3}-\frac{1}{9}+\frac{1}{27}-\frac{1}{81}+\cdots$
$=$ (by B, page 418) $\frac{\frac{1}{3}}{\frac{4}{3}}-\frac{\frac{1}{3}(-\frac{1}{3})^{\infty}}{\frac{4}{3}}=\frac{1}{4}-0=\frac{1}{4}.$

$(1+3)^{-1}=1^{-1}-1\cdot 1^{-2}\cdot 3+\frac{2}{\underline{|2}}\cdot 1^{-3}\cdot 3^2-\frac{6}{\underline{|3}}\cdot 1^{-4}\cdot 3^3+\cdots$
$=1-3+9-27+\cdots$;
$=$ no definite number.

17. $(2+1)^{-1}=2^{-1}-1\cdot 2^{-2}+1\cdot 2^{-3}-1\cdot 2^{-4}+1\cdot 2^{-5}-1\cdot 2^{-6}+\cdots$
$=\frac{1}{2}-\frac{1}{4}+\frac{1}{8}-\frac{1}{16}+\frac{1}{32}-\frac{1}{64}+\cdots$
$=$ (by B, page 418) $\frac{\frac{1}{2}-0}{1-(-\frac{1}{2})}=\frac{1}{3}.$

$(1+2)^{-1}=1^{-1}-1\cdot 1^{-2}\cdot 2+1\cdot 1^{-3}\cdot 2^2-1\cdot 1^{-4}\cdot 2^3+1\cdot 1^{-5}\cdot 2^4-\cdots$
$=1-2+4-8+16-\cdots$
$=$ no definite number.

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1.

$$\begin{array}{rr|l}
 & x^3+3x^2+3x+1 & \underline{x+1} \\
 & x^3 & \\ \hline
3\cdot x^2= & 3x^2 & 3x^2+3x+1 \\
3x^2\div 3x^2=1 & & \\
3\cdot x\cdot 1= & 3x & \\
1^2= & 1 & \\ \hline
 & 3x^2+3x+1 & \underline{3x^2+3x+1}
\end{array}$$

2.

$$
\begin{array}{rr|l}
 & & 8x^3-12x^2+6x-1 \;\underline{|\,2x-1} \\
 & & 8x^3 \\ \hline
3(2x)^2= & 12x^2 & -12x^2+6x-1 \\
-12x^2\div 12x^2=-1 & & \\
3\cdot 2x\cdot(-1)= & -6x & \\
(-1)^2= & 1 & \\ \hline
 & 12x^2-6x+1 & -12x^2+6x-1 \\ \hline
\end{array}
$$

3.

$$
\begin{array}{rr|l}
 & & 27x^3+27x^2y+9xy^2+y^3 \;\underline{|\,3x+y} \\
 & & 27x^3 \\ \hline
3(3x)^2= & 27x^2 & 27x^2y+9xy^2+y^3 \\
27x^2y\div 27x^2=y & & \\
3\cdot 3x\cdot y= & 9xy & \\
y^2= & y^2 & \\ \hline
 & 27x^2+9xy+y^2 & 27x^2y+9xy^2+y^3 \\ \hline
\end{array}
$$

4.

$$
\begin{array}{rr|l}
 & & 64a^3-144a^2c+108ac^2-27c^3 \;\underline{|\,4a-3c} \\
 & & 64a^3 \\ \hline
3(4a)^2= & 48a^2 & -144a^2c+108ac^2-27c^3 \\
-144a^2c\div 48a^2=-3c & & \\
3\cdot 4a(-3c)= & -36ac & \\
(-3c)^2= & 9c^2 & \\ \hline
 & 48a^2-36ac+9c^2 & -144a^2c+108ac^2-27c^3 \\ \hline
\end{array}
$$

5.

$$
\begin{array}{rr|l}
 & & x^{12}-15x^{10}+75x^8-125x^6 \;\underline{|\,x^4-5x^2} \\
 & & x^{12} \\ \hline
3(x^4)^2= & 3x^8 & -15x^{10}+75x^8-125x^6 \\
-15x^{10}\div 3x^8=-5x^2 & & \\
3\cdot x^4(-5x^2)= & -15x^6 & \\
(-5x^2)^2= & 25x^4 & \\ \hline
 & 3x^8-15x^6+25x^4 & -15x^{10}+75x^8-125x^6 \\ \hline
\end{array}
$$

6.

$$
\begin{array}{rr|l}
 & & x^6-\frac{3x^4}{2}+\frac{3x^2}{4}-\frac{1}{8} \;\underline{|\,x^2-\frac{1}{2}} \\
 & & x^6 \\ \hline
3(x^2)^2= & 3x^4 & -\frac{3x^4}{2}+\frac{3x^2}{4}-\frac{1}{8} \\
-\frac{3x^4}{2}\div 3x^4=-\frac{1}{2} & & \\
3\cdot x^2\left(-\frac{1}{2}\right)= & -\frac{3x^2}{2} & \\
\left(-\frac{1}{2}\right)^2= & \frac{1}{4} & \\ \hline
 & 3x^4-\frac{3x^2}{2}+\frac{1}{4} & -\frac{3x^4}{2}+\frac{3x^2}{4}-\frac{1}{8} \\ \hline
\end{array}
$$

7.

$$
\begin{array}{rr|l}
 & \dfrac{x^3}{27}-\dfrac{2}{3}+\dfrac{4}{x^3}-\dfrac{8}{x^6} & \dfrac{x}{3}-\dfrac{2}{x^2} \\
 & \dfrac{x^2}{27} & \\
\hline
3\left(\dfrac{x}{3}\right)^2= & \dfrac{x^2}{3} & -\dfrac{2}{3}+\dfrac{4}{x^3}-\dfrac{8}{x^6} \\
-\dfrac{2}{3}\div\dfrac{x^2}{3}=-\dfrac{2}{x^2} & & \\
3\cdot\dfrac{x}{3}\left(-\dfrac{2}{x^2}\right)= & -\dfrac{2}{x} & \\
\left(-\dfrac{2}{x^2}\right)^2= & \dfrac{4}{x^4} & \\
\hline
 & \dfrac{x^2}{3}-\dfrac{2}{x}+\dfrac{4}{x^4} & -\dfrac{2}{3}+\dfrac{4}{x^3}-\dfrac{8}{x^6} \\
\hline
\end{array}
$$

8.

$$
\begin{array}{rr|l}
 & \dfrac{a^3}{c^6}-\dfrac{3}{c^3}+\dfrac{3}{a^3}-\dfrac{c^3}{a^6} & \dfrac{a}{c^2}-\dfrac{c}{a^2} \\
 & \dfrac{a^3}{c^6} & \\
\hline
3\left(\dfrac{a}{c^2}\right)^2= & \dfrac{3a^2}{c^4} & -\dfrac{3}{c^3}+\dfrac{3}{a^3}-\dfrac{c^3}{a^6} \\
-\dfrac{3}{c^3}\div\dfrac{3a^2}{c^4}=-\dfrac{c}{a^2} & & \\
3\cdot\dfrac{a}{c^2}\left(-\dfrac{c}{a^2}\right)= & -\dfrac{3}{ac} & \\
\left(-\dfrac{c}{a^2}\right)^2= & \dfrac{c^2}{a^4} & \\
\hline
 & \dfrac{3a^2}{c^4}-\dfrac{3}{ac}+\dfrac{c^2}{a^4} & -\dfrac{3}{c^3}+\dfrac{3}{a^3}-\dfrac{c^8}{a^6} \\
\hline
\end{array}
$$

9.

$$
\begin{array}{rr|l}
 & x^{\frac{3}{2}}-\dfrac{6x}{a}+\dfrac{12x^{\frac{1}{2}}}{a^2}-\dfrac{8}{a^3} & x^{\frac{1}{2}}-\dfrac{2}{a} \\
 & x^{\frac{1}{2}} & \\
\hline
3(x^{\frac{1}{2}})^2= & 3x & -\dfrac{6x}{a}+\dfrac{12x^{\frac{1}{2}}}{a^2}-\dfrac{8}{a^3} \\
-\dfrac{6x}{a}\div 3x=-\dfrac{2}{a} & & \\
3\cdot x^{\frac{1}{2}}\left(-\dfrac{2}{a}\right)= & -\dfrac{6x^{\frac{1}{2}}}{a} & \\
\left(-\dfrac{2}{a}\right)^2= & \dfrac{4}{a^2} & \\
\hline
 & 3x-\dfrac{6x^{\frac{1}{2}}}{a}+\dfrac{4}{a^2} & -\dfrac{6x}{a}+\dfrac{12x^{\frac{1}{2}}}{a^2}-\dfrac{8}{a^3} \\
\hline
\end{array}
$$

10.

$$
\begin{array}{r|l}
 & a^3+3a^2b+3ab^2+b^3-3a^2-6ab-3b^2+3a+3b-1 \;\underline{\lfloor a+b-1} \\
 & a^3 \\
\hline
3a^2 = 3a^2 & 3a^2b+3ab^2+b^3-3a^2-6ab-3b^2+3a+3b-1 \\
3a^2b \div 3a^2 = b & \\
3\cdot a\cdot b = 3ab & \\
b^2 = b^2 & \\
\hline
3a^2+3ab+b^2 & 3a^2b+3ab^2+b^3 \\
\hline
3(a+b)^2 = 3a^2+6ab+3b^2 & -3a^2-6ab-3b^2+3a+3b-1 \\
-3a^2 \div 3a^2 = -1 & \\
3(a+b)(-1) = \quad -3a-3b & \\
(-1)^2 = \quad 1 & \\
\hline
3a^2+6ab+3b^2-3a-3b+1 & -3a^2-6ab-3b^2+3a+3b-1 \\
\hline
\end{array}
$$

11.

$$
\begin{array}{r|l}
 & \qquad\qquad\qquad\qquad\qquad\qquad \underline{\lfloor x^2-3x+2} \\
 & x^6-9x^5+33x^4-63x^3+66x^2-36x+8 \\
 & x^6 \\
\hline
3(x^2)^2 = \quad 3x^4 & -9x^5+33x^4-63x^3+66x^2-36x+8 \\
-9x^5 \div 3x^4 = -3x & \\
3\cdot x^2(-3x) = \quad -9x^3 & \\
(-3x)^2 = \quad 9x^2 & \\
\hline
3x^4-9x^3+9x^2 & -9x^5+27x^4-27x^3 \\
\hline
3(x^2-3x)^2 = 3x^4-18x^3+27x^2 & 6x^4-36x^3+66x^2-36x+8 \\
6x^4 \div 3x^4 = 2 & \\
3(x^2-3x)\cdot 2 = \quad 6x^2-18x & \\
2^2 = \quad 4 & \\
\hline
3x^4-18x^3+33x^2-18x+4 & 6x^4-36x^3+66x^2-36x+8 \\
\hline
\end{array}
$$

12. To find the sixth root take the square root of the cube root.

$$
\begin{array}{r|l}
 & \qquad\qquad\qquad\qquad\qquad\qquad\qquad \underline{\lfloor x^2-4x+4} \\
 & x^6-12x^5+60x^4-160x^3+240x^2-192x+64 \\
 & x^6 \\
\hline
3(x^2)^2 = \quad 3x^4 & -12x^5+60x^4-160x^3+240x^2-192x+64 \\
-12x^5 \div 3x^4 = -4x & \\
3\cdot x^2(-4x) = \quad -12x^3 & \\
(-4x)^2 = \quad 16x^2 & \\
\hline
3x^4-12x^3+16x^2 & -12x^5+48x^4-64x^3 \\
\hline
3(x^2-4x)^2 = 3x^4-24x^3+48x^2 & 12x^4-96x^3+240x^2-192x+64 \\
12x^4 \div 3x^4 = 4 & \\
3(x^2-4x)\cdot 4 = \quad 12x^2-48x & \\
4^2 = \quad 16 & \\
\hline
3x^4-24x^3+60x^2-48x+16 & 12x^4-96x^3+240x^2-192x+64 \\
\hline
\end{array}
$$

The square roots of $x^2-4x+4 = \pm(x-2)$.

13.

$$
\begin{array}{rr|l}
 & 1+3x & \underline{1+x-x^2} \\
 & \underline{1} & \\
3\cdot 1^2 = & 3 & 3x \\
3x \div 3 = x & & \\
3\cdot 1\cdot x = & 3x & \\
x^2 = & \underline{x^2} & \\
 & 3+3x+x^2 & \underline{3x+3x^2+x^3} \\
3(1+x)^2 = & 3+6x+3x^2 & -3x^2-x^3 \\
-3x^2 \div 3 = -x^2 & & \\
3(1+x)(-x^2) = & -3x^2-3x^3 & \\
(-x^2)^2 = & \underline{x^4} & \\
 & 3+6x-3x^3+x^4 & \underline{-3x^2-18x^3+3x^5-3x^6}
\end{array}
$$

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1.

$$
\begin{array}{rr|l}
 & 15'625 & \underline{20+5=25} \\
(20)^3 = & \underline{8\,000} & \\
3(20)^2 = & 1200 & 7\,625 \\
7625 \div 1200 = 5 & & \\
3\cdot 20\cdot 5 = & 300 & \\
5^2 = & \underline{25} & \\
 & 1525 & \underline{7\,625}
\end{array}
$$

2.

$$
\begin{array}{rr|l}
 & 12'167 & \underline{20+3=23} \\
(20)^3 = & \underline{8\,000} & \\
3(20)^2 = & 1200 & 4\,167 \\
4167 \div 1200 = 3 & & \\
3\cdot 20\cdot 3 = & 180 & \\
3^2 = & \underline{9} & \\
 & 1389 & \underline{4\,167}
\end{array}
$$

3.

$$
\begin{array}{rr|l}
 & 1'404'928 & \underline{100+10+2=112} \\
(100)^3 = & \underline{1\,000\,000} & \\
3(100)^2 = & 30000 & 404\,928 \\
404928 \div 30000 = 10 & & \\
3\cdot 100\cdot 10 = & 3000 & \\
(10)^2 = & \underline{100} & \\
 & 33100 & \underline{331\,000} \\
3(110)^2 = & 36300 & 73\,928 \\
73928 \div 36300 = 2 & & \\
3\cdot 110\cdot 2 = & 660 & \\
2^2 = & \underline{4} & \\
 & 36964 & \underline{73\,928}
\end{array}
$$

4.

		13′481′272	$200 + 30 + 8 = 238$
$(200)^3 =$		8 000 000	
$3\,(200)^2 =$	120000	5 481 272	
$5481272 \div 120000 = 30$			
$3 \cdot 200 \cdot 30 =$	18000		
$(30)^2 =$	900		
	138900	4 167 000	
$3\,(230)^2 =$	158700	1 314 272	
$1314272 \div 158700 = 8$			
$3 \cdot 230 \cdot 8 =$	5520		
$8^2 =$	64		
	164284	1 314 272	

5.

		41′063.625	$30 + 4 + .5 = 34.5$
$(30)^3 =$		27 000.000	
$3\,(30)^2 =$	2700	14 063.625	
$14063 \div 2700 = 4$			
$3 \cdot 30 \cdot 4 =$	360		
$4^2 =$	16		
	3076	12 304.000	
$3\,(34)^2 =$	3468	1 759.625	
$1759.625 \div 3468 = .5$			
$3 \cdot 34 \cdot .5 =$	51		
$.5^2 =$	.25		
	3519.25	1 759.625	

6.

		1.052′8	$1 + .01 + .007 = 1.017 +$
$1^3 =$		1	
$3 \cdot 1^2 =$	3	.052 800	
$.0528 \div 3 = .01$			
$3 \cdot 1 \cdot .01 =$	.03		
$(.01)^2 =$	.0001		
	3.0301	.030 301	
$3\,(1.01)^2 =$	3.0603	.022 499	
$.022499 \div 3.0603 = .007$			
$3 \cdot 1\ .01 \cdot .007 =$	.02121		
$(.007)^2 =$	.000049		
	3.081559	.021 570 913	

7.

$$\begin{array}{rll}
 & & .017'3 \;\underline{|\,.2 + .05 + .008 = .258 +} \\
.2^3 = & & .008 \\
3 \cdot .2^2 = & .12 & .009\ 300 \\
.0093 \div .12 = & .05 & \\
3 \cdot .2 \cdot .05 = & .03 & \\
(.05)^2 = & .0025 & \\
 & .1525 & .007\ 625 \\
3\,(.25)^2 = & .1875 & .001\ 675\ 000 \\
.001675 \div .1875 = & .008 & \\
3 \cdot .25 \cdot .008 = & .006 & \\
(.008)^2 = & .000064 & \\
 & .188164 & .001\ 507\ 312
\end{array}$$

8.

$$\begin{array}{rll}
 & & .004'913 \;\underline{|\,.1 + .07 = .17} \\
.1^3 = & & .001 \\
3 \cdot .1^2 = & .03 & .003\ 913 \\
.003913 \div .03 = & .07 & \\
3 \cdot .1 \cdot .07 = & .021 & \\
(.07)^2 = & .0049 & \\
 & .0559 & .003\ 913
\end{array}$$

9.

$$\begin{array}{rll}
 & & .000'062 \;\underline{|\,.03 + .009 = .039 +} \\
(.03)^3 = & & .000\ 027 \\
3\,(.03)^2 = & .0027 & .000\ 035\ 000 \\
.000035 \div .0027 = & .009 & \\
3 \cdot .03 \cdot .009 = & .00081 & \\
(.009)^2 = & .000081 & \\
 & .003591 & .000\ 032\ 319
\end{array}$$

10.

		35 | 3 + .2 + .07 + .001 = 3.271 +
$3^3 =$		27
$3 \cdot 3^2 =$	27.	8.000
$8 \div 27 = .2$		
$3 \cdot 3 \cdot .2 =$	1.8	
$.2^2 =$	.04	
	28.84	5.768
$3\,(3 \cdot 2)^2 =$	30.72	2.232 000
$2 \cdot 232 \div 30.72 = .07$		
$3 \cdot 3.2 \cdot .07 =$	.672	
$(.07)^2 =$	.0049	
	31.3969	2.197 783
$3\,(3.27)^2 =$	32.0787	.034 217 000
$.034217 \div 32.0787 = .001$		
$3 \cdot 3.27 \cdot .001 =$	.00981	
$(.001)^2 =$	.000001	
	32.088511	.032 088 511

11. $\sqrt[3]{\frac{2}{3}} = \sqrt[3]{.66666+}$

		.666′666′666 | .8 + .07 + .003 = .873+
$.8^3 =$		.512
$3 \cdot .8^2 =$	1.92	.154 666 666
$.1546 \div 1.92 = .07$		
$3 \cdot .8 \cdot .07 =$	.168	
$(.07)^2 =$	.0049	
	2.0929	.146 503
$3\,(.87)^2 =$	2.2707	.008 163 666
$.0081636 \div 2.2707 = .003$		
$3 \cdot .87 \cdot .003 =$	.00783	
$(.003)^2 =$	.000009	
	2.278539	.006 835 587

12. The edge is $\sqrt[3]{5832}$ inches long.

		5′832 | 10 + 8 = 18
$(10)^3 =$		1 000
$3\,(10)^2 =$	300	4 832
$4832 \div 300 = 8$		
$3 \cdot 10 \cdot 8 =$	240	
$8^2 =$	64	
	604	4 832

13. The diagonal is the hypotenuse of a right triangle whose one leg is the edge of the cube and whose other leg is the diagonal of a face of the cube.

$$\begin{array}{rr|l}
 & & 46'656 \;\; \underline{|\,30+6=36} \\
(30)^3 & & 27\,000 \\
\hline
3(30)^2 = & 2700 & 19\,656 \\
19656 \div 2700 = 6 & & \\
3\cdot 30\cdot 6 = & 540 & \\
6^2 = & \underline{36} & \\
 & 3276 & 19\,656
\end{array}$$

The diagonal $= \sqrt{\left(\sqrt{(36)^2+(36)^2}\right)^2+(36)^2} = \sqrt{2\,(36)^2+(36)^2} = 36\sqrt{3}.$

$$\begin{array}{r|l}
 & 3.00'00'00'00'00 \;\; \underline{|\,1.7320+} \\
 & 1 \\
\hline
2.7 & 2.00 \\
 & 1.89 \\
\hline
343 & .1100 \\
 & .1029 \\
\hline
3.462 & .007100 \\
 & .006924 \\
\hline
3.4640 & .0001760000
\end{array}$$

The diagonal equals $36 \cdot 1.7320 + = 62.35 +$ meters.

14. The sixth root equals the square root of the cube root.

$$\begin{array}{rr|l}
 & & 46'656'000 \;\; \underline{|\,300+60=360} \\
(300)^3 = & & 27\,000\,000 \\
\hline
3(300)^2 = & 270000 & 19\,656\,000 \\
19656000 \div 270000 = 60 & & \\
3\cdot 300\cdot 60 = & 54000 & \\
(60)^2 = & \underline{3600} & \\
 & 327600 & 19\,456\,000
\end{array}$$

$$\begin{array}{r|l}
 & 3'60.000000 \;\; \underline{|\,+18.973+} \\
 & 1 \\
\hline
28 & 260 \\
 & 224 \\
\hline
36.9 & 36.00 \\
 & 33.21 \\
\hline
37.87 & 2.7900 \\
 & 2.6509 \\
\hline
37.943 & .139100 \\
 & .113829 \\
\hline
 & .026271
\end{array}$$

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1. $$x^2 + 4x - 7 = 0. \quad (1)$$

Let the roots of (1) be r_1 and r_2.

$$(r_1 + r_2)^2 = r_1^2 + r_2^2 + 2r_1r_2. \quad (2)$$

But $r_1 + r_2 = -4$, and $r_1r_2 = -7$.

Substituting in (2), $(-4)^2 = r_1^2 + r_2^2 + 2(-7)$.

Hence $r_1^2 + r_2^2 = 30$.

2. $$x^3 - 3x - 2 = 0. \quad (1)$$

Let the roots of (1) be r_1 and r_2.

$$(r_1 + r_2)^3 = r_1^3 + r_2^3 + 3r_1r_2(r_1 + r_2). \quad (2)$$

But $r_1 + r_2 = 3$, and $r_1r_2 = -2$.

Substituting in (2), $3^3 = r_1^3 + r_2^3 + 3(-2)3$.

Whence $r_1^3 + r_2^3 = 45$.

3. $$x^2 + 7x + 1 = 0. \quad (1)$$

Let the roots of (1) be r_1 and r_2.

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{r_1 + r_2}{r_1r_2}. \quad (2)$$

But $r_1 + r_2 = -7$, and $r_1r_2 = 1$.

Substituting in (2), $\frac{1}{r_1} + \frac{1}{r_2} = -7$.

4. $$x^2 - x - 5 = 0. \quad (1)$$

Let the roots of (1) be r_1 and r_2.

$$(r_1 + r_2)^2 = r_1^2 + r_2^2 + 2r_1r_2. \quad (2)$$

But $r_1 + r_2 = 1$, and $r_1r_2 = -5$.

Substituting in (2), $1^2 = r_1^2 + r_2^2 + 2(-5)$. (3)

Whence $r_1^2 + r_2^2 = 11$. (4)

Let the required equation be

$$x^2 + bx + c = 0. \quad (5)$$

Then $b = -(r_1^2 + r_2^2) = -11$,

and $c = (r_1r_2)^2 = 25$.

Hence the required equation is

$$x^2 - 11x + 25 = 0.$$

5. Let r_1 and r_2 be the roots of the equation

$$x^2 + x - 3 = 0.$$

Then $r_1 + r_2 = -1$, and $r_1r_2 = -3$.

Let the required equation be

$$x^2 + bx + c.$$

Then $$b = -\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = -\frac{r_1 + r_2}{r_1 r_2} = \frac{1}{3},$$

and $$c = \frac{1}{r_1 r_2} = -\frac{1}{3}.$$

Hence the required equation is

$$x^2 + \tfrac{1}{3}x - \tfrac{1}{3} = 0.$$

6. If r_1 and r_2 are the roots of the equation

$$x^2 - 3x + 5 = 0,$$

$$r_1 + r_2 = 3, \text{ and } r_1 r_2 = 5.$$

$$r_1^2 r_2 + r_1 r_2^2 = r_1 r_2 (r_1 + r_2). \qquad (1)$$

Substituting in (1), $$r_1^2 r_2 + r_1 r_2^2 = 5 \cdot 3 = 15.$$

ANNOUNCEMENTS